CHAPTER

17

PRACTICAL GEOMETRY –TRIANGLES

Animation 17.1: Practical Geometry – Triangles
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Students Learning Outcomes

After studying this unit, the students will be able to:

- Construct a triangle having given: two sides and the included angle, one side and two of the angles, two of its sides and the angle opposite to one of them and two of them angles, two of its sides and the angle opposite to one of them (with all the three possibilities).
- Draw: angle bisectors, altitudes, perpendicular bisectors, medians, of a given triangle and verify their concurrency.
- Construct a triangle equal in area to a given quadrilateral. Construct a rectangle equal in area to a given triangle. Construct a square equal in area to a given rectangle. Construct a triangle of equivalent area on a base of given length.

Introduction

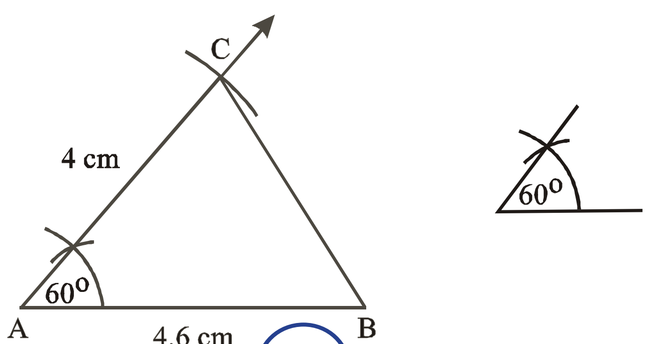
In this unit we shall learn to construct different triangles, rectangles, squares etc. The knowledge of these basic constructions is very useful in every day life, especially in the occupations of wood-working, graphic art and metal trade etc. Intermixing of geometrical figures is used to create artistic look. The geometrical constructions are usually made with the help of a pair of compasses, set squares, dividers and a straight edge.

Observe that

If the given line segments are too big or too small , a suitable scale may be taken for constructing the figure.

17.1 Construction of Triangles

(a) To construct a triangle, having given two sides and the included angle.



Given

Two sides, say

$m\overline{AB}$ = 4.6cm and $m\overline{AC}$ = 4cm and the included angle, $m\angle A = 60^\circ$.

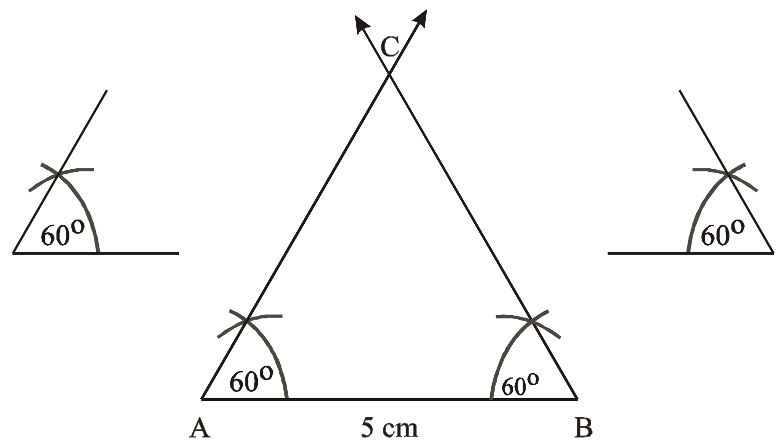
Required

To construct the $\triangle ABC$ using given information of sides and the included angle = $\angle 60^0$

Construction:

(i) Draw a line segment $m\overline{AB}$ = 4.6cm
(ii) At the end A of $\overline{AB}$ make $m\angle BAC = \angle 60^0$
(iii) Cut off $m\overline{AC}$ = 4cm from the terminal side of $\angle 60^0$.
(iv) Join BC
(v) Then ABC is the required $\triangle$.

(b) To construct a triangle, having given one side and two of the angles.



Given

The side $m\overline{AB}$ = 5cm, say and two of the angles, say

$m\angle A = 60^\circ$ and $m\angle B = 60^\circ$.

Required

To construct the $\triangle ABC$ using given data.

Construction:

(i) Draw a line segment $m\overline{AB} = 5cm$

(ii) At the end A of $\overline{AB}$ make $m\angle BAC = \angle 60^0$

(iii) At the end point B of $\overline{BA}$ make $m\angle ABC = \angle 60^0$

(iv) The terminal sides of these two angles meet at C.

(v) Then ABC is the required $\triangle$.

Observe that

When two angles of a triangle are given, the third angle can be found from the fact that the sum of three angles of triangle is 180^0. Thus two angles being known, all the three are known, and we can take any two of these three angles as the base angles with given side as base.

(c) Ambiguous Case

To construct a triangle having given two of its sides and the angle opposite to one of them.

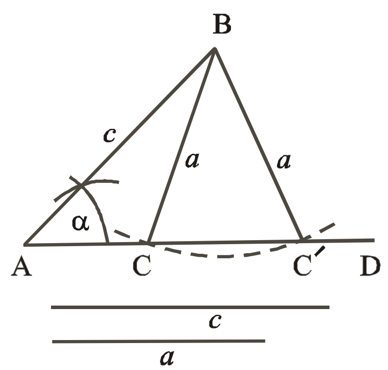


Figure (a)

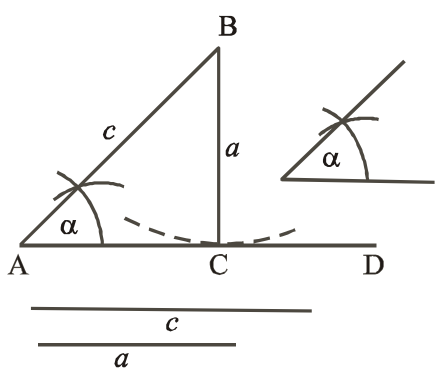


Figure (b)

Given

Two sides a, c and $m\angle A = \alpha$ opposite to one of them, say a.

Required

To construct a triangle having the given parts.

Construction:

(i) Draw a line segment AD of any length.

(ii) At A make $m\angle DAB = m\angle A = \alpha$

(iii) Cut off $\overline{AB} = c$.

(iv) With centre B and radius equal to a, draw an arc.

Three cases arise.

Case I

When the arc with radius a cuts $\overline{AD}$ in two distinct points C and C′ as in Figure (a). Joint $\overline{BC}$ and $\overline{BC}'$.

Then both the triangles ABC and ABC′ have the given parts and are the required triangles.

Case II

When the arc with radius a only touches $\overline{AD}$ at C, as in Figure (b). Join $\overline{BC}$.

Then $\triangle ABC$ is the required triangle angled at C.

Case III

When the arc with radius a neither cuts nor touches $\overline{AD}$ as in Figure (c).

There will be no triangle in this case.

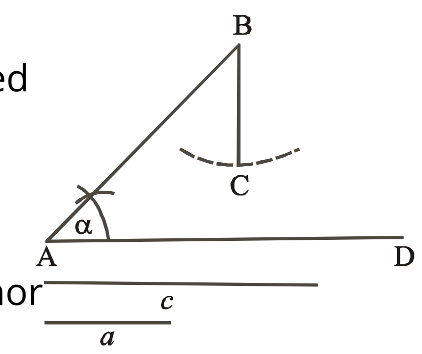


Figure (c)

Note: Recall that in a $\triangle ABC$ the length of the side opposite to $\angle A$ is denoted by a, opposite to $\angle B$ is denoted by b and opposite to $\angle C$ is denoted by c.

EXERCISE 17.1

1. Construct $\triangle ABC$ in which

(i)	$m\overline{AB} = 3.2cm$,	$m\overline{BC} = 4.2cm$,	$m\overline{CA} = 5.2cm$
(ii)	$m\overline{AB} = 4.2cm$,	$m\overline{BC} = 3.9cm$,	$m\overline{CA} = 3.6cm$
(iii)	$m\overline{AB} = 4.8cm$,	$m\overline{BC} = 3.7cm$,	$m\angle B = 60°$
(iv)	$m\overline{AB} = 3cm$,	$m\overline{AC} = 3.2cm$,	$m\angle A = 45°$

(v) $m\overline{AB} = 4.2$cm, $m\overline{CA} = 3.5$cm, $m\angle C = 75^\circ$
(vi) $m\overline{AB} = 2.5$cm, $m\angle A = 30^\circ$, $m\angle B = 105^\circ$
(vii) $m\overline{AB} = 3.6$cm, $m\angle A = 75^\circ$, $m\angle B = 45^\circ$

2. Construct $\triangle XYZ$ in which
   (i) $m\overline{YZ} = 7.6$cm, $m\overline{XY} = 6.1$cm and $m\angle X = 90^\circ$
   (ii) $m\overline{ZX} = 6.4$cm, $m\overline{YZ} = 2.4$cm and $m\angle X = 90^\circ$
   (iii) $m\overline{XY} = 5.5$cm, $m\overline{ZX} = 4.5$cm and $m\angle Z = 90^\circ$.
3. Construct a right-angled $\triangle$ measure of whose hypotenuse is 5 cm and one side is 3.2 cm. (Hint: Angle in a semi-circle is a right angle).
4. Construct a right-angled isosceles triangle whose hypotenuse is
   (i) 5.2 cm long
   [Hint: A point on the right bisector of a line segment is equidistant form its end points.]
   (ii) 4.8 cm (iii) 6.2 cm (iv) 5.4 cm
5. (Ambiguous Case) Construct a $\triangle ABC$ in which
   (i) $m\overline{AC} = 4.2$cm, $m\overline{AB} = 5.2$cm, $m\angle B = 45^\circ$ (two Δs)
   (ii) $m\overline{AC} = 2.5$cm, $m\overline{AB} = 5.0$cm, $m\angle A = 30^\circ$ (one Δs)
   (iii) $m\overline{BC} = 5$cm, $m\overline{AB} = 3.5$cm, $m\angle B = 60^\circ$

Definitions

Three or more than three lines are said to be concurrent, if they all pass through the same point. The common point is called the point of concurrency of the lines. The point of concurrency has its own importance in geometry. They are given special names.

(i) The internal bisectors of the angles of a triangle meet at a point called the incentre of the triangle.
(ii) The point of concurrency of the three perpendicular bisectors of the sides of a $\triangle$ is called the circumcentre of the $\triangle$.
(iii) The point of concurrency of the three altitudes of a $\triangle$ is called its orthocentre.
(iv) The point where the three medians of a $\triangle$ meet is called the centroid of the triangle.

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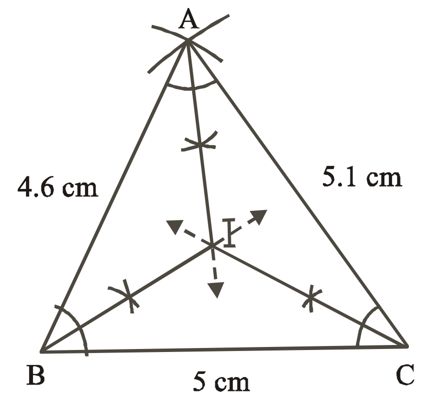
17.1.1 Drawing angle bisectors, altitudes etc.

(a) Draw angle bisectors of a given triangle and verify their concurrency.

Example

(i) Construct $\triangle ABC$ having given $m\overline{AB} = 4.6$cm, $m\overline{BC} = 5$cm and $m\overline{CA} = 5.1$cm.
(ii) Draw its angle bisectors and verify that they are concurrent.



Given

The side $m\overline{AB} = 4.6$cm, $m\overline{BC} = 5$cm and $m\overline{CA} = 5.1$cm of a ΔABC.

Required

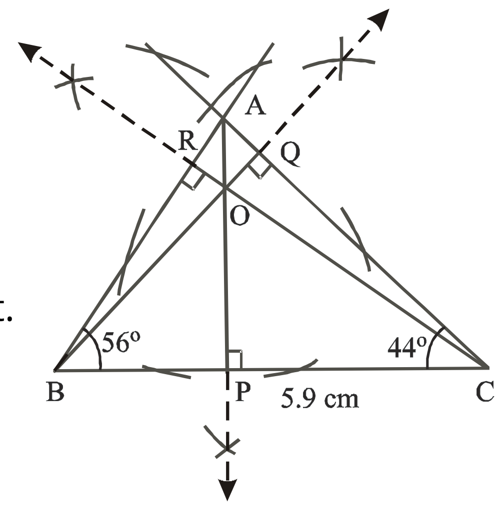
(i) To construct $\triangle ABC$.
(ii) To draw its angle bisectors and verify their concurrency.

Construction

(i) Take $m\overline{BC} = 5$cm.
(ii) With B as centre and radius $m\overline{BA} = 4.6$cm draw an arc.
(iii) With C as centre and radius $m\overline{CA} = 5.1$cm draw another arc which intersects the first arc at A.
(iv) Join $\overline{BA}$ and $\overline{CA}$ to complete the $\triangle ABC$.
(v) Draw bisectors of $\angle B$ and $\angle C$ meeting each other in the point I.
(vi) Now draw bisector of the third $\angle A$.
(vii) We observe that the third angle bisector also passes through the point I.
(viii) Hence the angle bisectors of the $\triangle ABC$ are concurrent at I, which lies within the $\triangle$.

Note: Recall that the point of concurrency of bisectors of the angles of triangle is called its **incentre.**

(b) Draw altitudes of a given triangle and verify their concurrency.



Example

(i) Construct a triangle ABC in which $m\overline{BC}$ = 5.9cm, $m\angle B = 56^0$ and $m\angle C = 44^0$.

(ii) Draw the altitudes of the triangle and verify that they are concurrent.

Given

The side $m\overline{BC}$ = 5.9cm and $m\angle B = 56^0$, $m\angle C = 44^0$.

Required

(i) To Construct △ABC.

(ii) To draw its altitudes and verify their concurrency.

Construction

(i) Take $m\overline{BC}$ = 5.9cm.

(ii) Using protractor draw $m\angle CBA = 56^0$ and $m\angle BCA = 44^0$ to complete the △ABC

(iii) From the vertex A drop $\overline{AP} \perp \overline{BC}$.

(iv) From the vertex B drop $\overline{BQ} \perp \overline{CA}$. These two altitudes meet in the point O inside the △ABC.

(v) Now from the third vertex C, drop $\overline{CR} \perp \overline{AB}$.

(vi) We observe that this third altitude also passes through the point of intersection O of the first two altitudes.

(vii) Hence the three altitudes of △ABC are concurrent at O.

Note: Recall that the point of concurrency of the three altitudes of a triangle is called its **orthocentre.**

(c) Draw perpendicular bisectors of the sides of a given triangle and verify their concurrency.

Example

(i) Construct a △ABC having given $m\overline{AB}$ = 4cm, $m\overline{BC}$ = 4.8cm and $m\overline{AC}$ = 3.6cm.

(ii) Draw perpendicular bisectors of its sides and verify that they are concurrent.

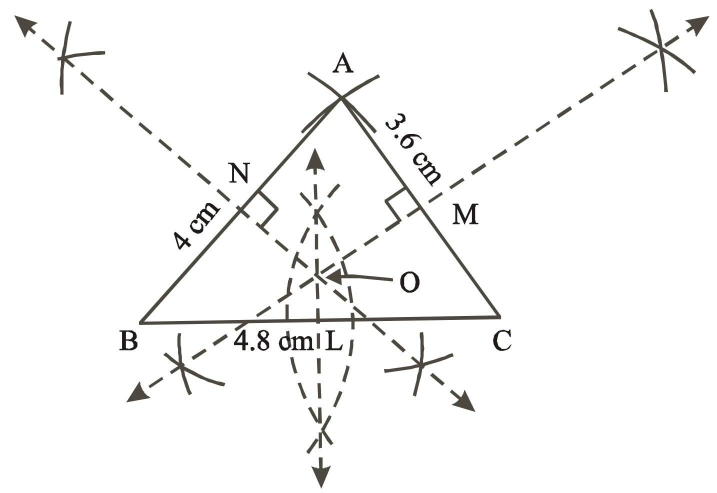
Given

Three sides $m\overline{AB}$ = 4cm, $m\overline{BC}$ = 4.8cm and $m\overline{AC}$ = 3.6cm of a △ABC.

Required

(i) To Construct △ABC.

(ii) To draw perpendicular bisectors of its sides and to verify that they are concurrent.



Construction

(i) Take $m\overline{BC}$ = 4.8cm.

(ii) With B as centre and radius $m\overline{BA}$ = 4cm draw an arc.

(iii) With C as centre and radius $m\overline{CA}$ = 3.6cm draw another arc that intersects the first arc at A.

(iv) Join $\overline{BA}$ and $\overline{CA}$ to complete the △ABC.

(v) Draw perpendicular bisectors of $\overline{BC}$ and $\overline{CA}$ meeting each other at the point O.

(vi) Now draw the perpendicular bisector of third side $\overline{AB}$.

(vii) We observe that it also passes through O, the point of intersection of first two perpendicular bisectors.

(viii) Hence the three perpendicular bisectors of size of △ABC are concurrent at O.

Note: Recall that the point of concurrency of the perpendicular bisectors of the sides of a triangle is called its **circumcentre.**

(d) Draw medians of a given triangle and verify their concurrency

Example

(i) Construct a △ABC in which $m\overline{AB}$ = 4.8cm, $m\overline{BC}$ = 3.5cm and $m\overline{AC}$ = 4cm.

(ii) Draw medians of △ABC and verify that they are concurrent at a point within the triangle. By measurement show that the medians divide each other in the ratio 2 : 1.

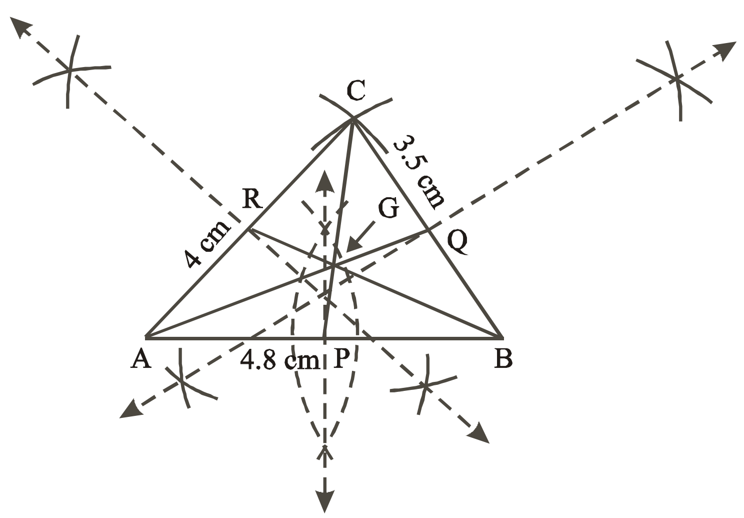
Given

Three side $m\overline{AB}$ = 4.8cm, $m\overline{BC}$ = 3.5cm and $m\overline{AC}$ = 4cm of a △ABC.

Required

(i) To Construct △ABC.

(ii) Draw its medians and verify their concurrency.



Construction

(i) Take $m\overline{AB}$ = 4.8cm.

(ii) With A as centre and $m\overline{AC}$ = 4cm as radius draw an arc.

(iii) With B as centre and radius $m\overline{BC}$ = 3.5cm draw another arc which intersects the first arc at C.

(iv) Join $\overline{AC}$ and $\overline{BC}$ to get the △ABC.

(v) Draw perpendicular bisectors of the sides $\overline{AB}$, $\overline{BC}$ and $\overline{CA}$ of the △ABC and mark their mid-points P, Q and R respectively.

(vi) Join A to the mid-point Q to get the median $\overline{AQ}$.

(vii) Join B to the mid-point R to get the median $\overline{BR}$.

(viii) The medians $\overline{AQ}$ and $\overline{BR}$ meet in the point G.

(ix) Now draw the third median $\overline{CP}$.

(x) We observe that the third median also passes through the point of intersection G of the first two medians.

(xi) Hence the three medians of the △ABC pass through the same point G. That is, they are concurrent at G. By measuring, $\overline{AG}$: $\overline{GQ}$ = 2 : 1 etc.

Note: Recall that the point of concurrency of the three medians of a triangle is called the **centroid** of the △ABC.

EXERCISE 17.2

1. Construct the following △'s ABC. Draw the bisectors of their angles and verify their concurrency.

(i)	$m\overline{AB}$ = 4.5cm,	$m\overline{BC}$ = 3.1cm,	$m\overline{CA}$ = 5.2cm
(ii)	$m\overline{AB}$ = 4.2cm,	$m\overline{BC}$ = 6cm,	$m\overline{CA}$ = 5.2cm
(iii)	$m\overline{AB}$ = 3.6cm,	$m\overline{BC}$ = 4.2cm,	m∠B = 75°.

2. Construct the following △'s PQR. Draw their altitudes and show that they are concurrent.

(i)	$m\overline{PQ}$ = 6cm,	$m\overline{QR}$ = 4.5cm,	$m\overline{PR}$ = 5.5cm
(ii)	$m\overline{PQ}$ = 4.5cm,	$m\overline{QR}$ = 3.9cm,	m∠R = 45°
(iii)	mRP = 3.6cm,	m∠Q = 30°,	m∠P = 105°.

3. Construct the following triangles ABC. Draw the perpendicular bisectors of their sides and verify their concurrency. Do they meet inside the triangle?

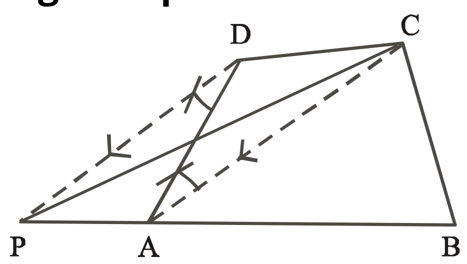
(i)	$m\overline{AB}$ = 5.3cm,	m∠A = 45°,	m∠B = 30°
(ii)	$m\overline{BC}$ = 2.9cm,	m∠A = 30°,	m∠B = 60°
(iii)	$m\overline{AB}$ = 2.4cm,	$m\overline{AC}$ = 3.2cm,	m∠A = 120°.

4. Construct the following $\triangle$s XYZ. Draw their three medians and show that they are concurrent.

(i)	$m\overline{YZ} = 4.1cm$,	$m\angle Y = 60°$,	$m\angle X = 75°$
(ii)	$m\overline{XY} = 4.5cm$,	$m\overline{YZ} = 3.4cm$,	$m\overline{ZX} = 5.6cm$
(iii)	$m\overline{ZX} = 4.3cm$,	$m\angle X = 75°$,	$m\angle Y = 45°$

17.2 Figures with Equal Areas

(i) Construct a triangle equal in area to a given quadrilateral.



Given

A quadrilateral ABCD.

Required

To construct a $\triangle$ equal in area to quadrilateral ABCD.

Construction

(i) Join AC.
(ii) Through D draw DP || CA, meeting BA produced at P.
(iii) Join PC.
(iv) Then PBC is the required triangle.

Observe that

$\triangle$s APC, ADC stand on the same base AC and between the same parallels AC and PD.

Hence $\triangle APC = \triangle ADC$

$\triangle APC + \triangle ABC = \triangle ADC + \triangle ABC$ or $\triangle PBC$ = quadrilateral ABCD.

EXERCISE 17.3

1. (i) Construct a quadrilateral ABCD, having $m\overline{AB} = m\overline{AC} = 5.3cm$, $m\overline{BC} = m\overline{CD} = 3.8cm$ and $m\overline{AD} = 2.8cm$.
   (ii) On the side BC construct a $\triangle$ equal in area to the quadrilateral ABCD.
2. Construct a $\triangle$ equal in area to the quadrilateral PQRS, having

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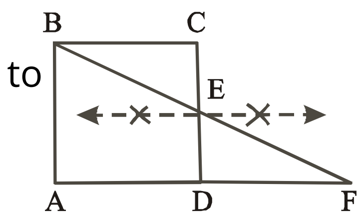
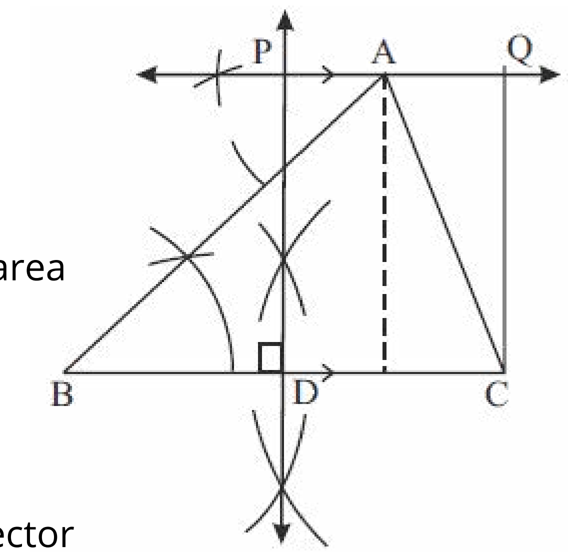
$m\overline{QR} = 7cm$, $m\overline{RS} = 6cm$, $m\overline{SP} = 2.75cm$. $m\angle QRS = 60^0$, and $m\angle RSP = 90^0$.

[Hint: $2.75 = \frac{1}{2} \times 5.5$]

3. Construct a $\triangle$ equal in area to the quadrilateral ABCD, having $m\overline{AB} = 6cm$, $m\overline{BC} = 4cm$, $m\overline{AC} = 7.2cm$, $m\angle BAD = 105^0$, and $m\overline{BD} = 8cm$.
4. Construct a right-angled triangle equal in area to a given square.



(ii) Construct a rectangle equal in area to a given triangle.



Given

$\triangle ABC$

Required

To construct a rectangle equal in area to $\triangle ABC$.

Construction

(i) Take a $\triangle ABC$.
(ii) Draw $\overleftrightarrow{DP}$, the perpendicular bisector of $\overline{BC}$.
(iii) Through the vertex A of $\triangle ABC$ draw $\overleftrightarrow{PAQ} \parallel \overline{BC}$ intersecting $\overleftrightarrow{PD}$ at P.
(iv) Take $m\overline{PQ} = m\overline{DC}$.
(v) Join Q and C.
(vi) Then CDPQ is the required rectangle.

Example

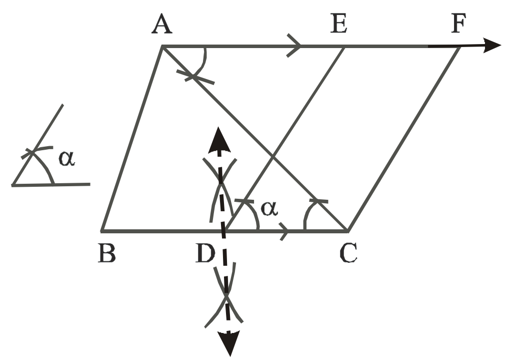
Construct a parallelogram equal in area to a given triangle having one angle equal to a given angle.

Given

$\triangle ABC$ and $\angle\alpha$.

Required

To construct a parallelogram equal in area to $\triangle ABC$ and having one angle = $\angle\alpha$



Construction

(i) Bisect $\overline{BC}$ at D.

(ii) Draw $\overline{DE}$ making $\angle CDE = \angle\alpha$

(iii) Draw $\overrightarrow{AEF} \parallel$ to $\overline{BC}$ cutting $\overline{DE}$ at E.

(iv) Cut off $\overline{EF} = \overline{DC}$. Join C and F.

Then CDEF is the required parallelogram.

EXERCISE 17.4

1. Construct a $\triangle$ with sides 4 cm, 5 cm and 6 cm and construct a rectangle having its area equal to that of the $\triangle$. Measure its diagonals. Are they equal?
2. Transform an isosceles $\triangle$ into a rectangle.
3. Construct a $\triangle ABC$ such that $m\overline{AB} = 3$cm, $m\overline{BC} = 3.8$cm, $m\overline{AC} = 4.8$cm. Construct a rectangle equal in area to $\triangle ABC$, the and measure its sides.

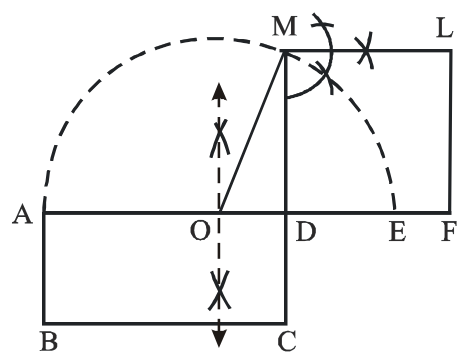
(iii) Construct a square equal in area to a given rectangle.

Given

A rectangle ABCD.

Required

To construct a square equal in area to rectangle ABCD.



Construction

(i) Produced $\overline{AD}$ to E making $m\overline{DE} = m\overline{CD}$.

(ii) Bisect $\overline{AE}$ at O.

(iii) With centre O and radius $\overline{OA}$ describe a semi - circle.

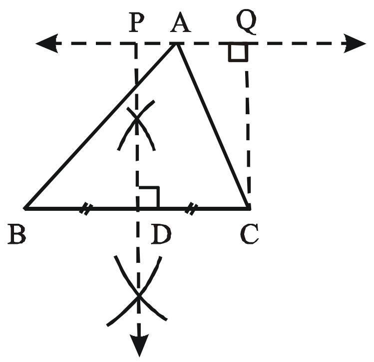
(iv) Produced $\overline{CD}$ to meet the semi - circle in M.

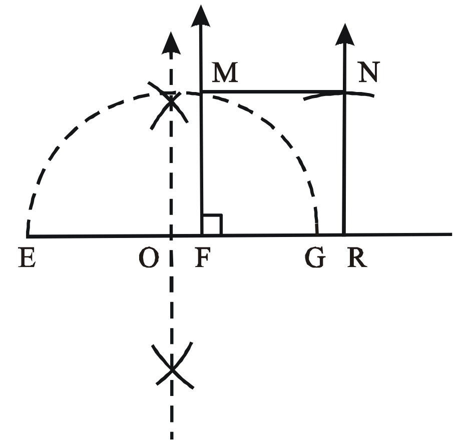
(v) On $\overline{DM}$ as a side construct a square DFLM.

This shall be the required square.

Example

Construct a square equal in area to a given triangle.





Given

$\triangle ABC$.

Required

To construct a square equal in area to $\triangle ABC$.

Construction

(i) Draw $\overleftrightarrow{PAQ} \parallel \overline{BC}$.

(ii) Draw the perpendicular bisector of $\overline{BC}$, bisecting it at D and meeting $\overleftrightarrow{PAQ}$ at P.

(iii) Draw $\overline{CQ} \perp \overline{PQ}$ meeting it in Q.

(iv) Take a line EFG and cut off $\overline{EF} = \overline{DP}$ and $\overline{FG} = \overline{DC}$.

(v) Bisect $\overline{EG}$ at O.

(vi) With O as centre and radius = $\overline{OE}$ draw a semi - circle.

(vii) At F draw $\overline{FM} \perp \overline{EG}$ meeting the semi - circle at M.

(viii) With $\overline{MF}$ as a side, complete the required square FMNR.

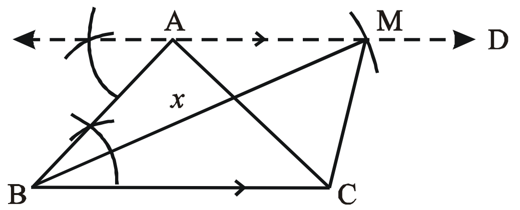
(iv) Construct a triangle of equivalent area on a base of given length.

Given

△ABC

Required

To construct a triangle with base x and having area equivalent to area △ABC.



Construction

(i) Construct the given △ABC.
(ii) Draw $\overleftrightarrow{AD} \parallel \overline{BC}$.
(iii) With B as centre and radius = x, draw an arc cutting $\overleftrightarrow{AD}$ in M.
(iv) Join $\overline{BM}$ and $\overline{CM}$.
(v) Then BCM is the required triangle with base $\overline{BM} = x$ and area equivalent to △ABC.

EXERCISE 17.5

1. Construct a rectangle whose adjacent sides are 2.5 cm and 5 cm respectively. Construct a square having area equal to the given rectangle.
2. Construct a square equal in area to a rectangle whose adjacent sides are 4.5 cm and 2.2 cm respectively. Measure the sides of the square and find its area and compare with the area of the rectangle.
3. In Q.2 above verify by measurement that the perimeter of the square is less than that of the rectangle.
4. Construct a square equal in area to the sum of two squares having sides 3 cm and 4 cm respectively.
5. Construct a △ having base 3.5 cm and other two sides equal to 3.4 cm and 3.8 cm respectively. Transform it into of a square equal square area.

6. Construct a △ having base 5 cm and other sides equal to 5 cm and 6 cm. Construct a square equal in area to given △.

REVIEW EXERCISE 17

1. Fill in the following blanks to make the statement true:
(i) The side of a right angled triangle opposite to 90° is called ......
(ii) The line segment joining a vertex of a triangle to the mid-point of its opposite side is called a ......
(iii) A line drawn from a vertex of a triangle which is ...... to its opposite side is called an altitude of the triangle.
(iv) The bisectors of the three angles of a triangle are ......
(v) The point of concurrency of the right bisectors of the three sides of the triangle is ...... from its vertices.
(vi) Two or more triangles are said to be similar if they are equiangular and measures of their corresponding sides are ....
(vii) The altitudes of a right triangle are concurrent at the ...... of the right angle.

2. Multiple Choice Questions. Choose the correct answer.

3. Define the following

(i)	Incentre	(ii)	Circumcentre
(iii)	Ortho centre	(iv)	Centroid
(v)	Point of concurrency		

SUMMARY

In this unit we learnt the construction of following figures and relevant concepts:

- To construct a triangle, having given two sides and the included angle.

- To construct a triangle, having given one side and two of the angles.
- To construct a triangle having given two of its sides and the angle opposite to one of them.
- Draw angle bisectors of a given triangle and verify their concurrency.
- Draw altitudes of a given triangle and verify their concurrency.

- Draw perpendicular bisectors of the sides of a given triangle and verify their concurrency.
- Draw medians of a given triangle and verify their concurrency.
- Construct a triangle equal in area to a given quadrilateral.
- Construct a rectangle equal in area to a given triangle.
- Construct a square equal in area to a given rectangle.
- Construct a triangle of equivalent area on a base of given length.
- Three or more than three lines are said to be concurrent if these pass through the same point and that point is called the point of concurrency.
- The point where the internal bisectors of the angles of a triangle meet is called incentre of a triangle.
- Circumscentre of a triangle means the point of concurrency of the three perpendiculars bisectors of the sides of a triangle.
- Median of a triangle means a line segment joining a vertex of a triangle to the midpoint of the opposite side.
- Orthocentre of a triangle means the point of concurrency of three altitudes of a triangle.