TEST-2
(Ch.2, 9 (Th#3, 4), Def. of Ch-2, 8 & 9)

MATHEMATICS (SCIENCE)
Paper: (Objective Type)
Time Allowed: 20 Minutes -019 (10^{th} Class) **Marks: 15**

Note: For possible answer A, B,C and D to each question are given. The choice which you think is correct, fill that circle in front of that question with marker or Pen ink cutting of filing two or more circle will Result in zero mark in that questions.

1. 1. $b^2 - 4ac$ is called:
(a) Solution set (b) Discriminant (c) Value of x (d) None of these
2. The length of the diameter of a circle is how many times the radius of circle.
(a) 1 (b) 2 (c) 3 (d) 4
3. Cube roots of -1 are:
(a) $-1, -\omega, -\omega^2$ (b) $-1, \omega, -\omega^2$ (c) $-1, -\omega, \omega^2$ (d) $1, -\omega, \omega^2$
4. Complete circle is divided into.
(a) 360° (b) 270° (c) 180° (d) 90°
5. $\omega^{-13} + \omega^{-17} =$ __________.
(a) ω (b) ω^2 (c) -1 (d) 1
6. The chord passing through the centre of circle is.
(a) Radius (b) Diameter (c) Secant (d) Circumference
7. If α, β are the roots of an equation then the standard form of quadratic equation is:
(a) $x^2 - (\alpha\beta)x + \alpha\beta = 0$ (b) $x^2 - (\alpha + \beta)x + (\alpha + \beta) = 0$
(c) $x^2 - \alpha\beta(x) + \alpha\beta = 0$ (d) $x^2 - (\alpha + \beta)x + \alpha\beta = 0$
8. The roots of the quadratic equation $ax^2 + bx + c = 0$ are:
(a) $\dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ (b) $\dfrac{-b \pm \sqrt{b^2 + 4ac}}{2a}$
(c) $\dfrac{b \pm \sqrt{b^2 - 4ac}}{2a}$ (d) $\dfrac{-b \pm \sqrt{b^3 - 4ac}}{2a}$
9. Two square roots of unity are.
(a) 1,-1 (b) $1,\omega$ (c) $1,-\omega$ (d) ω, ω^2
10. $b^2 - 4ac < 0$ then the roots of $ax^2 + bx + c = 0$ are:
(a) Imaginary (b) Irrational and unequal (c) Real (d) Rational and equal
11. $1 + \omega =$ __________. ok
(a) ω (b) ω^2 (c) $-\omega^2$ (d) 1
12. Product of cube roots of unity is.
(a) 0 (b) 1 (c) -1 (d) 3
13. $b^2 - 4ac > 0$ and is a perfect square then the roots of $ax^2 + bx + c = 0$ are: ok
(a) Rational (real) and unequal (b) Irrational (real) and unequal
(c) Imaginary (d) Irrational and equal
14. If $\alpha = \dfrac{-b + \sqrt{b^2 - 4ac}}{2a}$ and $\beta = \dfrac{-b - \sqrt{b^2 - 4ac}}{2a}$. The sum of the roots of $ax^2 + bx + c = 0$ is:
(a) $\dfrac{b}{a}$ (b) $-\dfrac{b}{a}$ (c) $\dfrac{c}{a}$ (d) $\dfrac{a}{c}$
15. Through how many no-collinear points a circle can pass.
(a) One (b) Two (c) Three (d) Four

TEST NO.2
(Ch.2, 9 (Th#3, 4), Def. of Ch-2, 8 & 9)

MATHEMATICS (SCIENCE)
Paper : (Essay Type) **-019 (10^{TH} CLASS)** **Time Allowed: 2:10 hours**
Maximum Marks: 60

(PART – 1)

2. Write short answer to any SIX (6) questions: **12**

(i) Find the discriminant $2x^2 + 3x - 1 = 0$
(ii) Define a circle.
(iii) Solve $(1 - \omega - \omega^2)^7$
(iv) Define Circumference of a circle.
(v) Define discriminant of equation.
(vi) Define Chord.
(vii) Find the cube roots of -27
(viii) Define segment of a circle.
(ix) Evaluate $(9 + 4\omega + 4\omega^2)^3$

3. Write short answer to any SIX (6) questions: **12**

(i) Differentiate between a chord and diameter of a circle.
(ii) Write the quadratic equation with roots 2 and -6
(iii) Evaluate $(1 - 3\omega - 3\omega^2)^5$.
(iv) Differentiate between interior and exterior of a circle and illustrate by diagram.
(v) If α, β are the roots of the equation $x^2 + px + q = 0$ *the evaluate* $\alpha^2 + \beta^2$
(vi) If $\alpha\beta$ are root of the equation $lx^2 + mx + n = 0$ Find value of $\alpha^3\beta^2 + \alpha^2\beta^3$
(vii) Differentiate between sector of circle and its segment.
(viii) Find 'K' if sum of squares of the roots of the equation $4kx^2 + 3kx - 8 = 0$ is 2'.
(ix) Define Radial segment

4. Write short answer to any SIX (6) questions: **12**

(i) Evaluate $\dfrac{\left(-1+\sqrt{-3}\right)^9}{2} + \dfrac{\left(-1-\sqrt{-3}\right)^9}{2}$.
(ii) Write area of the circle.
(iii) Using discriminant, find the nature of the root and verify the results by solving the equation $x^2 - 5x + 5 = 0$.
(iv) Define projection.
(v) Find sum and product of roots of equation. $x^2 - 5x + 3 = 0$
(vi) What is radius of a circle?
(vii) Differentiate between a circle and circumference.
(viii) Define Synthetic division.
(ix) Define quadratic equation.

(PART – 2)

Note: **Attempt THREE questions in all.**

5. **(a)** Find 'P' if the sum of squares of roots of the equation $4x^2 + 3px + p^2 = 0$ is unity. 4
(b) Prove that $(1+\omega)(1+\omega^2)(1+\omega^4)(1+\omega^8)$ . . . 2n factors = 1. 4

6. **(a)** Find 'h', if sum of the squares of the roots of the equation $x^2 - hx + 10 = 0$ is 4

differ by 3.

(b) If α *and* β are the roots of the equation $lx^2 + mx + n = 0, (l \neq 0)$ find the value of $\frac{1}{\alpha^2} + \frac{1}{\beta^2}$ 4

7. **(a)** Use synthetic division to find the values of l and m, if . l *and* m, *if* $(x+3)$ *and* $(x2)$ *are* the factors of the polynomial $x^3 + 4x^2 + 2\,lx + m$ 4
(b) Find the nature of the root of equation $x^2 - 23x + 120 = 0$ and verify the result by solving the equations. 4

8. **(a)** Find the dimension of a rectangle , whose perimeter is 80 cm and its area is 375 cm2. 4
(b) Find m, if the roots of the equation 3x2 – 2 x + 7m + 2 = 0 satisfy the relation $7\alpha - 3\beta = 18$

4

9. Prove that perpendicular from the center of a circle on a chord bisects it. 8

OR

Prove that If two chords of circle are congruent then they will be equidistant from the center.

Test : 2

MATHEMATICS (SCIENCE)

(PART - 1)

2. Write short answer to any Six (6) question:

(i) Find the discriminant

$$2x^2+3x-1=0$$

Solution: $2x^2+3x-1=0$

Here, $a=2, b=3, c=-1$

Discriminant $= b^2-4ac$

$= (3)^2-4(2)(-1)$

$= 9+8 = 17$ Ans

(ii) **Define a circle?**

A "circle" is locus of a moving point in a plane which is equidistant from a fixed point. The fixed point is called "centre" of the circle.

(iii) Solve:

$$(1-\omega-\omega^2)^7$$

Solution: $(1-\omega-\omega^2)^7$

$= [1-(\omega+\omega^2)]^7$

$= [(1-(-1)]^7$ $\left[\text{using } 1+\omega+\omega^2=0 \quad \omega+\omega^2=-1\right]$

$= (1+1)^7$

$= (2)^7$

$= 128$ Ans.

(iv) Define Circumference of a circle?

The perimeter or length of the boundary of the circle is called the circumference

(v) Define discriminant of equation?

The expression "b^2-4ac" of the quadratic expression ax^2+bx+c is called discriminant.

(vi) Define chord?

The join of any two points on the circumference of the circle is called it's chord.

(vii) Find the cube roots of -27.

Let $x^3 = -27$

$x^3 + 27 = 0$

$x^3 + 3^3 = 0$

$(x+3)(x^2 - 3x + 9) = 0$ which gives

$x + 3 = 0$ i.e, $x = -3$

and $x^2 - 3x + 9 = 0$

Here, $a = 1, b = -3, c = 9$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1\times 9)}}{2(1)}$$

$$x = \frac{3 \pm \sqrt{9 - 36}}{2}$$

$$x = \frac{3 \pm \sqrt{-27}}{2}$$

$$x = \frac{3 \pm 3\sqrt{-3}}{2}$$

$$x = \frac{3(1 \pm i\sqrt{3})}{2}$$

$x = -3\omega, -3\omega^2$

Roots are: $-3, -3\omega, -3\omega^2$

(viii) Define Segment of a circle?

A Segment is the portion of a circle bounded by an arc and a corresponding chord

(ix) Evaluate:

$$(9+4\omega+4\omega^2)^3$$

Solution: $(9+4\omega+4\omega^2)^3$

$= [9+4(\omega+\omega^2)]^3$

$= [9+4(-1)]^3$ [as $\omega+\omega^2=-1$]

$= (9-4)^3$

$= 5^3 = 125$ Ans

3- Write short answer to any six (6) questions:

(i) Differentiate between a chord and diameter of a circle?

Chord	diameter
The join of any two points on the circumference of the circle is called it's chord.	A chord which passes through the centre of the circle is called diameter of the Circle

(ii) Write the quadratic equation with roots 2 and -6.

Solution:

$S =$ Sum of roots $= 2 + (-6) = 2 - 6$

$= -4$

$P =$ Product of roots $= (2)(-6) = -12$

Equation is: $x^2 - Sx + P = 0$

Putting values of S and P

$x^2 - (-4)x + (-12) = 0$

$x^2 + 4x - 12 = 0$

(iii) Evaluate:

$$(1 - 3\omega - 3\omega^2)^5$$

Solution: $(1 - 3\omega - 3\omega^2)^5$

$= [1 - 3(\omega + \omega^2)]^5$

$= [1 - 3(-1)]^5 \quad [\omega + \omega^2 = -1]$

$= (1 + 3)^5$

$= (4)^5$

$= 1024$ Ans

$$\text{Prod} = \alpha\beta = \frac{c}{a} = \frac{-8}{4k} = \frac{-2}{k}$$

Now $\alpha^2+\beta^2 = 2$

$$(\alpha+\beta)^2 - 2\alpha\beta = 2$$

Putting value of $\alpha+\beta, \alpha\beta$

$$\left[-\frac{3}{4}\right]^2 - 2\left[\frac{-2}{k}\right] = 2$$

$$\frac{9}{16} + \frac{4}{k} = 2$$

$$\frac{4}{k} = 2 - \frac{9}{16}$$

$$\frac{4}{k} = \frac{32-9}{16}$$

$$\frac{4}{k} = \frac{23}{16}$$

$$k = \frac{4\times 16}{23}$$

$$k = \frac{64}{23}$$

(ix) Define Radical segment:

Radical segment of a circle is a line segment detemind by the centre and a point on the circle.

4- Write short answer to any Six (6) questions.

(i) Evaluate $\left[\frac{-1+\sqrt{-3}}{2}\right]^9 + \left[\frac{-1-\sqrt{-3}}{2}\right]^9$

Solution: $\left[\frac{-1+\sqrt{-3}}{2}\right]^9 + \left[\frac{-1-\sqrt{-3}}{2}\right]^9$

$$= \omega^9 + (\omega^2)^9$$
$$= \omega^9 + \omega^{18}$$
$$= (\omega^3)^3 + (\omega^3)^6$$
$$= 1^3 + 1^6$$
$$= 1 + 1$$
$$= 2 \text{ Ans.}$$

(ii) Write the area of the circle?

Area of a circle $= \pi r^2$

(iii) Using discriminant find the nature of the root and verify the result by solving the equation $x^2 - 5x + 5 = 0$

Solution:

$$x^2 - 5x + 5 = 0$$

Here $a = 1, b = -5, c = 5$

$$\text{Disc} = b^2 - 4ac$$
$$= (-5)^2 - 4(1)(5)$$
$$= 25 - 20$$
$$= 5$$

Verification:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{+5 \pm \sqrt{(-5)^2 - 4(1)(5)}}{2(1)}$$

$$x = \frac{5 \pm \sqrt{25 - 20}}{2}$$

$$x = \frac{5 \pm \sqrt{5}}{2}$$

(iv) **Define projection:**

The projection of a given point on a line is the foot of $\perp$ drawn from the point on that line. However the projection of given point P on a line AB is the point P itself.

(v) **find sum and product of roots of equation:**

$$x^2 - 5x + 3 = 0$$

Here $a = 1$, $b = -5$, $c = 3$

Let $\alpha\beta$ be it's roots then

$$\text{Sum} = \alpha + \beta = \frac{-b}{a} = \frac{-(-5)}{1} = 5$$

$$\text{prod} = \alpha\beta = \frac{c}{a} = \frac{3}{1} = 3.$$

(vi) **Define radius of a circle:**

The distance from the centre of the circle to ~~the~~ the any point on the circle is called radius of the circle

(vii) Differentiate between a circle and circumference?

Circle

A circle is locus of a moving point in a plane which is equidistant from a fixed point

Circumferrence.

The perimeter or length of the boundary of the circle is called circumferrence.

(viii) Define synthetic division?

Synthetic division is the process to finding the quotient and remainder when a pholynomial is divided by a linear polynomial

(ix) Define Quadratic Equation.

The equation whose degree is 2 is called quadratic Equation.

$$ax^2 + bx + c = 0$$

(PART - 2)

Attempt any three question. Each question carries (8) but question number is compulsory.

Q NO 5 (a)

Find p, if the roots of the equation $x^2 - x + p^2 = 0$ differ by unity:

Solution: Let α, and $\alpha - 1$ be the roots of $x^2 - x + p^2 = 0$

Then $\alpha + \alpha - 1 = \frac{-b}{a} = \frac{-(-1)}{1} = 1$

$2\alpha - 1 = 1 \Rightarrow 2\alpha = 1 + 1 \Rightarrow 2\alpha = 2 \Rightarrow \alpha = 1 \quad (i)$

and $\alpha(\alpha - 1) = \frac{c}{a} = \frac{p^2}{1} = p^2$

or $\alpha(\alpha - 1) = p^2 \quad (ii)$

Putting value of α from equation (i) in equation (ii) we get

$1(1 - 1) = p^2$

$1(0) = p^2$

$p^2 = 0$

$p = 0$

Q No 5 (b)

Prove that

$(1+\omega)(1+\omega^2)(1+\omega^4)(1+\omega^8)$ ..... $2n$ factors $=1$

Solution: $\{(1+\omega)(1+\omega^2)\}\{(1+\omega\cdot\omega^3)(1+\omega^6\cdot\omega^2)\}$ ..... $2n$

$=\{(1+\omega)(1+\omega^2)\}\{(1+\omega)(1+\omega^2)\}$ ..... n factors

(as $\omega^3=\omega^6=1$)

$=\{(1+\omega)(1+\omega^2)\}^n$

$=\{1+\omega^2+\omega+\omega^3\}^n$

$=\{(1+\omega+\omega^2)+\omega^3\}^n$

$=\{0+1\}^n$

$=(1)^n$

$=1$ proved.

Q No 6 (a)

Find h, if the roots of the equation $x^2-hx+10=0$ differ by 3.

Solution:

Let a and $a-3$ be the roots of

$x^2-hx+10=0$

Then $a+a-3=-\frac{b}{a}=-\left[\frac{-h}{1}\right]=h$

$2a-3=h \Rightarrow 2a=h+3 \Rightarrow a=\frac{h+3}{2}$

and $\alpha(\alpha-3) = \frac{c}{a} = \frac{10}{1} = 10$

or $\alpha(\alpha-3) = 10$

Putting value of α from equation (i) in equation (ii) we get

$\left[\frac{h+3}{2}\right]\left[\frac{h+3}{2}-3\right] = 10 \Rightarrow \left[\frac{h+3}{2}\right]\left[\frac{h+3-6}{2}\right] = 10$

$\left[\frac{h+3}{2}\right]\left[\frac{h-3}{2}\right] = 10 \Rightarrow h^2-9 = 40$ that is

$h^2 = 49 \Rightarrow h = \pm 7$

Q No 6 (b)

find m if, the roots of the equation $3x^2-2x+7m+2=0$ satisfy the relation $7\alpha-3\beta = 18$

$3x^2-2x+7m+2=0$

Let α, β be it's roots then

$S = \alpha+\beta = \frac{b}{a} = -\frac{(-2)}{3} = \frac{2}{3}$

$P = \alpha\beta = \frac{c}{a} = \frac{7m+2}{3}$ (ii)

$7\alpha-3\beta = 18$ (give relation) (iii)

$\beta = \frac{2}{3} - \alpha$ from (i)

putting $\beta = \frac{2}{3} - \alpha$ from (ii)

$7\alpha - 3\left(\frac{2}{3} - \alpha\right) = 18$

$7\alpha - 2 + 3\alpha = 18$

$10\alpha = 18 + 2 = 20$

$\alpha = 2$

putting $\alpha = 2$ in (i)

$2 + \beta = \frac{2}{3}$

$\beta = \frac{2}{3} - 2 = \frac{2 - 6}{3} = \frac{-4}{3}$

$\alpha\beta = \frac{7m + 2}{3}$

Putting value of α, β in it

$2\left[\frac{-4}{3}\right] = \frac{7m + 2}{3}$

$\frac{-8}{3} = \frac{7m + 2}{3}$

$7m + 2 = -8$

$7m = -8 - 2$

$7m = -10 \Rightarrow m = \frac{-10}{7}$

Q No 7 (a)

$(x-1)$ and $(x+1)$ are the factors of the polynomial $x^3 - 3l x^2 + 2mx + 6$

Solution

$$P(x) = x^3 - 3l x^2 + 2mx + 6$$

$$x - a = x - 1$$

$$\therefore a = 1$$

Using synthetic division

$$\begin{array}{r|cccc} & 1 & -3l & 2m & 6 \\ 1 & & 1 & -3l+1 & 2m-3l+1 \\ \hline & 1 & -3l+1 & 2m-3l+1 & \big| \; 2m-3l+7 \end{array}$$

$$R = 2m - 3l + 7 = 0$$

$$P(x) = x^3 - 3l x^2 + 2mx + 6$$

$$x = a = x + 1$$

$$\therefore a = -1$$

Using synthetic division

$$\begin{array}{r|cccc} & 1 & -3l & 2m & 6 \\ & & -1 & 3l+2 & -2m-3l-1 \\ \hline & 1 & -3l-1 & 2m+3l+1 & \big| \; -2m-3l+5 \end{array}$$

$$R = -2m - 3l + 5 = 0$$

$$4m + 2 = 0$$

$$4m = -2$$

$$m = -\frac{2}{4} \Rightarrow m = -\frac{1}{2}$$

Putting $m = -\frac{1}{2}$ in (i)

$$2\left[-\frac{1}{2}\right] - 3l + 7 = 0$$

$$-1 - 3l + 7 = 0$$

$$-3l = -7 + 1$$

$$-3l = -6$$

$$l = \frac{-6}{-3} = 2$$

$$l = 2, \quad m = -\frac{1}{2}$$

Q No 7 (b)

find the condition that the roots of the equation $(mx + c)^2 - 4ax = 0$ are equal

$$x^2 + (mx + c)^2 = a^2$$

$$x^2 + m^2x^2 + 2mcx + c^2 = a^2$$

$$x^2 + m^2x^2 + 2mcx + c^2 - a^2 = 0$$

$$(1 + m^2)x^2 + 2mcx + (c^2 - a^2) = 0$$

Disc = must be zero for equal roots.

$a' = 1 + m^2$, $b' = 2mc$, $c' = c^2 - a^2$

$$\text{Disc} = (b')^2 - 4a'c' = 0$$

Putting value of a', b', c'

$$(2mc)^2 - 4(1+m^2)(c^2 - a^2) = 0$$

$$4m^2c^2 - 4(c^2 - a^2 + m^2c^2 - m^2a^2) = 0$$

$$4m^2c^2 - 4c^2 + 4a^2 - 4m^2c^2 + 4m^2a^2 = 0$$

$$-4c^2 = -4m^2a^2 - 4a^2$$

$$c^2 = m^2a^2 + a^2$$

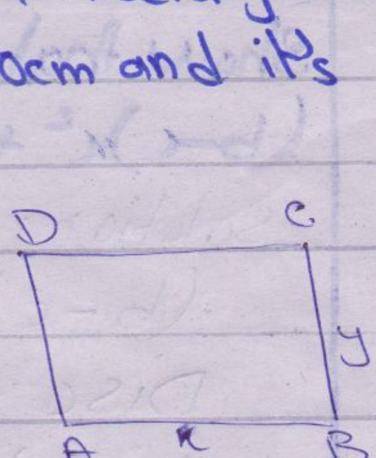
$$c^2 = a^2(m^2 + 1)$$

$$c^2 = a^2(1 + m)$$ Hence the result.

QNo 8 (a)

Find the dimensions of a rectangle whose perimeter is 80cm and it's area is 375cm^2

Let x and y be the length and width respectively of the rectangle.



According to the given condition.

$$2x + 2y = 80$$

$$x + y = 40 \quad \text{.......(A)}$$

$$xy = 375 \quad \text{.......(B)}$$

$$x = 40 - y \quad \text{.......(from A)}$$

$x = 40 - y$ in (B)

$$(40 - y)y = 375$$

$$40y - y^2 = 375$$

$$y^2 - 40y + 375 = 0$$
$$y^2 - 25y - 15y + 375 = 0$$
$$y(y-25) - 15(y-25) = 0$$
$$y - 15 = 0 \text{ gives } y = 15$$
Putting $y = 15$ in (A)
$$x + 15 = 40$$
$$x = 40 - 15 = 25$$
Length $= 25$ cm
Breadth $= 15$ cm.

Q no 8 (b)

Show that the roots of the equation $(b-c)x^2 + (c-a)x + (a-b) = 0$

Solution:
$$(b-c)x^2 + (c-a)x + (a-b) = 0$$
Disc: $(b')^2 - 4a'c'$
$$= (c-a)^2 - 4(b-c)(a-b)$$
$$= c^2 + a^2 - 2ca - 4(ab - b^2 - ca + bc)$$
$$= c^2 + a^2 - 2ca - 4ab + 4b^2 + 4ca - 4bc$$
$$= c^2 + a^2 + 4b^2 - 2ca - 4bc - 4ab$$
$$= [(c)^2 + (a)^2 + (2b)^2 - 2(c)(a) - 2(a)(2b) - 2(c)(2b)]$$
$$= [c + a - 2b]^2$$
which is always positive
Hence the roots are real.