Domain 5 Statistics and Probability

Sub-Domain (i): Information Handling

Students' Learning Outcomes

After completing this domain, the students will be able to:

- recall difference between discrete and continuous data and grouped and ungrouped data
- reinforce representing the discrete data using suitable graph such as
  - ❖ line graph
  - ❖ bar graph
  - ❖ multiple bar graph
  - ❖ pie chart

Frequency Distribution

- construct cumulative frequency distribution
- represent frequency distribution by constructing:
  - ❖ histogram
  - ❖ frequency polygon

Measure of Central Tendency

- solve real life situations involving mean of grouped and ungrouped data

Advanced/ Additional

- calculate the median and mode for ungrouped data
- solve real life problems involving median and mode of grouped data

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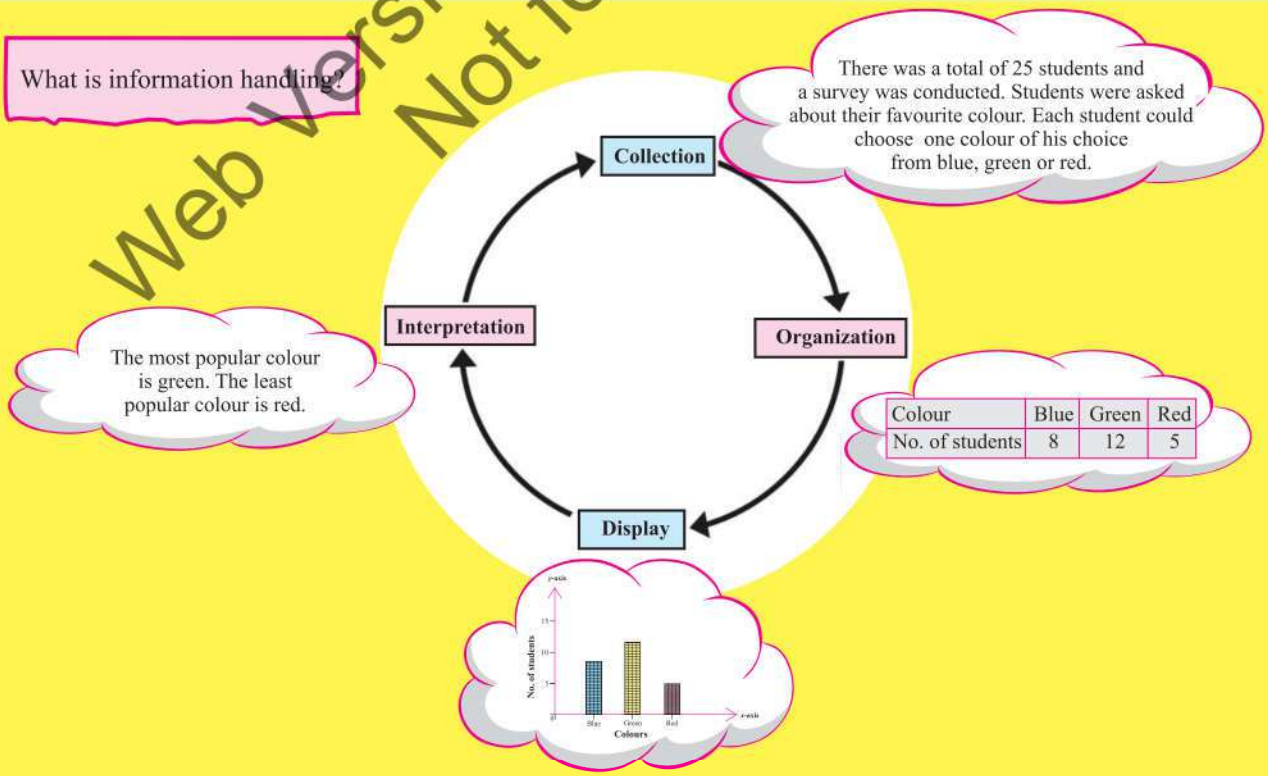
What is information handling?



Data

The collection of information in the form of facts and figures is called data e.g., the height of 20 students of your class, and the marks obtained by the students of your class.

Keep in mind!

Discrete and continuous data is quantitative data.

Type of Data

Discrete data:

It can take only some specific values. Whole numbers are used to write the discrete data e.g.,

- 5 flowers
- 20 students
- 30 caps

Continuous data:

It can take every possible value in a given interval, decimal numbers are used to write the continuous data. e.g.,

- 5.5 *kg*
- 15.5 litre
- 7.5 *cm*

Discrete data is further classified into two groups.

Continuous data is also classified into two groups.

Grouped data | Ungrouped data | Grouped data | Ungrouped data

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Ungrouped Data

Data which is not arranged in any systematic order (groups or classes) is called ungrouped data.

For example, The marks obtained out of 10 in a test by 20 students of a class

3, 10, 9, 8, 7, 6, 5, 4, 1, 8, 9, 5, 6, 7, 8, 5, 6, 7, 8, 5

Grouped Data

Data which is arranged in systematic order (groups or classes) is called grouped data.

For example, 30 students got admission in Grade-8. This data may be divided into 5 sections according to their scores. The detail is given below.

Scores	*50-60*	*60-70*	*70-80*	*80-90*	*90-100*
No of students	*5*	*7*	*12*	*2*	*4*

Explain to the students, the difference between the discrete and continuous data (ungrouped and grouped) by using real life examples.

Difference Between Discrete and Continuous Grouped and Ungrouped Data

Discrete Ungrouped and Grouped Data

Let us consider the number of pair of shoes sold by the shopkeeper in a month

10, 11, 12, 10, 11, 14, 15, 13, 14, 10, 11, 11, 12, 13, 15, 15, 14, 10, 11, 13, 10, 10, 10, 14, 13, 15, 14, 14, 15, 10. The data is called discrete ungrouped data.

If we arrange the above given data in groups or classes, then it is called discrete grouped data.

Pair of shoes	Tally marks
10	𝍸 |||
11	𝍸
12	||
13	||||
14	𝍸 |
15	𝍸

Continuous Ungrouped and Grouped Data

Let us consider the distances (in km) of 20 employees from their residence to the factory.

3, 3.5, 4, 4.7, 9.5, 10, 15.5, 16, 11, 10.2, 8.5,12,13, 13.5, 7, 8.5, 8, 9, 10.2, 15. The data is called Continuous ungrouped data.

If we arrange the above given data in groups or classes, then it is called continuous grouped data.

Class limits	Tally marks
$3 \leq x < 6$	||||
$6 \leq x < 9$	||||
$9 \leq x < 12$	𝍸 |
$12 \leq x < 15$	|||
$15 \leq x < 17$	|||

Graphical Representation

Graph is another way to represent the data. In graphical representation, the data is shown briefly.

The following graphs are used to represent a data in a meaningful way:

- Line Graph
- Bar Graph
- Multiple Bar Graph
- Pie Chart.

i. Line Graph

Usama sold the following length of cloth (in metre) in five weeks.

Weeks	1st	2nd	3rd	4th	5th
Length of cloth (*m*)	200	400	100	500	600

Now, we represent the above information by using a line graph.

Key fact!

In line graph, we use line segments to represent the data and line graph is used for continuous data.

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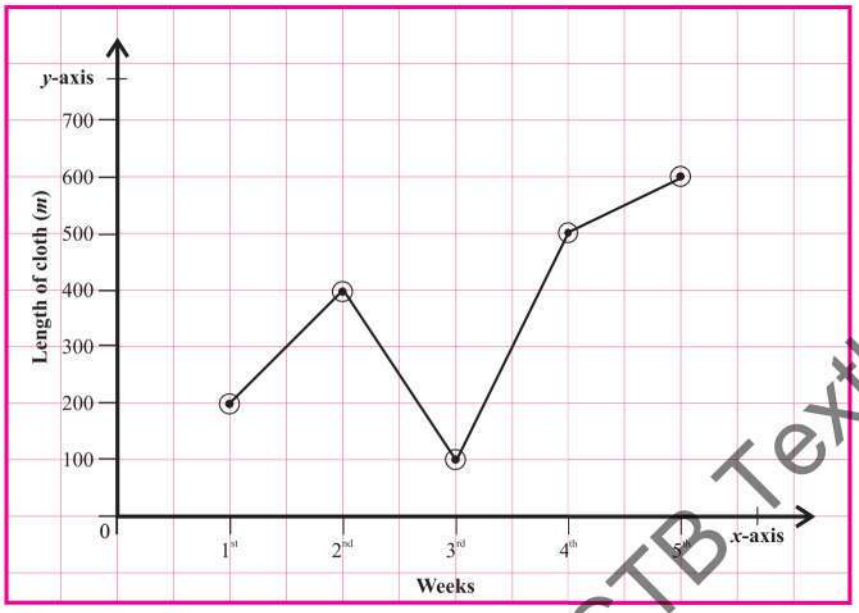
Title Graph showing the length of cloth (m) sold by Usama in 5 weeks.



Scale

Along x-axis:
2 squares represent 1 week

Along y-axis:
1 squares represents 100 metre

Description:

- Usama sold maximum length of cloth on the 5th week.
- Usama sold the least length of cloth in the 3rd week.

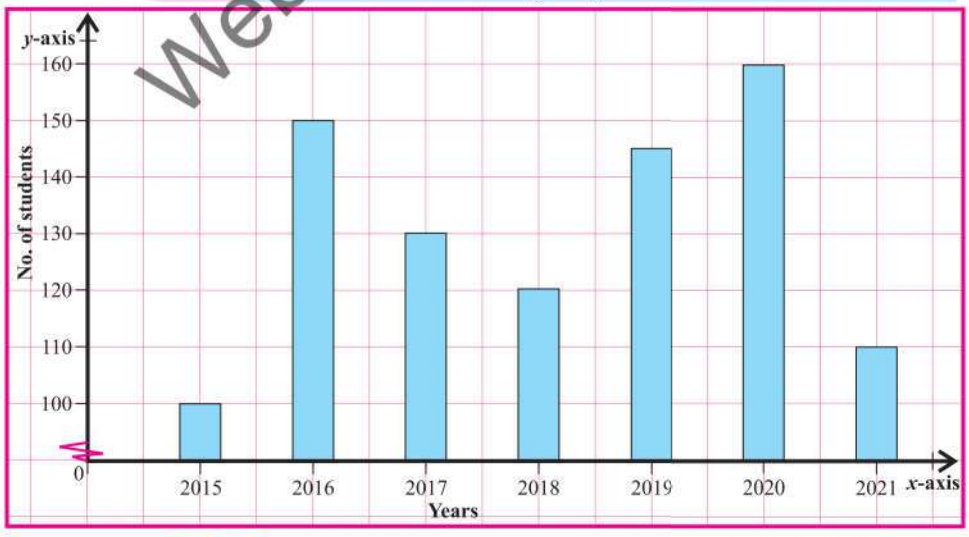
ii. Bar Graph

The following data represents the number of students of a high school who passed matriculation examination in the first division during the years 2015-2021.

Years	2015	2016	2017	2018	2019	2020	2021
No. of Students	100	150	130	120	145	160	110

Now, draw a bar graph for the given data.

Title Graph showing number of students who passed examination in the first division during the years 2015-2021



Key facts!

- In bar graph, we use bars having same width.
- Discrete data is mostly represented by using bar graph.

Think!

In which year, the maximum number of students passed the examination?

Scale

On x-axis:
2 squares represent 1 year

On y-axis:
1 square represents 10 students

Keep in mind!

The height of each bar depends upon frequency.

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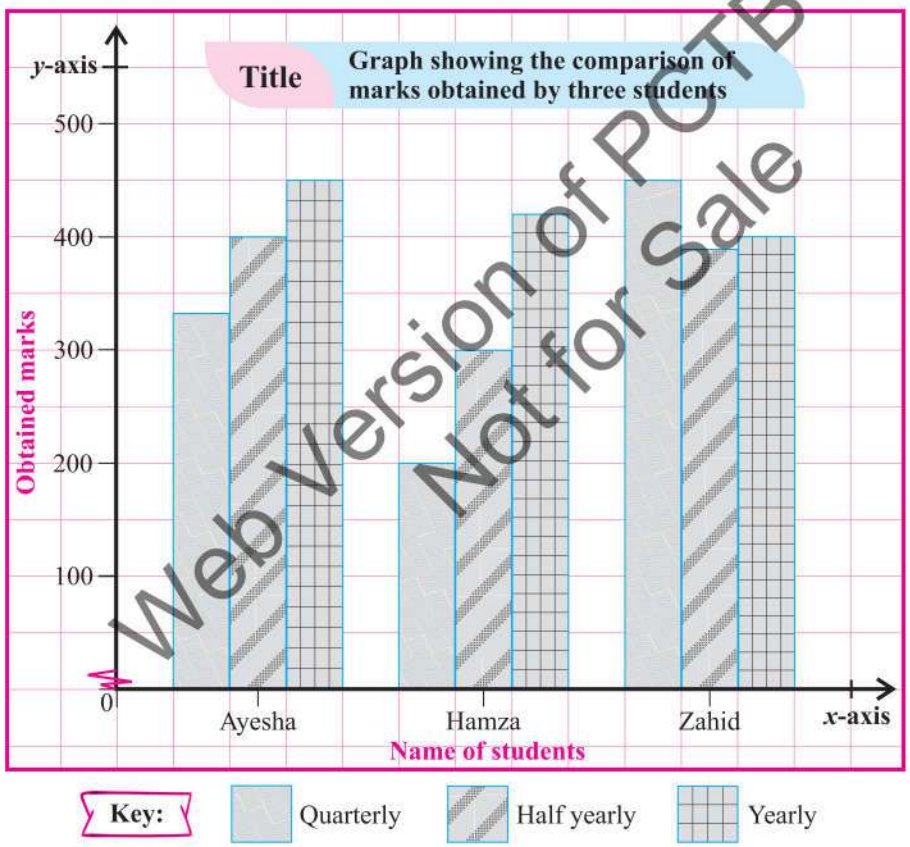
ii. Multiple Bar Graph

The following are the marks obtained out of 500 by three students in quarterly, half yearly and yearly examinations:

Multiple bar graph is an extension of bar graph. It is used for the comparison of two or more variable characteristics.

	Marks out of 500		
Name of Students	**Quarterly**	**Half Yearly**	**Yearly**
Ayesha	*330*	*400*	*450*
Hamza	*200*	*300*	*420*
Zahid	*450*	*390*	*400*

Now, draw a multiple bar graph for the given data.



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Remember!

The given graph is called vertical multiple bar graph.

Scale

On *x*-axis:
4 squares represent 1 student
On *y*-axis:
1 square represents 50 students

Try yourself!

Multiple bar graph can also be drawn horizontally. Can you draw it horizontally?

iv. Pie Graph

The table given below shows the total number of students in six grades of a high school.

Pie chart is used for the comparison of values of different items by making the sectors of a circle. Angles are used to represent the sectors.

Classes	V	VI	VII	VIII	IX	X	Total
No. of students	300	210	162	95	60	23	850

Need to know!

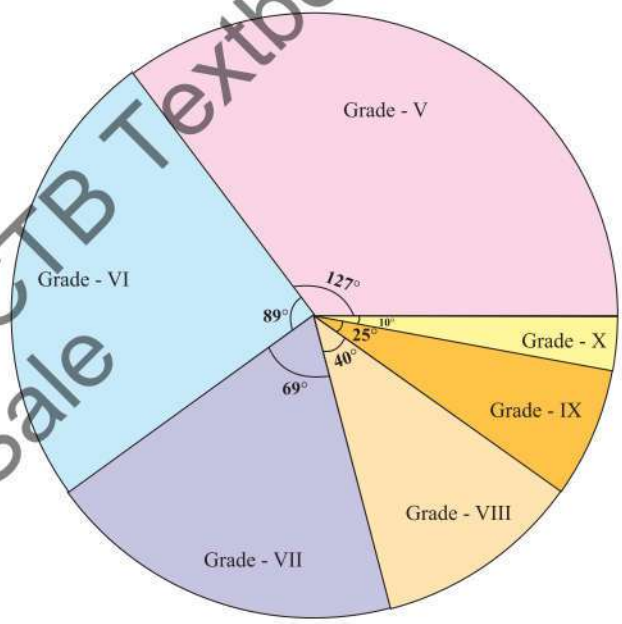
Pie chart is also known as circular graph.

Now, we represent the data by using a pie chart.

Keep in mind!

Pie graph can be made clockwise or counter clockwise.

Grades	No of Students	Angle of the sectors
V	300	$\frac{300}{850}\times 360^0 = 127^0$
VII	210	$\frac{210}{850}\times 360^0 = 89^0$
VII	162	$\frac{162}{850}\times 360^0 = 69^0$
VIII	95	$\frac{95}{850}\times 360^0 = 40^0$
IX	60	$\frac{60}{850}\times 360^0 = 25^0$
X	23	$\frac{23}{850}\times 360^0 = 10^0$
Total	**850**	**360°**



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Exercise 5.1

1. The marks of a student "Zeeshan" in the grade 6 examinations were 50, 80, 60, 50, 85, 90. Show these marks by drawing an appropriate graph.

2. A shopkeeper sold the following number of toys in 6 weeks.

Weeks	I	II	III	IV	V	VI
No. of Toys	120	80	90	125	110	75

Represent the above information by a simple bar graph. Also tell:

(i) How many toys were sold by the shopkeeper altogether?

(ii) In which week the maximum and least number of toys were sold?

3. Draw an appropriate graph to show the points out of 100 scored by four students in three games.

Students	Game-I	Game-II	Game-III
Ali	72	60	85
Anam	48	50	75
Zara	80	30	60
Umer	50	80	50

4. The following are the runs made in a two days cricket match by the students in a high school. Draw an appropriate graph for the following data.

Name	Javed	Hamza	Husnain	Jamshaid	Arsalan
1st day	55	60	35	80	65
2nd day	90	25	45	50	30

5. Mrs. Imran purchased the following goods and services from a grocery store for her kitchen in the last month. Graph the data using a pie graph.

Goods and Services	Rice	Floor	Sugar	Soap	Oil	Miscellaneous
Expenditure (in Rs.)	600	500	300	150	800	250

6. A shirt making company prepared the following number of shirts with different colours. Draw a pie chart for the following data:

Colour	Red	Blue	Green	Black	White	Yellow
No of Shirts	50	60	35	40	70	80

7. A factory makes fertilizer and sold in 5 cities i.e., 15% sold in city A, 20% sold in city B, 25% sold in city C, 35% sold in city D and 5% sold in city E. Draw a suitable graph to represent this information.

8. The distribution of masses (Ibs) of 100 students is given. Draw a suitable graph to represent the data.

Midpoints of masses	114.5	124.5	134.5	144.5	154.5	164.5
Frequency	35	20	10	5	15	15

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Frequency Distribution

Frequency

The number of times a value occurs in a data is called the **frequency** of that value.

Frequency Distribution

To write a data in the form of a table in such a way that the frequency of each class is observed at once is called frequency distribution.

Cumulative Frequency

Cumulative frequency is calculated by adding each frequency from a frequency distribution to the sum of its predecessors.

Construction of Frequency and Cumulative Frequency Distribution

The following are the masses in kg of 50 students selected from a school:

35, 30, 32, 36, 31, 40, 35, 42, 35, 45, 37, 41, 33, 37, 30, 28, 29, 30, 32, 33, 31, 35, 36, 30, 28, 37, 39, 28, 31, 34, 39, 45, 38, 36, 35, 28, 31, 34, 30, 41, 35, 36, 41, 28, 31, 34, 30, 29, 28, 37.

Now, we arrange the data in groups or classes or the form of table as below:

i. Look for the largest and smallest value i.e., 45 and 28 respectively.

ii. Number of classes to be made is 6.

iii. To find out the size of class interval use the given formula.

$$\text{Size of class interval} = \frac{\text{largest value} - \text{smallest value}}{\text{number of classes}}$$

$$= \frac{45 - 28}{6} = \frac{17}{6} \cong 2.8 \approx 3$$

Keep in mind!

The table below, is called frequency and cumulative frequency distribution.

Class Limits	Tally Marks	Frequency (f)	Cumulative Frequency ($c.f.$)
28–30	𝍸 𝍸 ||||	14	14
31–33	𝍸 ||||	9	9 + 14 = 23
34–36	𝍸 𝍸 |||	13	13 + 23 = 36
37–39	𝍸 ||	7	7 + 36 = 43
40–42	𝍸	5	5+ 43 = 48
43–45	||	2	2 + 48 = 50
Total	———	**50**	

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Example 1: Listed below are the scores of 50 students in a 60 marks test.

25, 33, 26.5, 34, 28, 35, 29, 36, 30, 54, 30, 39, 36, 37, 39, 40.5, 37, 34, 27, 41, 37, 41, 38, 42.5, 48, 51, 40, 51, 43, 40, 41, 39, 48, 51, 53, 41, 37.5, 52, 28, 46, 44,37, 39, 52, 51.5, 40, 45, 46, 43, 53

Make a frequency distribution table taking 6 classes of equal size by tally marks. Also make a cumulative frequency distribution.

Solution:

The lowest value = 25

The highest value = 54

Total classes to be made = 6

We take the size of class $= \frac{54 - 25}{6}$

$= \frac{29}{6} = 5$ *(approx.)*

Class Limits	Tally Marks	Frequency	Cumulative Frequency
25 – 29	𝍸 |	*6*	*6*
30 – 34	𝍸	*5*	*5 + 6 = 11*
35 – 39	𝍸 𝍸 |||	*13*	*13 + 11 = 24*
40 – 44	𝍸 𝍸 ||	*12*	*12 + 24 = 36*
45 – 49	𝍸	*5*	*5 + 36 = 41*
50 – 54	𝍸 ||||	*9*	*9 + 41 = 50*
	Total:	***50***	

Graphs of Frequency Distribution

The following are the type of graphs which can be used to represent a frequency distribution on a graph.

i. Histogram ii. Frequency polygon

Construction of Histogram

We are familiar with pie and bar graphs. Another common graphic way of representing data is by means of a histogram. A histogram is similar to bar graph but it is constructed for a frequency table.

In a histogram, the values of the data (classes) are represented along the horizontal axis and the frequencies are shown by bars perpendicular to the horizontal axis. Bars of equal width are used to represent individual classes of frequency table.

To draw a histogram from a grouped data, the following steps are followed:

(i) Draw *X-axis* and *Y-axis*.

(ii) Mark class boundaries of the classes along *X-axis*.

(iii) Mark frequencies along *Y-axis*.

(iv) Draw a bar for each class so that the height of the bar drawn for each class is equal to the frequency of the class.

How to calculate class boundaries?

- Lower class boundaries are obtained by "minus on-half" from the lower class limits.
- Upper class boundaries are obtained by "plus one half" to the upper limits.

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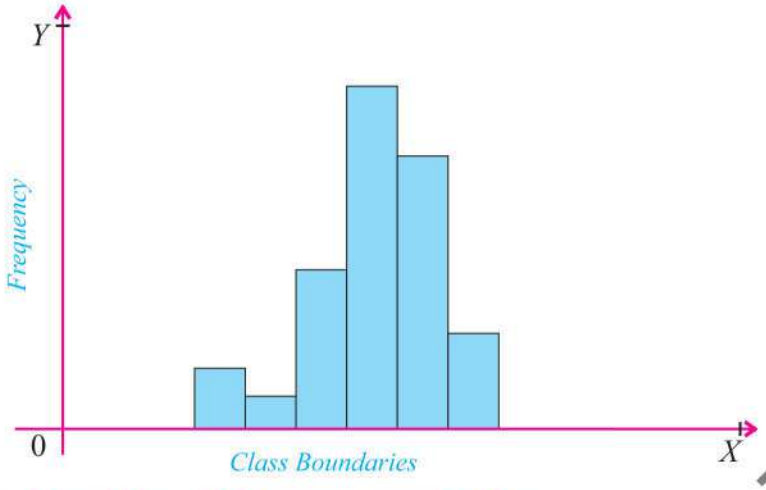
The graph is shown below:



Key fact!

Continuous data is mostly represented by using histogram and frequency polygon.

Construction of Frequency Polygon

To draw a frequency polygon from a grouped data, the following steps are followed:

i. Draw *X*-axis and *Y*-axis.
ii. Take midpoints on *X*-axis and class frequency on *Y*-axis.
iii. Put a dot mark against each midpoint of the class limits corresponding to its class frequency.
iv. Join all the dots by a line segment to get the required frequency polygon.

The graph is shown below.

Keep in mind!

Midpoint is obtained by finding out the average of class limits,
e.g., $\frac{\text{Lower limit + Upper limit}}{2}$

Do you know?

Midpoints are also known as class marks.

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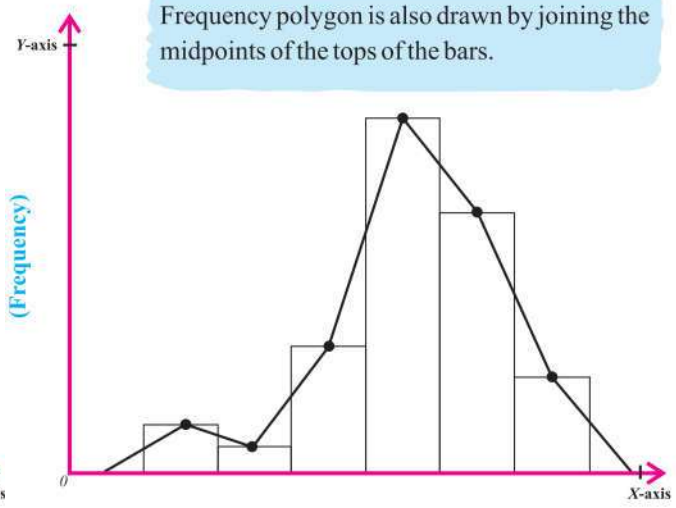
Midpoints (x)

Frequency Polygon

Frequency polygon is also drawn by joining the midpoints of the tops of the bars.



Class Boundaries (C.B)

Frequency Polygon on Histogram

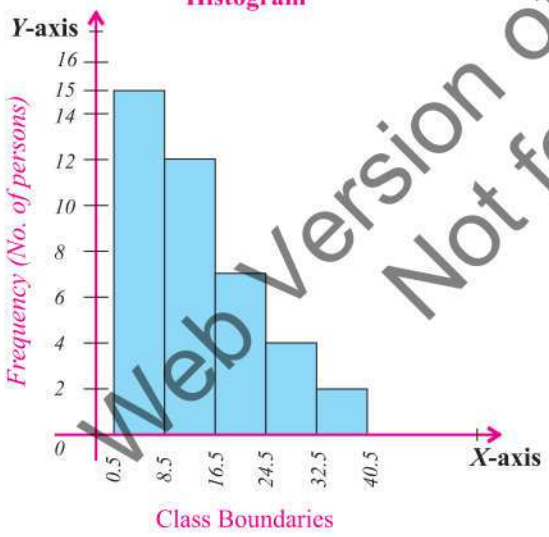
Example 2: The detail of distances travelled daily by the residents of a locality are given below. Construct a histogram and frequency polygon for the following frequency table.

Distance travelled (in *km*)	*1-8*	*9-16*	*17-24*	*25-32*	*33-40*
Number of persons	*15*	*12*	*7*	*4*	*2*

Solution: Frequency distribution table is:

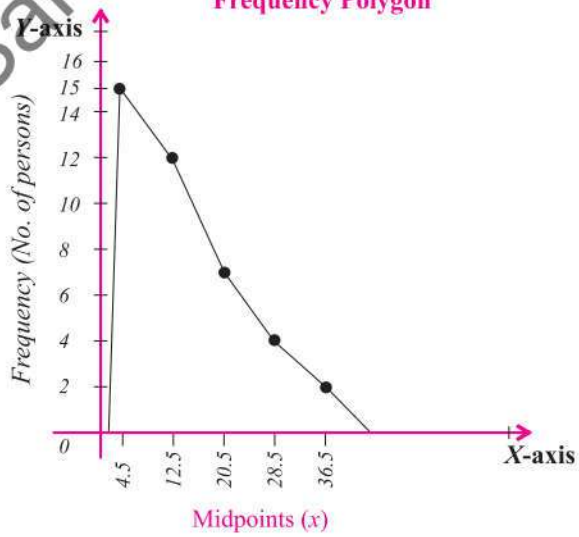
Distance travelled (km)	**Frequency (No. of Persons)**	**Class boundaries**	**Midpoints (x)**
1-8	*15*	*0.5 - 8.5*	$\frac{1+8}{2} = 4.5$
9-16	*12*	*8.5 - 16.5*	$\frac{9+16}{2} = 12.5$
17-24	*7*	*16.5 - 24.5*	$\frac{17+24}{2} = 20.5$
25-32	*4*	*24.5 - 32.5*	$\frac{25+32}{2} = 28.5$
33-40	*2*	*32.5 - 40.5*	$\frac{33+40}{2} = 36.5$
Total:	***40***		

Histogram



Class Boundaries

Frequency Polygon



Midpoints (x)

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Exercise 5.2

1. The following data displays the number of draws of different categories of bonds.

35, 55, 64, 70, 99, 89, 87, 65, 67, 38, 62, 60, 70, 78, 69, 86, 39, 71, 56, 75, *51, 99, 68, 95, 86, 53, 59, 50, 47, 55, 81, 80, 98, 51, 63, 66, 79, 85, 83, 70*

Construct a frequency distribution and cumulative frequency distribution for the above data, with seven classes of equal size of class interval 10.

2. Listed below are the number of electricity units consumed by *50* households in a low income group in a locality of Lahore.

55, 45, 64,130, 66,155, 80,102, 62, 60,101, 58, 75, 81, 111, 90, 55,151, 66, 139, 77, 99, 67, 51, 50, 125, 83, 55,136, 91, 86, 54, 78,100, 113, 93, 104, 111, 113, 96, 96, 87,109, 94,129, 99, 69, 83, 97, 97

With *12* classes of equal width of *10,* construct a frequency and cumulative frequency distribution for the electricity units consumed.

3. The following list, is of scores in a mathematics examination. Using the starting class *40 - 44* , set up a frequency and cumulative frequency distribution. List the class boundaries.

63, 88, 79, 92, 86, 87, 83, 78, 40, 67, 68, 76, 46, 81, 92, 77, 84, 76, 70, 66, 77, 75, 98, 81, 82, 81, 87, 78, 70, 60, 94, 79,52,82, 77,81,

4. Construct a frequency and cumulative frequency distribution for the following numbers using *1-10* as the starting class. List the class boundaries.

54, 67, 63, 64, 57, 56, 55, 53, 53, 54, 44, 45, 45, 46, 47, 37, 23, 34, 44,27, 36, 45, 34, 36,15, 23, 43,16, 44, 34, 36, 35, 37, 24, 24,14, 43, 37, 27, 36, 33, 25, 36, 26, 5, 44, 13, 33, 33, 17

5. Following are the number of days that 36 tourists stayed in Hunza valley.

1, 6, 16, 21, 41, 21, 5, 31, 20, 27, 17, 10, 3, 32, 2, 48, 8, 12, 21, 44, 1, 36, 5, 12, 3, 13, 15, 10, 18, 3, 1, 11, 14, 12, 64, 10.

Construct a frequency and cumulative frequency distribution starting with the class *1-7*. Also list the class boundaries.

6. Construct a histogram and frequency polygon for each of the frequency tables in questions *1-5*.

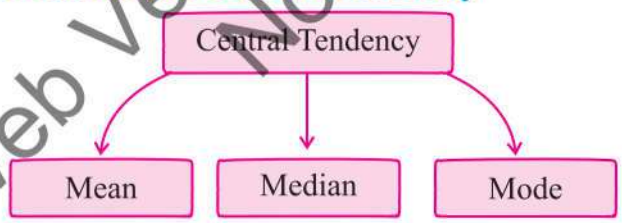
Measures of Central Tendency

Measures of Central Tendency help us to find out the centre of the data or distribution.

Key fact!

The measure that gives the centre of the data is called Measure of Central Tendency.

Description of Measures of Central Tendency



Calculation of Measures of Central Tendency

Mean (Average) for Ungrouped Data

Key fact!

The average is calculated by adding all the values and then dividing it by the total number of values. It is denoted by $\bar{X}$.

Let $x_1, x_2, \ldots, x_n$ be n given quantities. Then their average is the value representing the tendency of these quantities and is called their Mean value or Arithmetic Mean. It can be calculated by the formula:

$$\bar{X} = \frac{x_1 + x_2 + \ldots, + x_n}{N} = \frac{\Sigma X}{N} \quad \text{(Direct method)}$$

$$\bar{X} = \frac{sum\ of\ all\ values}{number\ of\ values}$$

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Example 3 The scores of a student in eight papers are ***58, 72,65,85,94, 78,87,85.*** Find the mean score.

$$\bar{X} = \frac{58 + 72 + 65 + 85 + 94 + 78 + 87 + 85}{8}$$

$$\bar{X} = \frac{624}{8} = 78$$

Hence, mean score is ***78***

Mean for Grouped Data

Let $x_1, x_2, x_3, \ldots x_n$ be the midpoints of the class limits with corresponding frequencies say $f_1, f_2, f_3, \ldots f_n$. Then the arithmetic mean is obtained by dividing the sum of the products of f and x by the total of all the frequencies.

Example 4: Given below is a frequency distribution of the masses in kg of 130 students. Find the mean.

Masses (kg)	*40-44*	*44-48*	*48-52*	*52-56*	*56-60*	*60-64*
Frequency	*5*	*13*	*45*	*32*	*26*	*9*

Solution: To calculate the mean of masses, we have to find out the midpoint of each class.

Masses (kg)	**Frequency (*f*)**	**Midpoints(*x*)**	***fx***
40-44	*5*	*42*	*210*
44-48	*13*	*46*	*598*
48-52	*45*	*50*	*2250*
52-56	*32*	*54*	*1728*
56-60	*26*	*58*	*1508*
60-64	*9*	*62*	*558*
	$\sum f = 130$	—	$\sum fx = 6852$

Mean (for grouped data) $\bar{X} = \frac{\sum fx}{\sum f}$

$$\bar{X} = \frac{6852}{130}$$

$$\bar{X} = 52.707 \approx 52.71\text{kg}$$

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Short Formula for Computing Arithmetic Mean

The computation of arithmetic mean, using direct method for ungrouped data as well as for grouped data is no doubt easy for small values of x and f. But when the values of x and f become very large, it becomes difficult to deal with the questions. So, to minimize our time and calculations, we take deviations from an assumed or provisional mean. Let A be considered as the provisional mean, (may be any value from the values of x or any number) and D denotes the deviation of X from A i.e., $D = X - A$. For $X = D + A$, the formula of arithmetic mean becomes:

$$\bar{X} = A + \frac{\Sigma D}{N} \qquad \text{(For ungrouped data)} \qquad (1)$$

$$\bar{X} = A + \frac{\Sigma fD}{\Sigma f} \qquad \text{(For grouped data)} \qquad (2)$$

Example 5: Find the arithmetic mean using short formula for the runs made by a batsman Saad in 8 one-day matches. **Runs:** 40, 45, 50, 52, 50, 60, 56, 70.

Solution: Taking deviations from A = 52

X	40	45	50	52	50	60	56	70	
$D = X - A$	−12	−7	−2	0	−2	8	4	18	$\Sigma D = 7$

Now $\Sigma D = 7$

$\because \quad \bar{X} = A + \frac{\Sigma D}{N}$

$\therefore \quad \bar{X} = 52 + \frac{7}{8} = 52 + 0.875 = 52.88$ or 53 runs

Example 6: Deviations from 12.5 of ten different values are 6, –2, 3.5, 9, 8.7, –5.5, 14, 11.3, –6.8, –4.2. Find the arithmetic mean.

Solution: Deviations from 12.5 are:

$D = 6, -2, 3.5, 9, 8.7, -5.5, 14, 11.3, -6.8, -4.2$

Now, $\Sigma D = 34$. Also, A = 12.5, using the formula (1) we have,

$$\bar{X} = A + \frac{\Sigma D}{n} = 12.5 + \frac{34}{10}$$

or $\bar{X} = 12.5 + 3.4 = 15.9$

Example 7: The heights (in inches) of 200 school boys are recorded in the following frequency distribution. Find the mean height by short formula.

Height (inches) X	51	52	53	54	55	56	57	58	59	60
Frequency (f)	2	5	8	24	55	45	38	16	6	1

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Solution:

Height (inches) X	Frequency (f)	$A = 55$ $D = X - A$	fD
51	2	−4	−8
52	5	−3	−15
53	8	−2	−16
54	24	−1	−24
A←55	55	0	0
56	45	1	45
57	38	2	76
58	16	3	48
59	6	4	12
60	1	5	5
Total	$\Sigma f = 200$		$\Sigma fD = 135$

Now using the formula (2), we get

$$\bar{X} = A + \frac{\Sigma fD}{\Sigma f}$$

or $$\bar{X} = 55 + \frac{135}{200}$$

or $$\bar{X} = 55 + 0.675$$

$$\therefore \bar{X} = 55.68 \text{ inches approx.}$$

Coding Method of Finding Arithmetic Mean

In fact, this method is applicable when all the class intervals h are of equal size, and hence, the calculation of arithmetic mean becomes more easy.

Now applying the coding variable U, stated as

$$U = \frac{X - A}{h}$$

or $$hU = X - A$$

or $$hU + A = X$$

Where A is an assumed mean and h is the class interval size.

The formula of Arithmetic mean becomes:

$$\bar{X} = A + \frac{\Sigma U}{N} \times h \quad \text{(for ungrouped data)} \qquad (3)$$

$$\bar{X} = A + \frac{\Sigma fU}{\Sigma f} \times h \quad \text{(for grouped data)} \qquad (4)$$

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Example 8: Five workers do their job at a big general store. Their daily wages are Rs. 300, 350, 400, 450, 500. Calculate the mean wage by coding method.

Solution: By using the formula (3), we have

$$\bar{X} = A + \frac{\Sigma U}{N} \times h$$

Here, let $A = 450$, and $h = 50$. Then

X	300	350	400	450	500	---
$U = \frac{X - A}{h}$	−3	−2	−1	0	1	$\Sigma U = -5$

$$\bar{X} = A + \frac{\sum U}{n} \times h$$

or $$\bar{X} = 450 + \frac{-5}{5} \times 50$$

$\therefore$ $$\bar{X} = 450 - 50 = \text{Rs. } 400$$

Example 9: Given below is a frequency distribution of the masses in kg of 130 students. Find the mean of masses using coding method.

Mass (kg)	40-44	44-48	48-52	52-56	56-60	60-64
Frequency	5	13	45	32	26	9

Solution:

Mass (kg)	**Frequency (f)**	X	$U = \frac{X - A}{h}$	fU
40-44	5	42	−2	−10
44-48	13	46	−1	−13
48-52	45	50 (A)	0	0
52-56	32	54	+1	32
56-60	26	58	+2	52
60-64	9	62	+3	27
---	**$\Sigma f = 130$**	--	--	**$\Sigma fU = 88$**

Now using the formula (4), we get

$$\bar{X} = A + \frac{\Sigma fU}{\Sigma f} \times h$$

Taking $A = 50$ and $h = 4$, we have

$$\bar{X} = 50 + \frac{88}{130} \times 4$$

or $\bar{X} = 50 + 2.71$ or $\bar{X} = 52.71\,kg$

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Example 10: For the following information, find the arithmetic mean.

(i) If $D = X - 10, \Sigma D = 50$ and $n = 5$

(ii) If $U = \frac{X-15}{3}$, $\Sigma U = 25$ and $n = 5$

(iii) If $D = X - 100, \Sigma fD = 200.$ $\Sigma f = 50$

(iv) If $U = \frac{X-100}{5}$, $\Sigma fU = 240$ and $\Sigma f = 100$

Solution: (i) Since $D = X - A$, therefore, $A = 10$

Hence, $\bar{X} = A + \frac{\Sigma D}{N} = 10 + \frac{50}{5} = 20$

(ii) Since $U = \frac{X-A}{h}$ therefore, $A = 15$ and $h = 3$

Hence $\bar{X} = A + \frac{\Sigma U}{N} \times h = 15 + \frac{25}{5} \times 3 = 15 + 15 = 30$

(iii) Since $D = X - A$, therefore, $A = 100$

Hence $\bar{X} = A + \frac{\Sigma fD}{\Sigma f} = 100 + \frac{200}{50} = 104$

(iv) Since $U = \frac{X-A}{h}$ therefore, $A = 100$ and $h = 5$

Hence $\bar{X} = A + \frac{\Sigma fU}{f} \times h = 100 + \frac{240}{100} \times 5$

$\therefore \quad \bar{X} = 100 + 12 = 112$

Remember!

Merits of mean

- It is the most simple and reliable form of an average.
- It is easy to find and understand.
- We need only the sum of all observations and number of observations.

Demerits of mean

- It is highly affected by extreme values.
- It provides a high value on having one very large value in the data.

Median for Ungrouped Data

If a data is arranged in ascending or descending order, then median of the data is:

(a) the middle value of the data, if it consists of odd number of values.

Median $= \left(\frac{n+1}{2}\right)^{th} item$ (when n is odd number)

(b) the mean of the two middle values of the data, if the number of values in a data is even.

Median $= \frac{1}{2}\left(\left(\frac{n}{2}\right)^{th} item + \left(\frac{n+2}{2}\right)^{th} item\right)$ (when n is even number)

Example 11: The masses in kg of 9 students are as under. Find the median:

29, 32, 45, 27, 30, 47, 35, 37, 33

Solution: Arranging these values in descending order:

47, 45, 37, 35, 33, 32, 30, 29, 27

The middle value is *33*

So, Median = *33* Kg

$\left(\frac{9+1}{2} = 5^{th} item\right)$

Try yourself!

Calculate the median when n is even number.

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Median for Grouped Data

The median for grouped data is obtained by using the following formula:

$$\text{Median } (\tilde{X}) = \ell + \frac{h}{f}\left(\frac{n}{2} - c\right)$$

Where, ℓ = the lower class boundary of median class.

h = the size of class of median class.

f = the frequency of the median class.

$n = \Sigma f$ = Total frequency

c = cumulative frequence of preceding class of the median class.

Keep in mind!

To calculate the median, we need:

- class boundaries
- cumulative frequency
- locate median class $\frac{n^{th}}{2}$ value in *c.f.*
- underline the median class for taking the values of *f* and *h*.
- It is denoted by $\tilde{X}$.

Example 12: Following are the masses (in pounds) of 50 students. Find the median of masses.

Mass (1bs)	*110-114*	*115-119*	*120-124*	*125-129*	*130-134*
No of students	*5*	*12*	*23*	*6*	*4*

Solution:

Mass (1bs)	Frequency	Class Boundaries	Cumulative Frequency (c. f)
110-114	*5*	*109.5-114.5*	*5*
115-119	*12*	*114.5-119.5*	*5+12=17*
120-124	*23*	*119.5-124.5*	*17+23=40* (Median class)
125-129	*6*	*124.5-129.5*	*40+6=46*
130-134	*4*	*129.5-134.5*	*46+4=50*
	$\Sigma f = 50$		$\Sigma f = 6852$

Here, $n = 50$ so, $\frac{n}{2} = \frac{50}{2} = 25$; 25th item lies in $119.5 - 124.5$

$$\text{Median } (\tilde{X}) = \ell + \frac{h}{f}\left(\frac{n}{2} - c\right)$$

$$= 119.5 + \frac{5}{23}(25 - 17) \text{ (after putting the values)}$$

$$= 119.5 + \frac{40}{23} = 119.5 + 1.74$$

$$\text{Median } (\tilde{X}) = 121.24 \text{ lbs}$$

Remember!

Merits of Median

- It is easy to find.
- It is not affected by extreme values.

Demerits of Median

- It cannot be calculated unless the data is arranged in some order.
- It is not based on all items.

Mode for Ungrouped Data

Mode is the value that occurs most frequently in a data . In case no value is repeated in a data, then the data has no mode. If two or more values occur with the same greatest frequency, then each is a mode. It is denoted by $(\hat{X})$.

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Example 13: Find the mode of the given data:
12,5,7,8,2,2,4,3,5,7, 2

Solution: The value *2* is repeated the most, so *2* is the mode of this data.

Example 14: Find the mode of the given data:
2,4, 6,8,10,12,14,16,20

Solution: This data has no mode because no value is repeated in the given data.

Example 15: Find the mode of the data given below:
1,2,2,2,3,4,5,5,5,6,7

Solution: Since **2** is repeated 3 times and 5 is also repeated 3 times so this data has two modes i.e., **2** and **5.**

Remember!

(i) A data can have more than one Mode.

(ii) A data may or may not have a Mode.

Remember!

Merits of Mode

- In certain cases mode can be located in the data.
- It is the most common value.

Demerits of Median

- Sometimes, data do not contain any mode.
- Data set may contain more than one mode. In this situation, mode is not most sitable measure.

Mode for Grouped Data

The mode is calculated by using the following formula:

$$\text{Mode } (\hat{X}) = \ell + \frac{f_m - f_1}{(f_m - f_1) + (f_m - f_2)} \times h$$

ℓ = the lower class boundary of modal class

f_m = the frequency of the modal class

f_1 = the frequency preceding the modal class

f_2 = the frequency following the modal class

h = the size of modal class.

Example 16: Following are the heights in (inches) of 40 students in class-8.

Heights (inches)	*48-50*	*50-52*	*52-54*	*54-56*	*56-58*	*58-60*
No of students	*5*	*7*	*10*	*9*	*6*	*3*

Find out the mode of the above given data.

Solution:

Heights (inches)	Frequency
48-50	*5*
50 52	*7*
52-54	*10*
54-56	*9*
56-58	*6*
58-60	*3*
Total	

← *Model Class*

We only assume the class with the highest frequency as the model class.

$$\text{Mode } (\hat{X}) = \ell + \frac{f_m - f_1}{(f_m - f_1) + (f_m - f_2)} \times h$$

$$\text{Mode } (\hat{X}) = 52 + \frac{10 - 7}{(10 - 7) + (10 - 9)} \times 2$$

$$\text{Mode } (\hat{X}) = 52 + \frac{3}{3 + 1} \times 2 = 52 + 1.5 = 53.5 \text{ inches}$$

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Exercise 5.3

1. What is the mean? Explain the methods of calculation of mean for ungrouped data.
2. Compute mean by direct and short method of the following data:
   (i) 10, 8, 6, 0, 8, 3, 2, 5, 8, 4
   (ii) 1, 3, 5, 3, 5, 3, 7, 5, 7, 5, 7
   (iii) 5, 4, 1, 4, 0, 3, 4, 11, 9
   (iv) 5, 9, 8, 6, 8, 7, 6, 7, 9, 8, 7, 9, 8, 7, 9, 7, 8, 5
   (v) –5, 4, 4, 0, –2, –3, –7, –10, 8, 9, –8
3. Calculate the mean from the following information:
   (i) If $D = x - 140$. $\Sigma D = 500$, and $n = 10$
   (ii) If $U = \frac{x - 130}{6}$, $\Sigma U = -500$, and $n = 15$
   (iii) If $U = \frac{x - 130}{6}$, $\Sigma U = 60$, and $n = 100$.
4. (i) The mean of 50 numbers is 75. Find their sum.
   (ii) The mean of 4, 5, 10, 9, 8, 11, 9, 11, 5, 7 and z is 8.8 find the value of z.
5. The deviations from 18.5 of 10 different values are 10.2, –17.5, –11.25, 15.50, 14.28, –13.75, 14.5, –12.50, 13.50 and 12.50. Calculate the mean.
6. The mean mass of 10 students is 30.5 *kg*. The masses of 9 students are 25 kg, 29 *kg*, 28 *kg*, 32 *kg*, 33.5 *kg*, 30.5 *kg*, 31.7 *kg*, 30.7 kg and 31.5 *kg*. What is the mass of 10^{th} student?
7. The mean height of 10 women is 1.5 *m*. Find the total height of 10 women.
8. The mean length of 5 ribbons is 15.5 *m*. The mean length of 4 ribbons is 14.75 *m*. Find the length of 5^{th} ribbon
9. Define and explain the method of calculation of mean, median and mode for ungrouped data.
10. Find mean, median and mode for the following data:
    (i) 62, 90, 71, 83, 75
    (ii) 45, 65, 80, 92, 80, 73, 56, 96, 62, 78
    (iii) Number of letters in the first 20 words in a book. 3, 2, 5, 3, 3, 2, 3, 3, 2, 4, 2, 2, 3, 2, 3, 5, 3, 4, 4, 5
    (iv) The number of calories in nine different beverages of 250 mm bottles:
    99, 106, 101, 103, 108, 107, 107, 106, 108
11. The following distances (in km) were travelled by 40 students to reach their school.
    2, 8, 1, 5, 9, 5, 14, 10, 31, 20, 15, 4, 10, 6, 5, 10, 5, 18, 12, 25, 30, 27, 20, 3, 9, 15, 15, 18, 10, 1, 1, 6, 25, 16, 7, 12, 1, 8, 21, 12.
    Compute the mean, median and mode of the distances travelled.
12. Explain the method of calculation of mean, median and mode for grouped data.
13. Compute the mean and mode of the following table:

Size of Family	*2*	*3*	*4*	*5*	*6*	*7*	*8*
Frequency	*51*	*31*	*27*	*12*	*4*	*1*	*1*

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14. The following is distribution of marks obtained by the class-VIII in a test. Find the mean, median and mode. Also verify $\bar{x} > \tilde{x} > \hat{x}$.

Marks/midpoints (x)	5	6	7	8	9	10
Frequency	15	16	18	10	9	5

15. Find the class mark, mean (by using direct and coding method), median and mode of the following frequency table:

Class limits	1-40	41-80	81-120	121-160	161-200
Frequency	17	41	80	99	4

16. Find the mean, median and mode of the following frequency table:

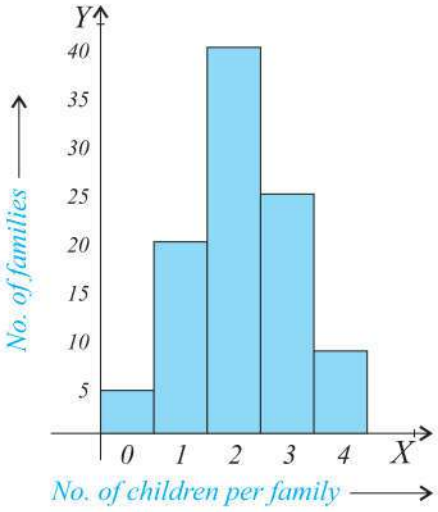
Class limits	1-5	6-10	11-15	16-20	21-25	26-30	31-35	36-40
Frequency	19	24	18	21	23	20	16	15

17. The diagram illustrates the number of children per family of a sample of *100* families in a certain housing estate:

(i) State the modal number of children per family.

(ii) Calculate the mean number of children per family.

(iii) Find the median number of children per family.



18. Given below is a frequency distribution:

Class limits	5-9	10-14	15-19	20-24	25-29
Frequency	1	8	18	11	2

Find mean (By direct, short and coding), median and mode of the frequency distribution.

19. The following table gives the distribution of marks obtained by two classes. Which class is better on the average?

Marks	30 – 40	40 – 50	50 – 60	60 – 70	70 – 80	80 – 90
Class A	20	32	35	40	15	8
Class B	15	30	40	42	20	10

20. If the mean of the following distribution is 3.78 find the value of a.

x	2	3	4	5	6
f	10	a	11	9	8

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Measure of Dispersion

The measure that is used to describe the spreadness of data or the data variation around a central value is called Measure of Dispersion.

The following formulas or techniques are used to measure the behaviour of each unit of the given data around its central value.

i. Range (R)
ii. Variance (S^2)
iii. Standard Deviation **(S.D.)**

Remember!

The purpose of measure of dispersion is to check the behaviour of each unit of the given data around the central value i.e., (Mean, Median and Mode).

i. Range (R):

It is the simplest measure of dispersion. Range is the difference between the largest and the smallest values of the data.

$$\text{Range (R)} = X_m - X_0$$

Where,

X_m = The largest value

X_0 = The smallest value

Keep in mind!

Dispersion is the state of getting spreading things..

Example 17: Find the range of the following data:

25, 35, 60, 48, 38, 15, 55

Keep in mind!

Range is calculated by using only two extreme data values.

Solution:

$\text{Range (R)} = X_m - X_0$

$X_m = 60$

$X_0 = 15$

$\text{Range} = 60 - 15$

$\text{Range} = 45$

Example 18:

Participant	1	2	3	4	5	6	7	8
Age	25	36	40	33	36	20	25	35

Solution:

$\text{Range (R)} = X_m - X_0$

$X_m = 40$

$X_0 = 20$

$\text{Range (R)} = 40 - 20$

$= 20$

The range of the data is 20 years.

ii. Variance (S^2)

The variance is a measure of variability. It is a single value, obtained by dividing the sum of the squares of the deviations taken from the mean by the number of observations in the data set.

Variance is the average of squared deviation from the mean.

Key fact!

Mostly, variance is used for comparing two sets of data. Smaller variance means that the data is more consistent.

It is denoted by S^2 and is defined for ungrouped data as:

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The formula of variance is:

$$S^2 = \frac{\sum(x-\bar{x})^2}{n} \quad \text{(By definition method)}$$

or

$$S^2 = \frac{\sum x^2}{n} - \left(\frac{\sum x}{n}\right)^2 \quad \text{(By direct method)}$$

Do you know?

- The data is widely spread when the variance of the data is larger.
- The data is clustered when the variance of the data is smaller.

Example 19:

Seven students took a test in Mathematics. They got marks (out of 15) as: 5, 7, 8, 11, 6, 9, 10. Find the variance in their marks by using both the methods.

Solution:

Here, $\bar{x} = \frac{5+7+8+11+6+9+10}{7}$

$\bar{x} = \frac{56}{7} = 8$ marks

$S^2 = \frac{\sum(x-\bar{x})^2}{n}$ (By definition)

(i) $S^2 = \frac{28}{7} = 4$ square marks

(ii) $S^2 = \frac{\sum x^2}{n} - \left(\frac{\sum x}{n}\right)^2$ (Direct method)

$S^2 = \frac{28}{7} - \left(\frac{28}{7}\right)^2$

$= 68 - (8)^2 = 68 - 64 = 4$

$S^2 = 4$ square marks

x	$x-\bar{x}$	$(x-\bar{x})^2$	x^2
5	−3	9	25
7	−1	1	49
8	0	0	64
11	3	9	121
6	2	4	36
9	1	1	81
10	2	4	100
$\sum x = 56$		$\sum(x-\bar{x})^2 = 28$	$\sum x^2 = 476$

(iii) Standard Deviation (S.D.)

Standard deviation is the square root of the average of squared deviation from the mean. It is abbreviated as S.D. and is denoted by S. It is calculated by the following formula for ungrouped data:

Keep in mind!

S.D. $= \sqrt{\text{variance}}$

$$S = \sqrt{\frac{\sum(x-\bar{x})^2}{n}} \quad \text{(By definition)}$$

$$S = \sqrt{\frac{\sum x^2}{n} - \left(\frac{\sum x}{n}\right)^2} \quad \text{(Direct method)}$$

Example 20: The height of the students in inches is given. Calculate the standard deviation for the given data.

x	$x-\bar{x}$	$(x-\bar{x})^2$	x^2
59	4	16	3481
56	1	1	3136
55	0	0	3025
53	2	4	2809
52	3	9	2704
Total 275	—	$\sum(x-\bar{x})^2 = 30$	$\sum x^2 = 15155$

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(i) $\bar{x} = \frac{\Sigma x}{n} = \frac{275}{5} = 55$ inches

$$S = \sqrt{\frac{\Sigma (x - \bar{x})^2}{n}}$$

$$= \sqrt{\frac{30}{5}}$$

$S = 2.45$ inches

(ii) $$S = \sqrt{\frac{\Sigma x^2}{n} - \left(\frac{\Sigma x}{n}\right)^2}$$

$$= \sqrt{\frac{15155}{5} - \left(\frac{275}{5}\right)^2}$$

$$= \sqrt{3031 - 3025} = \sqrt{6} = 2.45$$

$S = 2.45$ inches

Key fact!

- Both measures are used to check the variability in a data set.
- Variance and standard deviation of a constant is zero (0) e.g.,
  x: 5, 5, 5, 5, 5
  $S^2 = 0$; $S = 0$
- Variance and standard deviation is always greater than or equal to 0, i.e., $S^2 \geq 0$; $S \geq 0$
- If data is measured in inches, then the variance will be measured in square inches but the standard deviation will be measured in inches.

Exercise 5.4

1. Define range, variance and standard deviation and explain the purpose of measure of disperation.
2. Find the range, variance and standard deviation for the following sets of value.
   (i) 18, 19, 25, 22, 21, 19, 20, 27
   (ii) 75, 88, 90, 88, 81, 87, 82, 89
   (iii) 220, 225, 195, 320, 322, 235, 250, 253, 200, 310, 300
   (iv) 12.5, 15.25, 7.5, 9.25, 5.75, 10.5, 9.20, 8.53, 8.57, 10.35
3. In a data $n = 5$; $\Sigma x^2 = 10400$ and $\Sigma x = 50$. Find variance and standard deviation.
4. If variance = 5, $n = 7$, $\Sigma x^2 = 280$. Find the value of Σx.
5. The following are the marks of two student A and B in five test of Mathematics. Calculate mean, variance and standard deviation.

Student A	23	24	30	45	48
Student B	45	35	42	40	43

Hint!
Smaller variance means that data is more consistent.

(i) Who is better based on average?
(ii) Which student is more consistent in his performance?

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SUMMARY

- The collection of information in the form of facts and figures is called data.
- Data which is arranged in systematic order/groups or classes in called grouped data.
- In bar graph, we use bars having same width.
- Pie chart is used for comparison of values of different items by making the sectors of a circle. Angles are used to represent the sectors.
- The number of times a value occurs in a data is called the frequency of that value.
- To write a data in the form of a table in such a way that the frequency of each class can be observed at once is called frequency distribution.
- Cumulative frequency is calculated by adding each frequency from a frequency distribution to the sum of its predecessors.
- The mean is calculated by adding all the values and then dividing by the number of values.
- If a data is arranged in ascending or descending order then median of the data is:

(a) The middle value of the data, if it consists of odd number of values.

$$\text{Median} = \left(\frac{n+1}{2}\right)^{th} item \quad \text{(when } n \text{ is odd number)}$$

(b) The mean of the two middle values of the data, if the number of values in data is even.

$$\text{Median} = \frac{1}{2}\left(\left(\frac{n}{2}\right)^{th} item + \left(\frac{n+2}{2}\right)^{th} item\right) \quad \text{(when } n \text{ is even number)}$$

- Mode is the value that occurs most frequently in a data.
- Range is the difference between the largest and the smallest values of the data.
- Variance is a single value, obtained by dividing the sum of squares of the deviations taken from the mean by the numbers of observations in the data set.
- Standard deviation is the square root of the average of squared deviation from the mean.

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Sub-Domain (ii): Probability

Students' Learning Outcomes

After completing this domain, the students will be able to:

- determine that the probability of an event occurring is 'P' and an event 'not occurring is '1–P'

Experimental and Theoretical Probability

- perform probability experiments (for example tossing a coin, rolling a die, spinning a spinner etc. for each certain number of times) to estimate probability of a simple event
- compare experimental and theoretical probability in simple events
- explain and compute the probability of mutually exclusive, independent, and equally likely events
- calculate combining probabilities with "AND" and "OR"

Probability of a Simple Combined Events

- Predict the outcomes of simple combined events with the help of
  - sample space
  - tree diagram
- calculate probability of simple combined events
- apply the probability concept to real life situations

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How likely it is that an event will happen?

- **Certain / sure** to a ball
- **Likely** to a yellow ball
- **Unlikely** to a red ball
- **Impossible** to a green ball.

Girolamo Cardano is known as the father of probability.

Probability

Probability tells us how likely it is that an event will occur e.g., can you tell the probability of getting 4 in tossing a die?

Probability?

Probability is the chance of occurrence of an event.

$$Probability = \frac{Number\ of\ favourable\ outcomes}{Number\ of\ possible\ outcomes}$$

If Usman flips a coin, what will be the probability of occurrence of head?

The result of an experiment is called event.

When a coin is tossed, the sample space is given by:

S= {Head, Tail}

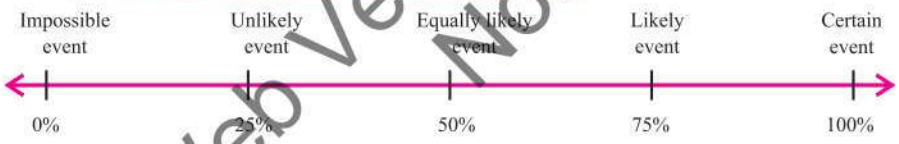
So, the chance of occurrence of head $= \frac{1}{2}$

P(head) $= \frac{1}{2} = 50\%$

The chance of occurrence of tail $= \frac{1}{2}$

P(tail) $= \frac{1}{2} = 50\%$

Notice that:
The chance of occurrence of head and tail are equal so, these events are called equally likely events.



If Zainab rolls a die, what will be the probability of getting 4?

When a die is rolled, the sample space is given by:

Sample Space $= S = \{1, 2, 3, 4, 5, 6\}$

Let A be the event of getting 4,

Where $n(A)$ = Number of elements in $A = 1$

$n(S)$ = Number of elements in sample space $= 6$

$$P(A) = \frac{n(A)}{n(S)}$$

The probability of getting 4 will be:

$$P(A) = \frac{1}{6}$$

Key fact!

The set of all possible outcomes is called sample space. It is denoted by S.

Remember!

The probability/chance of an event A is denoted by P (A).

Key fact!

The probability of an event always lies between 0 and 1 (both 0 and 1 inclusive).

Try yourself!

What will be the probability of getting 6?

Now think!

What will be the probability of not getting 4?

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The probability of not getting *4* is the complement of getting 4.
To find out the probability of not getting 4, we will subtract the probability of getting 4 from 1.

$$P(A') = 1 - P(A)$$
$$= 1 - \frac{1}{6}$$
$$= \frac{6-1}{6}$$
$$= \frac{5}{6}$$
$$P(A') = P(\text{not getting } 4) = \frac{5}{6}$$

Remember!

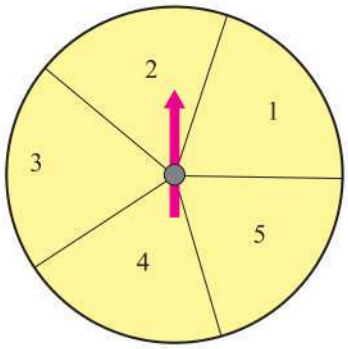
The complement of an event A consists of all outcomes that are not in the event A. It is denoted by $P(A')$. The sum of $P(A)$ and $P(A')$ is always equal to 1.

Need to know!

The sum of the probability of all the outcomes is always equal to one (1).

Example 1:

Ayesha! Look at the spinner given at the right. Find out the probability of getting 2.



Sample Space = $\{1,2,3,4,5\}$
No. of all possible outcomes = $n(S) = 5$
Let A be the event of getting 2.
No. of favourable outcomes = $n(A) = 1$

$$P(A) = P(\text{getting } 2) = \frac{\text{No. of favourable outcomes}}{\text{No. of all possible outcomes}}$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{1}{5}$$

Can you find out the probability of not getting 2?

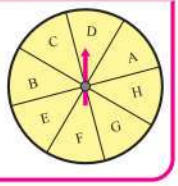
Yes, by subtracting the probability of getting 2 from 1, we can find out the probability of not getting 2.

$$P(\text{getting } 2) = \frac{1}{5}$$

$$P(A') = P(\text{not getting } 2) = 1 - \frac{1}{5}$$
$$= \frac{5-1}{5}$$
$$= \frac{4}{5}$$

Try yourself!

What is the probability of spinner not landing on D.



Key fact!

The event A and its complement are disjoint.

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Experimental and Theoretical Probability

The following methods are used to obtain the probabilities.

Types of Probabilities

Experimental Probability

It is based on actual results of an experiment

$$P(A) = \frac{\text{No. of favourable outcomes}}{\text{Total no. of trials}}$$

Example:
A coin is tossed 25 times. 16 times occurs head and 9 times occurs tail.

$$P(head) = \frac{16}{25}$$

$$P(tail) = \frac{9}{25}$$

Theoretical Probability

It is based on assumption

$$P(A) = \frac{\text{No. of favourable outcomes}}{\text{No. of all possible outcomes}}$$

Example:
A coin is tossed.

$$P(head) = \frac{1}{2}$$

$$P(tail) = \frac{1}{2}$$

Key fact!

Experimental probability is obtained by performing experiment. While the theoretical probability is obtained without performing any experiment.

Hina rolled a die 15 times. The given table is showing the outcomes of the experiment.

Die Outcomes						
Outcomes	1	2	3	4	5	6
No. of Trials	2	1	3	4	3	2

Can you tell the theoretical and experimental probabilities of the experiment?

The theoretical probability on each of the 6 digits is $\frac{1}{6}$

To calculate the experimental probabilities of each digit, we will use the outcomes which are given in the table.

$$P(event) = \frac{\text{No. of favourable outcomes}}{\text{Total no. of trials}}$$

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$P(1) = \frac{2}{15}$, $P(2) = \frac{1}{15}$, $P(3) = \frac{3}{15}$,

$P(4) = \frac{4}{15}$, $P(5) = \frac{3}{15}$, $P(6) = \frac{2}{15}$

Addition Law of Probability

Addition Law for Mutually Exclusive Events

If two events A and B are mutually exclusive, then the addition law of probability is,

$$P(A \cup B) = P(A) + P(B)$$

Or

$$P(A \text{ or } B) = P(A \cup B) = P(A) + P(B)$$

Need to know!

This law is also known as addition law of probability for mutually exclusive events.

Example 2:

A fair die is rolled. What will be the probability of getting 2 or 5?

Solution:

Sample Space = $\{1, 2, 3, 4, 5, 6\}$

$n(S) = 6$

Let A be the event of getting 2

$n(A) = 1$

$P(A) = \frac{n(A)}{n(S)} = \frac{1}{6}$

Let B be the event of getting 5

$n(B) = 1$; $n(S) = 6$

$P(B) = \frac{n(B)}{n(S)} = \frac{1}{6}$

As, events are mutually exclusive.

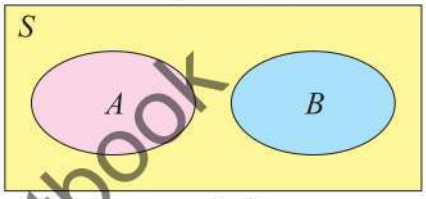
So, $P(A \text{ or } B) = P(A \cup B) = P(A) + P(B)$

$= \frac{1}{6} + \frac{1}{6} = \frac{2}{6} = \frac{1}{3}$

Activity

Toss a coin 50 times and find the experimental probabilities.

Disjoint sets



$A \cup B$

Key fact!

Mutually Exclusive Events:

- When two events do not occur together, these events are mutually exclusive events. i.e.
  $P(A \cap B) = 0$
- Mutually exclusive events are also known as disjoint sets.

Keep in mind!

The chance of occurrence of event A and B are equal. So, these events are called equally likely events.

Remember!

The events 2 and 5 cannot occur together that's why these events are mutually exclusive.
$P(A \cap B) = 0$

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Example 3: A bag contains 5 green balls, 6 yellow balls and 8 blue balls. One ball is selected at random, what will be probability of selecting a yellow ball or blue ball?

Solution: Let A be the event of getting a yellow ball and B be the event of getting a blue ball.

$$n \text{ (yellow balls)} = n(A) = 6$$

$$n \text{ (blue balls)} = n(B) = 8$$

$$n(s) = n(\text{Green balls}) + n(\text{yellow balls}) + n(\text{blue balls}) = 5 + 6 + 8 = 19$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{6}{19}$$

$$P(B) = \frac{n(B)}{n(S)} = \frac{8}{19}$$

As, events are mutually exclusive.

So, $P(A \text{ or } B) = P(A \cup B) = P(A) + P(B)$

$$= \frac{6}{19} + \frac{8}{19} = \frac{14}{19}$$

Multiplication Law of Probability

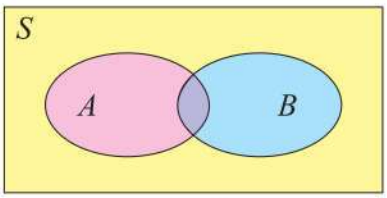
If two events A and B are independent, then the multiplication law of probability is,

$$P(A \cap B) = P(A) \cdot P(B)$$

Or

$$P(A \text{ and } B) = P(A) \cdot P(B)$$

Independent events:
Two or more events are said to be independent if the occurence of one event does not effect the occurence of another event.



$A \cap B$

Example 4: Let "A" and "B" are two independent events, $P(A) = 0.5$ and $P(B) = 0.38$. Find $P(A \cap B)$.

Solution: As, the events A and B are independent.

$$P(A \cap B) = P(A \text{ and } B) = P(A) \cdot P(B)$$

$$= (0.5) \cdot (0.38)$$

$$P(A \cap B) = 0.19$$

Keep in mind!
"OR" is used for union "AND" is used for intersection. $P(A \cap B)$ gives the combined probability of both events A and B.

Example 5: A fair dice is rolled and a fair coin is tossed. Find the probability of getting a tail and 4.

Solution: Let "A" is the probability of getting tail on a coin then,

$$P(A) = \frac{1}{2}$$

Let "B" is the probability of getting "4" on rolling a dice then,

$$P(B) = \frac{1}{6}$$

$$P(\text{Tail and 4}) = P(A \cap B) = P(A) \cdot P(B)$$

$$= \left(\frac{1}{2}\right) \cdot \left(\frac{1}{6}\right)$$

$$P(\text{Tail and 4}) = \frac{1}{12}$$

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Example 6: A jar has 2 red beans, 5 green beans and 6 blue beans. A bean is chosen at random. After replacing it, the second bean is chosen. Calculate:

(i) P(red and green beans) in same order

(ii) P(green and blue beans) in same order

(iii) P(red and blue beans) in any order

(iv) P(different colours) in any order

Solution:

Given,

Red beans $(Rb) = 2$; $n(Rb) = 2$

Green beans $(Gb) = 5$; $n(Gb) = 5$

Blue beans $(Bb) = 6$; $n(Bb) = 6$

$$n(S) = n(Rb) + n(Gb) + n(Bb)$$

$$= 2 + 5 + 6 = 13$$

$$P(Rb) = \frac{n(Rb)}{n(S)} = \frac{2}{13} \; ; \; P(Gb) = \frac{n(Gb)}{n(S)} = \frac{5}{13} \; ; \quad P(Bb) = \frac{n(Bb)}{n(S)} = \frac{6}{13}$$

(i) P(Rb and Gb) = ?

As, both the events are independent

(i) $P(Rb \text{ and } Gb) = P(Rb) \cdot P(Gb)$

$$= \left(\frac{2}{13}\right) \cdot \left(\frac{5}{13}\right) = \frac{10}{169}$$

(ii) $P(Gb \text{ and } Bb) = P(Gb) \cdot P(Bb)$

$$= \left(\frac{5}{13}\right) \cdot \left(\frac{6}{13}\right) = \frac{30}{169}$$

(iii) $P(Rb \text{ and } Bb) + P(Bb \text{ and } Rb) = P(Rb) \cdot P(Bb) + P(Bb) \cdot P(Rb)$

$$= \left(\frac{2}{13}\right) \cdot \left(\frac{6}{13}\right) + \left(\frac{6}{13}\right) \cdot \left(\frac{2}{13}\right) = \frac{12}{169} + \frac{12}{169} = \frac{24}{169}$$

(iv) P(different colours) = $P(Rb \text{ and } Gb) + P(Gb \text{ and } Rb) + P(GB \text{ and } Bb) +$
$P(Bb \text{ and } Gb) + P(Rb \text{ and } Bb) + P(Bb \text{ and } Rb)$

$$= \left(\frac{2}{13}\right) \cdot \left(\frac{5}{13}\right) + \left(\frac{5}{13}\right) \cdot \left(\frac{2}{13}\right) + \left(\frac{5}{13}\right) \cdot \left(\frac{6}{13}\right) + \left(\frac{6}{13}\right) \cdot \left(\frac{5}{13}\right) + \left(\frac{2}{13}\right) \cdot \left(\frac{6}{13}\right) + \left(\frac{6}{13}\right) \cdot \left(\frac{2}{13}\right)$$

$$= \left(\frac{10}{169}\right) + \left(\frac{10}{169}\right) + \left(\frac{30}{169}\right) + \left(\frac{30}{169}\right) + \left(\frac{12}{169}\right) + \left(\frac{12}{169}\right) = \frac{104}{169}$$

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Simple Combined Events (Tree Diagram)

If Zara toss a fair coin 2 times, what will be the possible outcomes?

The possible outcomes of that experiment can be written by using table.

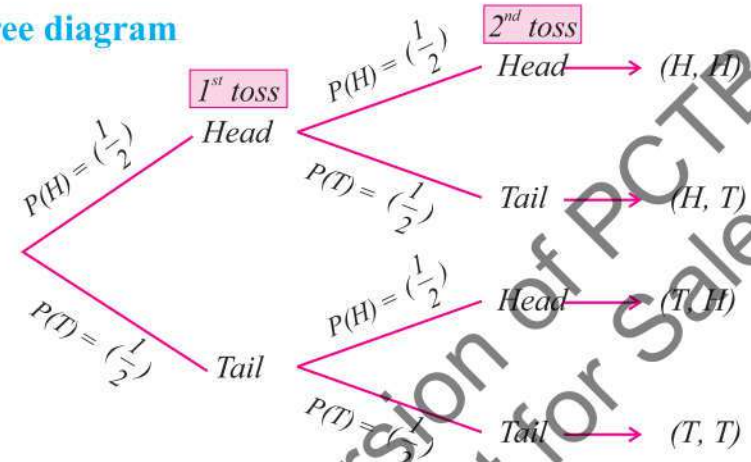
Flip Coin 2 / Flip Coin 1	H	T
H	(H, H)	(H, T)
T	(T, H)	(T, T)

The total number of possible outcomes of this experiment is 4. So, the sample space will be:

Simple space (S) = $\{(H,H), (H,T), (T,H), (T,T)\}$

$n(S) = 4$

Tree diagram



The number of possible outcomes of this experiment is *"4"*.

$S = \{(H,H), (H,T), (T,H), (T,T)\}$

$n(S) = 4$

Key fact!

Tree diagram is used to show the combination of two or more events.

Now, let's find out the probability of getting (i) exactly 2 heads (ii) 1 head and 1 tail.

(i) Exactly *2* heads

Let "A" be the event of getting *2* heads.

$A = \{(H,H)\}$: $n(A) = 1$; $n(S) = 4$

$$P(A) = \frac{n(A)}{n(S)} = \frac{1}{4}$$

(ii) Let "B" be the event of getting one head and one tail.

$B=\{(H,T),(T,H)\}$: $n(B)=2$; $n(S) = 4$

$$P(B) = \frac{n(B)}{n(S)} = \frac{2}{4} = \frac{1}{2}$$

By using combined events method

(i) $P(\text{Exactly } 2 \text{ heads}) = P(H \cap H) = P(H).P(H)$

$$= \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$$

(ii) $P(\text{head and tail}) = P(H \cap T) + P(T \cap H)$

$$= P(H)\ P(T) + P(T) \cdot P(H)$$

$$= \frac{1}{2} \cdot \frac{1}{2} + \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4} + \frac{1}{4}$$

$$= \frac{2}{4} = \frac{1}{2}$$

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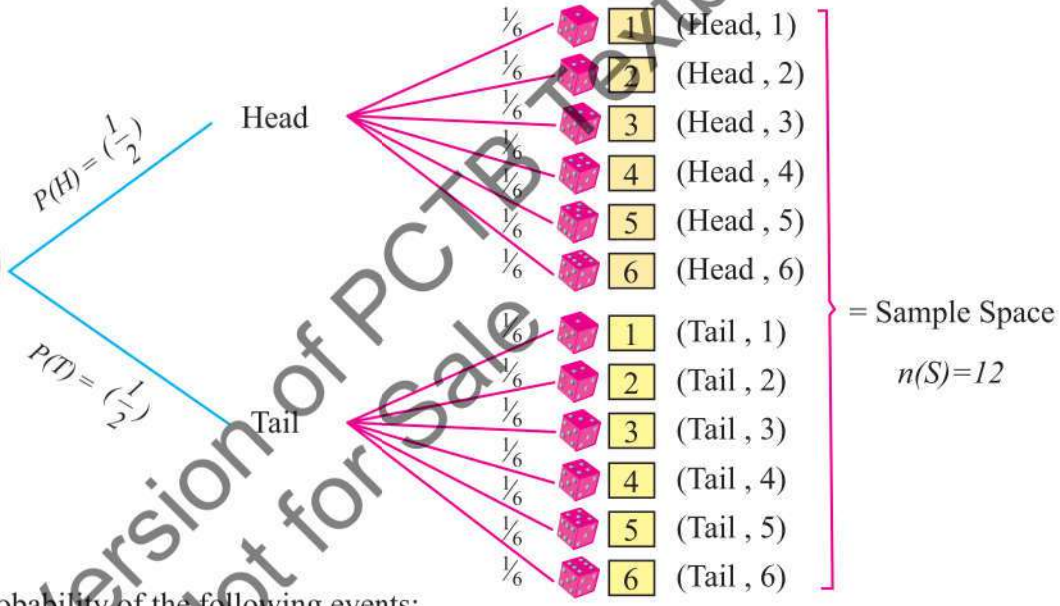
Hamza tossed a fair coin once and rolled a fair dice once.
Let's prepare a table of all possible outcomes.

Coin \ Dice	1	2	3	4	5	6
H	(H, 1)	(H, 2)	(H, 3)	(H, 4)	(H, 5)	(H, 6)
T	(T, 1)	(T, 2)	(T, 3)	(T, 4)	(T, 5)	(T, 6)

The total number of possible outcomes of this experiment is 12. So, the sample space will be:

Simple space (S) = $\{(H,1), (H,2),(H,3), (H,4), (H,5), (H,6), (T,1), (T,2),(T,3), (T,4), (T,5), (T,6)\}$

Let's prepare a tree diagram of all possible outcomes.



Now, calculate the probability of the following events:

(i) Head on coin and 5 on dice

Let "A" be the event of occurrence of head on coin and *5* on dice.

$A=\{(Head, 5)\}$; $n(A)=1$; $n(S) = 12$

$$P(A)= \frac{n(A)}{n(S)} = \frac{1}{12}$$

Try yourself!

Solve the given example by using combined events method (independent events).

(ii) Tail on coin and 2, *3, 4, 5, 6* on dice

Let "B" be the event of occurrence of tail on coin and *2, 3, 4, 5, 6* on dice

$$B = \{(tail,2), (tail,3), (tail,4), (tail,5), (tail,6)\}$$

$$n(B) = 5;\ n(S) = 12$$

$$P(B) = \frac{n(B)}{n(S)} = \frac{5}{12}$$

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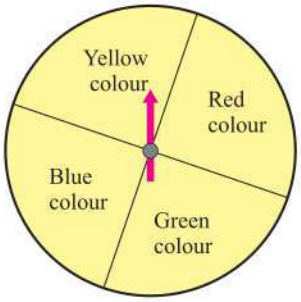
Exercise 5.5

1. Complete the following:

Sr. No.	*n(A)*	*n(S)*	*P(A)*
(i)	*5*	*15*	
(ii)		*25*	$\frac{17}{25}$
(iii)		*6*	$\frac{1}{6}$

2. Zara rolled a dice. What will be the probability of getting the digit "4". Also find the probability of not getting the digit "4".

3. A spinner is spun. Find the probability that the dice lands on red colour. Also find the probability that the dice lands on yellow or blue colour. (Hint: All events are equally likely).



4. A box contains 8 cards numbered 20 to 27. What is the probability of getting:
   (i) a number 22 from the box.
   (ii) a number 26 from the box
   (iii) all numbers 20 to 27 except 25 from the box

5. Find the probability of each of the following:
   (i) the digit "5" appears on rolling a dice.
   (ii) the digit "5" does not appear on rolling a dice.
   (iii) the digit "3" appears on rolling a dice.

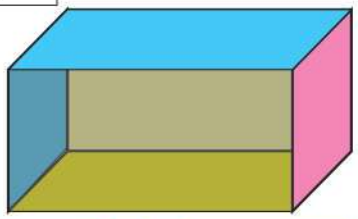
6. The probability that the team will win the match is 0.86. What is the probability that the team will not win the match?

7. The probability that tomorrow's temperature will be 0.45. Find out the probability that the temperature will not be 0.45.

8. Hamza throws the cube shaped cardboard 100 times. Each of the face of the cardboard having different colour.

Colours	Blue	Red	Green	Yellow	Orange	Purple
No. of trials	22	20	15	13	23	7

Find the probability of getting:
(i) red colour (ii) blue colour
(iii) red or orange colour (iv) not blue colour
(v) not yellow colour



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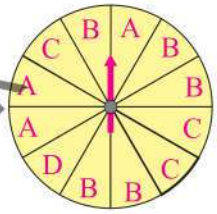
9. A group of 15 children consists of 7 boys and 18 girls. If a child is chosen at random from the group, what will be the probability that:
   (i) a boy is chosen (ii) a girl is chosen
   (iii) no boy is chosen (iv) no girl is chosen

10. A basket contains 5 red blocks, 3 green blocks, 2 blue blocks. If one block is picked at random, what will be the probability of getting.
   (i) Red block (ii) Blue block (iii) No blue block
   (iv) No green block (v) No red block



11. A spinner is spun. Find the probability that the alphabet on the landing is:
   (i) *B* (ii) not *B* (iii) *D*
   (iv) *A* (v) not *D* (vi) not *C*
   (**Hint:** all events are equally likely)

12. Three candidates A, B and C are in an examination. The probability of passing the candidate "A" is 0.25, the candidate "B" is 0.28 and the candidate "C" is 0.55. What will be the probability that A or C pass the exam? Also find out the probability of passing C or B.

13. A fair dice is rolled. Ahmad wins the game if the result is either odd or divisible by 6. What will be the probability of winning the game for Ahmad?

14. A dice is rolled. What will be the probability of getting 4 or 5?

15. A bag contains 3 red hairpins, 5 black hairpins, 5 green hairpins and 2 blue hairpins. Find the probability of selecting:
   (i) a red pin or a blue pin (ii) a green pin and a black pin (in same order)

16. A coin is tossed and a dice is rolled. Find the probability of getting head on the coin and the digit "4" on dice.

17. A jar contains 5 yellow marbles, 2 green marbles and 6 blue marbles. A marble is chosen at random from the jar. The second marble is chosen after replacing the first one. What is probability of choosing?
   (i) a green and a blue marble (in same order) (ii) a yellow and a green marble (in any order)
   (iii) *P*(different colours) in any order

18. A shopkeeper has 100 shirts. 40 are of red colour, 30 are of green colour, 20 are of black colour and 10 are of blue colour. A shirt is chosen at random by the customer. After replacing it, a second shirt is chosen. What is the probability of choosing?
   (i) a red colour shirt and a black colour shirt (in same order)
   (ii) a green colour shirt and a blue colour shirt (in any order)
   (iii) a red colour shirt and a green colour shirt (in any order)

19. Zainab rolled a dice 2 times.
   (i) List all the possible outcomes by using table and tree diagram.
   (ii) Also calculate the probability of getting:
      a) Exactly two 6's. b) Exactly two 4's. c) At least once time 4.

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20. Zeeshan spun a spinner having four different colours (red, green, blue and yellow) once and rolled a dice once.
    (i) List all the possible outcomes by using table and tree diagram.
    (ii) Also calculate the probability of getting:
        a) Green on the spinner and the digit "2" on the dice.
        b) Green or yellow on the spinner and the digit "6" on the dice.
        c) Red or blue on the spinner and 2,3 or 4 on the dice.

21. Saima wants to eat ice-cream. There are 3 flavours of ice-cream (Mango, Strawberry and Pistachio). Each flavour has 3 sizes (Small, Medium and large):
    (i) List all the possible outcomes by using table and tree diagram.
    (ii) Also calculate the probability of Selecting/Choosing:
        a) Mango flavour with medium size.
        b) Strawberry flavour with small or large sizes.
        c) Pistachio flavour with medium or large sizes.

SUMMARY

- Probability tells us how likely it is that an event will occur.
- The result of an experiment is called event.
- Probability is the chance of occurrence of an event.
- The set of all possible outcomes is called sample space. It is denoted by "S".
- Experimental probability is obtained by performing experiment. While the theoretical probability is obtained without performing any experiment.
- The probability/chance of an event "A" is denoted by $P(A)$.
- The probability of an event always lies between 0 and 1(both 0 and 1 inclusive).
- The sum of the probabilities of all the outcomes is always equal to one (1).
- Tree diagram is used to show the combination of two or more events.
- When two events do not occur together, these events are mutually exclusive events. i.e., $P(A \cap B) = 0$
- If two events A and B are mutually exclusive, then the addition law of probability is:

$$P(A \cup B) = P(A) + P(B)$$

- If two events A and B are independent, then the multiplication law of probability is:

$$P(A \cap B) = P(A) \cdot P(B)$$

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Review Exercise 5

1. Four options are given against each statement. Encircle the correct one.

i. What is the value of x i.e., 14, 24, x, 18, 30, when mean is 23.

(a) 28 (b) 29 (c) 30 (d) 31

ii. What is the value of x i.e., 40, 28, 16, 18, 37, 20, x, 35, when median is 26.

(a) 20 (b) 22 (c) 24 (d) 28

iii. A number which indicates how often a value occurs is called:

(a) frequency (b) mode (c) median (d) average

iv. An arrangement of the values that one or more variables taken in data is called:

(a) frequency (b) frequency distribution

(c) median (d) mode

v. A representation of a frequency distribution by means of rectangles whose widths represent class intervals and whose areas are proportional to the corresponding frequencies is called:

(a) histogram (b) bar chart (c) pie chart (d) line graph

vi. A measure of central tendency that attempts to describe a data by identifying the central position within that data:

(a) is a single value (b) are multiple values

(a) are duplicate values (d) are repeating values

vii. The statistical measure that identifies a single value as representative of an entire distribution is called:

(a) frequence distribution (b) histogram

(c) mean (d) central tendency

viii. The value which occupies the middle position when all the observations are arranged in an ascending/descending order is called:

(a) frequency distribution (b) median

(c) mode (d) mean

ix. The value that occurs most frequently in the data is called:

(a) frequency distribution (b) median

(c) Mode (d) mean

x. _____ is based on actual result of an experiment.

(a) probability (b) experimental

(c) theoretical probability (d) independent probability

xi. $P(A) = \frac{No.\ of\ favourable\ outcomes}{No.\ of\ all\ possible\ outcomes}$ is called:

(a) theoretical probability (b) experimental Probability

(c) combined probability (d) multiplicative probability

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xii. _____ can take only same specific values.

(a) data (b) continuous data

(c) discrete data (d) mode

xiii. _____ is an extension of bar graph

(a) line graph (b) pie chart

(c) multiple bar graph (d) circular graph

xiv. The number of times a value occurs in a data is called:

(a) frequency (b) cumulative frequency

(c) class boundaries (d) distribution

2. A survey was conducted from the students of Grade-VIII and asked about their favourite colour. Draw an appropriate graph for the following data and also interpret the data:

Colours	Blue	Red	Green	Yellow
No. of students	25	50	30	15

3. Following are the heights in inches of 12 students:

55, 53, 54, 58, 60, 61, 62, 56, 57, 52, 51, 63

Find the average height (direct and short method) of the given data.

4. Calculate the mean from the following information:

(i) if $U = \frac{X - 50}{}$; $\Sigma U = 25$; $n = 10$

(ii) if $U = \frac{X - 100}{2}$; $\Sigma U = 25$; $n = 10$

5. The price of *10* litres of drinking water was recorded at several stores, and the results are displayed in the table below:

Price *(Rs.)*	*74*	*75*	*76*	*77*	*78*	*79*	*80*

6. Calculate the mean, median and mode for each set of data given below:

(i) 3,6,3, 7,4,3,9 (ii) 11,10,12,12,9,10,14,12,9

(iii) 2,9,7,3,5,5,6,5,4,9 (iv) 6,8,11,5,2,9,7,8

(v) 153.8,154.7,156.9,154.3,152.3,156.1,152.3

Find the average price of drinking water by direct and short method.

7. On a prize distribution day, 50 students brought pocket money as under:

Rupees	5-10	10-15	15-20	20-25	25-30
Frequency	12	9	18	7	4

Calculated the mean, median and mode of the above data. Also draw histogram and frequency polygon.

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8. Find the range, variance and standard deviation for the following data.

110, 120, 115, 135, 127, 108, 113, 117, 118, 121, 122, 123, 127

9.

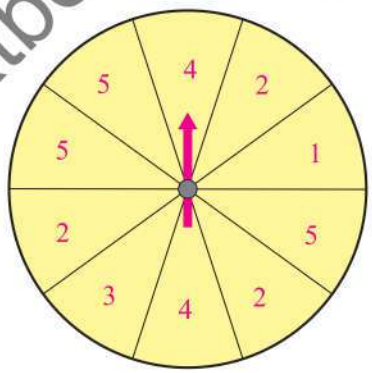
Company A	117	118	120	125	115	113
Company B	200	180	150	120	125	130

(i) Who is better based on average?

(ii) Which company is more consistent in his performance for producing the number of bags in a week.

10. Zara rolled a dice, what will be the probability of getting 1. Also tell what will be the probability of not getting 1.

11. Spin a spinner at right.



i. Write the sample space

ii. Find the probability of getting even numbers.

iii. P(2), P(1) and P(5). (**Hint:** All events are equally likely)

12. In a class of 30 students, 12 are used glasses. Find the probability of students who don't use glasses. Also calculate the probability of students who used glasses.

13. Twenty cards of whole numbers (10-29) are put in a basket. A card is selected at random from the basket. Find the probability that the number on the card.

i. Contains the number 12.

ii. Contains the number 20.

iii. Does not contain the number 20.

14. Imran rolled a dice once and tossed a coin once.

i. List all the possible outcomes by using tree diagram.

ii. Also calculate the probability of getting:

a) 5 on dice and tail on coin

b) 2, 4 or 6 on dice and head on coin

c) Odd numbers on dice and tail on coin.

15. The candidate "A" solved 75% of the problems of mathematics exercise. The candidate "B" solved 10% of the problems of the same exercise. What will be the probability that the candidate A or B solve the problems at random.

16. The probability that monkey will be alive about 25 years is 0.85 and 0.82 will be alive about 28 years. Find the probability that

i. Both of them will alive

ii. Both of them will die.

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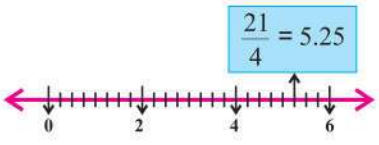
Answers

Exercise 1.1

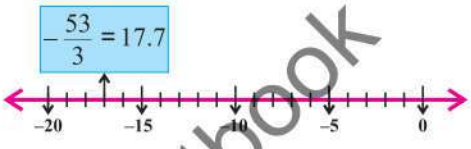
1. (iv), (viii), (ix), (x), (xi), (xii), (xiv), (xv), (xvii), (xviii) are terminating
(i), (iii), (vii), (xiii), (xvi) are recurring (ii) and (vi) are non-terminating

2. (i) and (vii) are irrational
(ii), (iii), (iv), (v) and (viii) are rational.

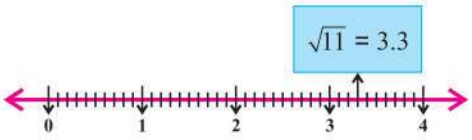
3. (i)



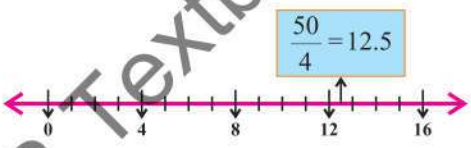
(ii)



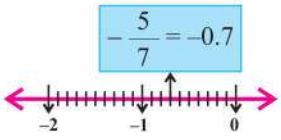
(iii)



(iv)



(v)



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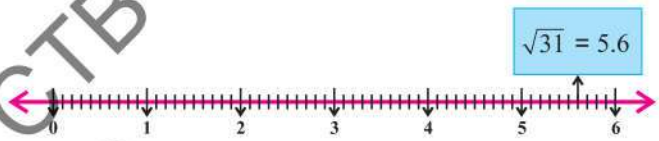

4. (i) 15 (ii) 21 (iii) 36 (iv) 9 (v) 100 (vi) 88

5. (i) 5 (ii) 11 (iii) 6 (iv) 5

6. (a) 9.2 ≈ 10 packets (b) 44 kg rice

7. (a) $12\frac{8}{15}$ *metres* (b) $28\frac{1}{5}$ *metres*

8. 92.3 ≈ 93 packets, 37.5 kg

9. (a) 25 bottles (b) 22 bottles (c) 112.5 litres

Exercise 1.2

1. (i) Identity property (ii) Associative property w.r.t "×" (iii) Additive inverse property
(iv) Commutative property w.r.t "+" (v) Closure property w.r.t "×" (vi) Multiplicative inverse property
(vii) Multiplicative identity property (viii) Multiplicative inverse property (ix) Associative property w.r.t "+"
(x) Associative property w.r.t "+" (xi) Multiplicative identity property (xii) Commutative property w.r.t. "×"
(xiii) Additive identity property (xiv) Commutative property w.r.t. "×" (xv) Additive inverse property

2.

	(i)	(ii)	(iii)	(iv)	(v)	(vi)	(vii)	(viii)	(ix)	(x)	(xi)
Additive inverse	−15	27	$-\frac{7}{9}$	$\frac{9}{16}$	$\frac{20}{23}$	−1	$\frac{2}{3}$	$-\frac{7}{2}$	$-\frac{5}{7}$	7	−12
Multiplicative inverse	$\frac{1}{15}$	$-\frac{1}{27}$	$\frac{9}{7}$	$-\frac{16}{9}$	$-\frac{23}{20}$	1	$-\frac{3}{2}$	$\frac{2}{7}$	$\frac{7}{5}$	$-\frac{1}{7}$	$-\frac{1}{12}$

3. (i) Trichotomy property (ii) Division property (iii) Subtraction property
(iv) Multiplicative property if c < 0 (v) Transitive property (vi) Multiplicative property if c > 0
(vii) Subtraction property (viii) Division property if c > 0

Exercise 1.3

1. (i) 4800 (ii) 785000 (iii) 8095000 (iv) 0.0289360 (v) 2346000 (vi) 0.780200

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2. (i) 2.79 (ii) 88.986 (iii) 15.35 (iv) 10.458
3. (i) –23.5 (ii) –45300 (iii) –17.79 (iv) 1172900
4. (i) 7.667 (ii) 18.33 (iii) 6.71 (iv) 3.667 (v) 0.86207 (vi) 0.009597
5. (i) 2 s.f (ii) 5 s.f (iii) 4 s.f (iv) 3 s.f
7. 21196.2 and 21196
8. 12700 and 12740

Exercise 1.4

1. (i) 49 (ii) 121 (iii) 361 (iv) 625 (v) 1369 (vi) 5625
2. (i) 1+2+3+4+5+6+5+4+3+2+1 (ii) 1+2+3+4+5+6+7+6+5+4+3+2+1 (iii) 1+2+3+4+3+2+1
   (iv) 1+2+3+4+5+4+3+2+1 (v) 1+2+3+2+1 (vi) 1+2+3+4+5+6+7+8+7+6+5+4+3+2+1

Exercise 1.5

1. (i) 28 (ii) 35 (iii) 50 (iv) 65 (v) 72 (vi) 88 (vii) 36
   (viii) 42 (ix) 171 (x) 227 (xi) 647 (xii) 490 2. (i) 37 (ii) 117 (iii) 225 (iv) 321

Exercise 1.6

1. (i) $\frac{7}{8}$ (ii) $\frac{11}{25}$ (iii) $\frac{14}{21}$ (iv) $1\frac{1}{6}$ (v) $\frac{26}{27}$ (vi) $3\frac{3}{5}$
2. (i) $\frac{4}{5}$ (ii) $\frac{13}{16}$ (iii) $2\frac{3}{8}$

Exercise 1.7

1. (i) 1.1 (ii) 0.8 (iii) 2.7 (iv) 1.2 (v) 1.3 (vi) 3.5
2. (i) 0.57 (ii) 0.72 (iii) 3.2 (iv) 4.53 (v) 25.47 (vi) 54.6
   (vii) 87.256 (viii) 0.0932 (ix) 48.73

Exercise 1.8

1. (i) 1.414 (ii) 1.732 (iii) 2.236 (iv) 2.646 (v) 3.317 (vi) 4.583
2. (i) 1.89 (ii) 2.52 (iii) 5.37 (iv) 7.95 (v) 30.35 (vi) 76.94

Exercise 1.9

1. (i) 3 (ii) 3 (iii) 3 (iv) 3 (v) 3 (vi) 4 (vii) 4 (viii) 4 (ix) 4 (x) 3 (xi) 4 (xii) 4

Exercise 1.10

1. 104976 cm^2
2. 1296 m^2
3. 62500 m^2 , Rs. 75000
4. 120 m
5. 2600 m
6. 350 trees
7. 464 m
8. 120m , 240 m
9. 187
10. 28 m
11. 350 m
12. 1000 m, Rs. 50, 000

Exercise 1.11

1. (i), (iii), (iv), (v)
2. (i) 9 (ii) 25 (iii) 24
3. (i) 2.744 (ii) 0.064 (iii) 0.512
4. (i) $\frac{1}{2}$ (ii) 33 (iii) 15
5. 64 cm^3
6. 512 cm^3
7. 15 cm
8. 9 m
9. 7 Cubical boxes

Review Exercise 1 (a)

1. (i) c (ii) (iii) b (iv) a (v) c (vi) c (vii) c (viii) c (ix) a
   (x) c (xi) c (xii) a (xiii) a (xiv) d
2. (i) terminating (ii) recurring (iii) terminating (iv) non- terminating
3. (i) rational (ii) rational (iii) rational (iv) irrational
4. (i) associative property w.r.t "+" (ii) additive inverse property

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(iii) Additive identity property (iv) Commutative property w.r.t. "×"

5. (i) 598200 (ii) 0.00256 (iii) 45.36 (iv) 3.13 (v) 2.14 (vi) 0.07895

6. (i) 104 *m* (ii) 104.0 = 104 *m*

7. Rs. 2800 8. (i) 2, 98 (ii) 3, 175 (iii) 3,112 (iv) 3,650

9. (i) $5\frac{1}{3}$ (ii) $4\frac{3}{17}$ (iii) $10\frac{1}{13}$ (iv) 0.231 (v) 0.452 (vi) 12.36

(vii) 0.5625 (viii) 6.17 (ix) 80.14 10. 402*m* 11. 14 marbles 12. (i) 12 (ii) 15

$\frac{6}{5}$

Exercise 1.12

1. (i) A = First five odd numbers. (ii) B = First five even numbers.
(iii) C = Set of integers between –10 and +10. (iv) D = Set of vowels of the English alphabet.
(v) E = Set of natural numbers equal to 100. (vi) F = Set of Prime numbers.

2. (i) A = {0, 1, 2, 3, …} (ii) B = {f, o, o, t, b, a, l, l} (iii) C = {0, 1, 2, 3, …}
(iv) D = {January, June, July} (v) E = {a, b, c, …, z} (vi) F = {5, 10, 15, 20, 25}

3. (i) $A = \{x \mid x \in w \in x \in 10\}$ (ii) $B = \{x \mid x \in D\}$
(iii) C = {*y* | *y* is a set of first five number divisible by 3} (iv) D = {*z* | *z* is a set of small English alphabet}
(v) E = {*x* | *x* is a set of days of the week} (vi) $F = \{x \mid x \in P \in 10 < x < 30\}$

4. Equivalent sets (ii) and (iii) 5. Equal sets (i) and (iii)

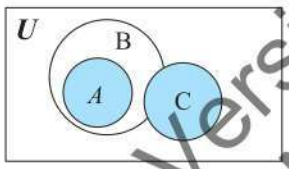
Exercise 1.13

1. (i) {ϕ} (ii) ϕ, {1} (iii) ϕ, {*a*}, {*b*}, {*a*, *b*}

2. (i) {ϕ, {–1}, {1}, {–1, 1}} (ii) {ϕ, {*a*}, {*b*}, {*c*}, {*a*, *b*}, {*a*, *c*}, {*b*, *c*}, {*a*, *b*, *c*}} (iii) {ϕ, {+}, {–}, {×}, {÷}, {+, –}, {+, ×}, {+, ÷}, {×, ÷}, {–, ×},{–, ÷},{+, –, ×}, {+, –, ÷}, {+, ×, ÷}, {–, ×, ÷}, {+, –, ×, ÷}}

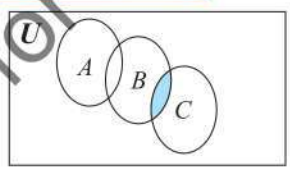
Exercise 1.15

3. (i)



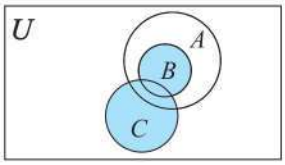
(A ∩ B) ∪ C

(ii)



(A ∪ B) ∩ C

(iii)



(A ∩ B) ∪ C

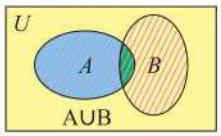
Exercise 1.16

1. (i) A ∪ B = {1, 2, 3, 4, 5, 6, 7, 8} (ii) C ∪ D = {*a*, *b*, *c*, *x*, *y*, *z*} (iii) E ∪ F = {3, 4, 6, 8, 9, 12}

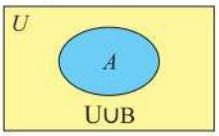
2. (i) A ∪ B = {0, 1, 2, 3} (ii) C ∪ D = {3, 9} (iii) E ∪ F = {c, d}

3. (i) A= {0, 1, 3, 5, 7, 9} (ii) $B' = \{0, 2, 4, 6, 8, 10\}$ (iii) $C' = \{1, 2, 4, 5, 7, 8, 10\}$
(iv) $A' = \{0, 1, 2, 3, 4\}$ 4. (i) A – B = {1, 3, 5} (ii) B – A = {8}

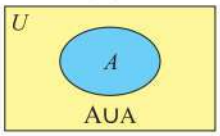
6. (i)



(ii)



(iii)



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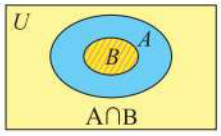

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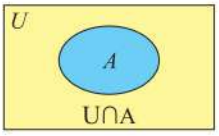
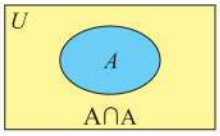


(vi)



7. 10 customers 8. 61 students

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Exercise 1.17

1. (i) 15 : 2 (ii) 1 : 10 (iii) 5 : 2 (iv) 3 : 4 (v) 3 : 2 (vi) 2 : 3 : 12
2. (i) P = 24 (ii) $x = \pm 12$ (iii) $m = 24$ (iv) $m = \pm 14$ (v) $x = 3$ (vi) $y = 9$ (vii) $p = 8$ (viii) $\frac{3}{2}$
3. 9 machines
4. 18 eggs
5. 45 labourers
6. 30 days
7. 280 m
8. 4 km
9. 50 minutes
10. 3.5 weeks
11. (i)

x	2	6	8	10	12	14
y	8	24	32	40	48	56

(ii)

x	5	10	15	20	25	30
y	100	200	300	400	500	600

12. (i)

x	1	2	4	8	10
y	80	40	20	10	8

(ii)

x	1	2	4	5	8	10
y	200	100	50	40	25	20

Exercise 1.18

1. 32 days
2. 36 *kg*
3. Rs.18, 864
4. Rs. 60, 000
5. 112 men
6. Rs. 10
7. 240 masons
8. 480 sweaters
9. 672 men
10. 400 bicycles

Exercise 1.19

1. US $ 385.25
2. UK £ 315.79
3. 1041.67 SAR
4. 23645.32 INR
5. 261.19 Australian Dollar
6. 3397.03 Chinese Yaun
7. 350.88 Canadian Dollar
8. 5718.95 Turkish Lira

Exercise 1.20

1. 10%
2. Rs. 2, 400
3. Rs. 3, 900
4. Rs. 1, 584
5. Discount Rs. 1600, Selling price Rs. 6400
6. Discount of food items = Rs. 187.50, Selling Price = Rs. 1062.50,
   Discount of other items = Rs. 150, Selling Price = Rs. 600
7. Rs. 575

Exercise 1.21

1. Rs. 4800
2. Rs. 4500
3. Rs. 14000
4. Rs. 75000
5. 3 %
6. $7\frac{1}{2}\%$
7. 3 years
8. 2 years

Exercise 1.22

1. Rs. 56250
2. Rs. 10900 included premium fee
3. Rs. 14170
4. Rs. 2992.50
5. Rs. 78039.36
6. Rs. 11970
7. 50,000

Exercise 1.23

1. Aslam's profit : Rs. 31500
   Akram's profit : Rs. 35000
2. Amina's profit : Rs. 3600
   Maryam's profit: Rs. 4800
3. 1st partner's share : Rs. 7320
   2nd partner's share: Rs. 4270
   Saeed's loss : Rs. 6000
4. Saad's loss : Rs. 3000
   Saud's loss : Rs. 4500
5. Akram's profit :Rs. 6000
   Asghar's profit :Rs. 8000
6. A's profit : Rs. 3000
   B's profit : Rs. 3600
   C's profit : Rs. 5400

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Exercise 1.24

1. Son's share Rs. 48000, Daughter's share Rs. 24000
2. Widow Rs. 100,000 Son Rs. 280,000, Daughter Rs. 140,000
3. Widow Rs. 87,500, Son Rs. 122,500, Daughter Rs. 61,250
4. Widow Rs. 9000, Son Rs. 42,000, Daughter Rs. 21,000
5. Son Rs 200,000 , Daughter Rs. 100,000
6. Son Rs. 40,000 , Daughter Rs. 20,000
7. Husband Rs. 45000, Son Rs. 67500 , Daughter Rs. 33750

Review Exercise 1 (b)

1. (i) c (ii) d (iii) c (iv) b (v) d (vi) a (vii) c (viii) d
   (ix) b (x) a (xi) b (xii) a (xiii) c (xiv) b (xv) a (xvi) c
3. (i) {ϕ,{1}, {2}, {3}, {4}, {1, 2}, {1, 3}, {1, 4}, {2, 3}, {2, 4}, {3, 4}, {1, 2, 3}, {1, 2, 4}, {1, 3, 4}, {2, 3, 4}, {1, 2, 3, 4}
   (ii) {ϕ, {a}, {b}, {c}, {a, b}, {a, c}, {b, c}, {a, b, c}
4. (i) ϕ, {2}, {4}, {6}, {2, 4}, {2, 6}, {4, 6}, {2, 4, 6}
   (ii) {ϕ, {+}, {–}, {×}, {÷}, {+, –}, {+ , ×}, {+, ÷},{– , ×},{–, ÷}, {×, ÷}, {+, –, ×}, {+, –, ÷}, {+, ×, ÷}, {–, ×, ÷}, {+, –, ×, ÷}}
6. 33 students
7. Rs. 5000
8. 4.5 litres
9. 127.5 ≈ 128 days
10. 20 lions
11. 120 items
12. 90 days
13. 28%
14. Rs.14550
15. Rs. 69105
16. Aslam's profit : Rs. 1400, Akram's profit : Rs. 120, Asghar's profit : 100
17. Widow Rs, 75000, Son Rs. 350000, Daughter Rs. 175000

Exercise 2.1

1. 28, 24, 20, 16, 12, 8
2. 1, 2, 4, 7, 11, 16, 22
3. 77, 66, 55, 44, 33, 22, 11
4. 50, 100, 150, 200, 250, 300, 350, 400
5. 2187, 729, 243, 81, 27, 9, 3
6. 2, 6, 11, 17, 24, 32, 41, 51

Exercise 2.2

1. (i) yes, d = 3 (ii) yes, d = –5 (iii) No (iv) yes, d = 0
2. (i) 1, 5, 9, … (ii) –3, –1, 1, … (iii) 3, 4, 5, … (iv) 2, 5, 8, …
3. (i) 7n – 1 (ii) 2n + 3 (iii) 5n – 1 (iv) 10n – 8
4. 31st
5. 21st
6. (i) 20, 26 (ii) 13, 17
7. (i) 87 (ii) 282
8. (i) 20 (ii) 20 (iii) 50 (iv) 51

Exercise 2.3

1. (i) No (ii) yes, r =10 (iii) yes, r = 2 (iv) yes, $r = \frac{1}{2}$ (v) No (vi) No
2. (i) 4^{n-1} (ii) $6(2)^{n-1}$ (iii) $15(3)^{n-1}$ (iv) $200\left(\frac{1}{2}\right)^{n-1}$
3. (i) 3, 6, 12, 24 (ii) 4, 8, 16, 32 (iii) $25, \frac{-25}{2}, \frac{25}{4}, \frac{-25}{8}$ (iv) $5, 1, \frac{1}{5}, \frac{1}{25}$
4. (i) 20000 (ii) 2187 (iii) 32 (iv) 3584
5. 61440 words
6. 1080°
7. 35 minutes

Exercise 2.4

1. Open sentences: i, ii, iv
   Closed sentences: iii, v, vi
2. Expressions: ii, iv
   Expressions: i, iii v, vi
3. Equations: iii, v
   Inequalities: i, ii, iv, vi
4. Expressions in (i), (ii), (v) and (vi) are polynomials and expressions in (iii) and (iv) are not polynomials.
5. (i) 7, –6 and 3 (ii) 5 and –3 (iii) 8, 2 and 5 (ii) 9, 3 and –2
6. (i) 1 (ii) 2 (iii) 3 (iv) 4
7. (i) Linear (ii) Quadratic (iii) Quadratic (iv) Linear (v) Cubic (vi) Biquadratic
   (vii) Biquadratic (viii) Quadratic

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Exercise 2.5

1. (i) $2x+1$ (ii) $4a^3+a^2-2a+4$ (iii) $3b^3+2ab^2$
2. (i) x^4-4x^3+1 (ii) $x-y+2$ (iii) $-5a^2b-2b^3$
3. $3a^4+4a^3-7a^3+7a-18$
4. $2x+2xy-y^2-3$
5. $3x^3+3x^2-3x-13$
6. (i) $3x^3+27$ (ii) $12x^4-34x^3+37x-17x+5$ (iii) $a^3+b^3+c^3-3abc$
7. $PQ=x^2y^2-x^3z-y^3z+xyz^2$, $QR=y^2z^2-xy^3-xz^3+x^2yz$ $PR=x^2z^2-x^3y-yz^3+xy^2z$
   $PQR=xyz(x^3+y^3+z^3)-(x^3y^3+y^3z^3+z^3x^3)$
8. (i) $x+3$ (ii) $x^2-3x-10$ (iii) x^2+xy+y^2 (iv) x^2-x-12
9. 3
10. P = 9

Exercise 2.6

1. (i) 2809 (ii) 5929 (iii) 259081 (iv) 1012036
2. (i) 3249 (ii) 9025 (iii) 357604 (iv) 3988009
3. (i) 2484 (ii) 39991 (iii) 999999 (iv) 0.9984
4. (i) 47 (ii) 11 (iii) 7

Exercise 2.7

1. $3(x-3y)$
2. $x(y+z)$
3. $2a(3b-7c)$
4. $3m^2n(mp-2)$
5. $15x(2x^2-3y)$
6. $17(x^2y^2-3)$
7. $x(4x+3x+2)$
8. $2p(p-2p^2+4)$
9. $xy(x^2-x+y)$
10. $7x(x^3-2xy+3y^3)$
11. $xyz(xyz-z+1)$
12. $4xy(x^2y-2+y^2)$
13. $xy^2(y^2-3y-6)$
14. $xyz(xy+xz+yz)$
15. $11xy(7x-3y-5xy)$
16. $5x^3(x^2+2x+3)$

Exercise 2.8

1. $(x-y)(a+b)$
2. $(a-3c)(2b-1)$
3. $(x-3)(x+2)$
4. $(x+5)(x-2)$
5. $(x+2)(x-7)$
6. $(x+3)(x-4)$
7. $(y-9)(y+3)$
8. $(x-8)(x-4)$
9. $(x-5)(x-7)$
10. $(x-13)(x-2)$
11. $(x-y)(a-b)$
12. $(y-a)(y-b)$
13. $(pq-rs)(a^2+b^2)$
14. $(x+y)(ab+cd)$

Exercise 2.9

1. $(x+7)^2$
2. $(3a+2b)^2$
3. $(4+3a)^2$
4. $(5x+8y)^2$
5. $7(a+6)$
6. $4(a+15)^2$
7. $(x-17)^2$
8. $(7x-6)^2$
9. $(x-9y)^2$
10. $(a^2-13)^2$
11. $2(a-16)^2$
12. $(1-3a^2b^2c^2)^2$
13. $x^2(2x-5yz)^2$
14. $\left(\frac{3}{4}x+\frac{2}{3}y\right)^2$
15. $\left(\frac{7}{8}x-\frac{8}{7}y\right)^2$
16. $\left(\frac{ax}{b}-\frac{cy}{d}\right)^2$
17. $4x^4(2x-1)^2$
18. $\left(a^2b^2x-c^2d^2y\right)^2$

Exercise 2.10

1. $(3-x)(3+x)$
2. $6(y-1)(y+1)$
3. $(4xy-5ab)(4xy+5ab)$
4. $xy(x-y)(x+y)$
5. $16(a-5b)(a+5b)$
6. $a^2b(b-8)(b+8)$
7. $7x(y-7)(y+7)$
8. $5x(x-3)(x+3)$
9. $11(a+b-3c)(a+b+3c)$
10. $3(5-a+b)(5+a-b)$
11. $\left(x-\frac{9}{5}+\frac{6}{5}y\right)\left(x-\frac{9}{5}-\frac{6}{5}y\right)$
12. $\left(9x+\frac{53}{4}\right)\left(x-\frac{3}{4}\right)$
13. $(11a-3b)(11b-3a)$
14. $\left(14x-\frac{23}{2}\right)\left(-2x+\frac{17}{2}\right)$
15. 121000
16. 348340
17. 1
18. 0.800

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Exercise 2.11

1. $(a+b-c)(a+b+c)$
2. $(a+3b+4c)(a+3b-4c)$
3. $(a+b+3ab)(a+b-3ab)$
4. $(x-2y-3xy)(x-2y+3xy)$
5. $(3a-b-4c)(3a-b+4c)$
6. $(a+b+2)(a-b-2)$

Exercise 2.12

1. $(x+3)(x+2)$
2. $(x+6)(x+4)$
3. $(x-7)(x-8)$
4. $(x+8)(x-5)$
5. $(2x+1)(2x+3)$
6. $(4x-y)(x+y)$
7. $(5x-9y)(2x+y)$
8. $(x+9)(x+3)$
9. $(x-10)(2x+3)$
10. $(3x-4y)(x-2y)$

Exercise 2.13

1. (i) $x^3+12x^2+48x+64$ (ii) $8m^3+12m^2+6m+1$ (iii) $a^3-6a^2b+12ab^2-8b^3$
   (iv) $125x^3-75x^2+15x-1$ (v) $8a^3+12a^2b+6ab^2+b^3$ (vi) $27x^3+270x^2+900x+1000$
   (vii) $8m^3+36m^2n+54mn^2+27n^3$ (viii) $64-144a+108a^2-27a^3$ (ix) $27x^3+81x^2y+81xy^2+27y^3$
   (x) $343+294b+84b^2+8b^3$ (xi) $64x^3-96x^2y+48xy^2-8y^3$ (xii) $125m^3+300m^2n+240mn^2+64n^3$
2. 488
3. 36
4. 322
5. 14
6. (i) 2197 (ii) 1092727 (iii) 0.970299

Exercise 2.14

1. (i) Base = 2, Exponent = 6 (ii) Base = 1, Exponent = 9 (iii) Base = t, Exponent = 9
   (iv) Base = m, Exponent = n
2. (i) 81 (ii) 343 (iii) 169 (iv) 256
3. (i) 5^4 (ii) 3^6 (iii) b^4 (iv) $u^3\times y^3$ (v) $7^2\times 11^4$ (vi) $a^3\times c^4\times d$
4. (i) 7^3 (ii) 2^9 (iii) 3^6 (iv) 5^5

Exercise 2.15

1. (i) $(10)^4$ (ii) $(2x-3y)^2$ (iii) 2^{12} (iv) $(ab)^6$ (v) 9^5 (vi) t^7 (vii) 3^0
   (viii) 2^5a^3 (ix) a^{13} (x) $(15)^0$ (xi) 5^3 (xii) xy
2. (i) $\frac{1}{4x^3y^7}$ (ii) $\frac{y^8}{x^4}$ (iii) $3x^3$ (iv) $\frac{1}{9}$ (v) $4x^4$ (vi) $729a^6$
   (vii) $4xy^3$ (viii) 4802 (ix) $9a^2b^2$

Exercise 2.16

1. (i) 51800000 (ii) 0.0000000009203 (iii) 216900000 (iv) 720000
   (v) 0.0000054 (vi) 0.0000000000054105
2. (i) 5.62×10^6 (ii) 1.4×10^{-6} (iii) 8.7402×10^8 (iv) 7.586×10^{15}
   (v) 4.4605×10^8 (vi) 9.95×10^{-11}

Exercise 2.17

1. (i) $y=-2x-7$, slope $=-2$, $y-$intrcept $=(0,-7)$
   (ii) $y=-\frac{1}{4}x+2$, slope $=-\frac{1}{4}$, $y-$intercept $=(0,2)$
   (iii) $y=8x+5$, slope $=8$, $y-$intercept $=(0,5)$
   (iv) $y=3x+4$, slope $=3$, $y-$intercept $=(0,4)$
   (v) $y=5x+3$, slope $=5$, $y-$intercept $=(0,3)$

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(vi) $y=-\frac{x}{4}+3$, slope $=-\frac{1}{4}$, $y-$intercept $=(0,3)$

2. $y=-22$ 3. $y=9$ 4. $x=-7$

5. (i) $y=m-s$ (ii) $y=2x-3$ (iii) $y=12-x$ (iv) $y=\sqrt{7-x}$

(v) $y=2(4x-7)$ (vi) $y=5-\frac{3}{2}x$ (vii) $y=\frac{1}{3}(ax+b-2)$ (viii) $y=\frac{x}{2}-3$

6. $b=p-a-c$ and $b=23$ 7. $h=\frac{2A}{a+b}$ and $h=4$ 8. $x=\frac{2}{3}y+3$ and $x=5$ 9. $y=95$

Exercise 2.18

1. J in quadrant IV, K in quadrant I, L in quadrant II, M in quadrant III,

Exercise 2.19

1. (i) $x-y=26$ (ii) $6x=y$ (iii) $x+3y=25$ (iv) $\frac{x+y}{x-y}=1$ (v) $2x+7=y$

2 (i) $x+y=10$, $x-y=2$ (ii) $2x+y=54$, $x+2y=92$ (iii) $x+2y=45$, $2x+3y=80$

Exercise 2.20

1. (i) $\{(2,-1)\}$ (ii) $\{(1,1)\}$ (iii) $\{(0,1)\}$ (iv) $\{(4,0)\}$

(v) $\left\{\left(\frac{30}{19},-\frac{18}{19}\right)\right\}$ (vi) $\{(5,2)\}$ 2. (i) $\left\{\left(\frac{8}{3},-\frac{1}{6}\right)\right\}$ (ii) $\left\{\left(\frac{7}{9},\frac{50}{9}\right)\right\}$

(iii) $\left\{\left(\frac{23}{25},-\frac{88}{25}\right)\right\}$ (iv) $\left\{\left(\frac{11}{19},-\frac{24}{19}\right)\right\}$ (v) $\left\{\left(\frac{5}{3},\frac{10}{3}\right)\right\}$ (vi) $\left\{\left(\frac{3}{5},\frac{9}{5}\right)\right\}$

3. (i) $\{(2,2)\}$ (ii) $\{(1,2)\}$ (iii) $\{(1,2)\}$ (iv) $\left\{\left(1,-\frac{1}{2}\right)\right\}$

Exercise 2.21

1. 2 2. –16 3. 3, 2 4. 11, 7 5. Samia's age = 14 years, Amna's Age = 7 years.

6. Ahsan's age = 61 years, Shakeel's Age = 13 years. 7. $\frac{3}{8}$

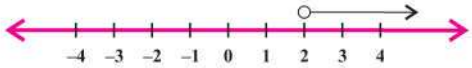
8. Price of melons = Rs. 50 per kg, price of mangoes = Rs. 80 per kg.

9. Football = Rs. 250, Basketball = Rs 180years. 10. $\frac{3}{5}$ 11. $\frac{4}{7}$

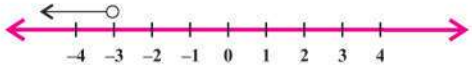
Exercise 2.22

1. (i) $\{x \mid x<5\}$ (ii) $\{x \mid x>-7\}$ (iii) $\{x \mid x\geq 4\}$ (iv) $\{x \mid x<-4\}$ (v) $\{x \mid x>6\}$ (vi) $\{x \mid x\geq -3\}$

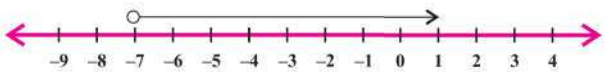
2. (i) $\{x \mid x>2\}$



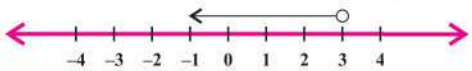
(ii) $\{x \mid x>-3\}$



(iii) $\{x \mid x>-7\}$



(iv) $\{x \mid x>3\}$



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Review Exercise 2

1. (i) b (ii) b (iii) d (iv) d (v) d (vi) b (vii) a (viii) d (ix) b (x) c (xi) c (xii) c (xiii) b (xiv) c (xv) d 2. (i) 26,32 (ii) 27,81 3. (i) 10,12,14,16 (ii) 6,18,54,162

4. 43 5. 1040 6. (i) $3x(y+2xy+3z)$ (ii) $(y^2-6)^2$ (iii) $(x^4+y^4)(x^2+y^2)(x+y)(x-y)$

(iv) $(x+9)(x-7)$ 7. (i) $11x^3-2x^2-35$ (ii) $10y^2-y-25$ (iii) x^2+x+2

8. (i) $\frac{16m^4}{n^4}$ (ii) $x^{12}y^8$ (iii) $6xy^2z^5$ 9. (i) $y=x+\frac{7}{3}$, Slope = 1, y-intercept = $\left(0,\frac{7}{3}\right)$

(ii) $y=\frac{1}{2}x-\frac{7}{4}$, Slope = $\frac{1}{2}$, y-intercept = $\left(0,-\frac{7}{4}\right)$ 10. $s=\frac{10-3t}{5}$ 11. $x=\frac{4}{y}$ and $x=6$

Exercise 3.1

1. (i) $13cm$ (ii) $2\sqrt{7}cm$ (iii) $12cm$ (iv) $25cm$ (v) $24cm$ (vi) $10cm$ 2. $7cm$

3. $8m$ 4. $10.12m$ 5. (i) is not right angled triangle 6. (i) $11cm$ (ii) $1cm$ (iii) $6\sqrt{5}m$

(iv) $10\sqrt{2}dm$ (v) $5\sqrt{5}dm$ 7. $\sqrt{a^2-25}$

Exercise 3.2

1. L = 5.59cm 2. L = 9.43cm 3. $r=3.8cm$ or $r=\frac{12}{\pi}$ 4. $x=900$ 5. L = 15.88units

6. 32.06cm 7. 15.71cm 8. Arc length = 63.55cm, Sector area = 889.77cm^2 9. Area =1848ft^2

10. Central angle(x) = 2600 11. Area = 64.108 Sq. inches

Exercise 3.3

1. (a) $75cm^2$ (b) $96cm^2$ (c) $180cm^2$ 2. $504cm^2$ 3. $2133\frac{1}{3}cm^3$, $1095.96cm^2$

4. $125cm^2$ 5. $160cm^2$ 6. $130cm^2$ 7. $320cm^3$ 8. $8cm$ 9. $54.93cm^2$

10. $21cm^2$ 11. $122\frac{2}{3}$ inch^2 12. $30cm^2$ 13. $61\frac{1}{3}cm^3$ 14. $2552.13m^3$

15. $5cm$ 16. $7.5cm$ 17. $5cm$

Exercise 3.4

1. (i) $154cm^2$ (ii) $98.56m^2$ (iii) $0.55m^2$ 2. (i) $3.5m$ (ii) $4.29m$ (iii) $4.95m$

3. (i) $817.6cm^3$ (ii) $2759.44cm^3$ (iii) $1437.33cm^3$ (iv) $164.70cm^3$ 4. (i) $4cm$, $268.19cm^3$

(ii) $0.44cm$, $0.36cm^3$ (iii) $7m$, $1437.33m^3$ 5. 1913030.67ℓ 6. (i) 4:1 (ii) 8:1

7. $V=2304\ \pi cm^3$, 13824(approx.) 8. $900cm$

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Exercise 3.5

1. $942.86cm^2$ 2. $8ft$ 3. 24cm 4. $84.86cm^2$ 5. $5.276cm$ 6. (a) $718.66cm^3$
(b) $2425.5cm^3$ (c) $65.49cm^3$ 7. $15.12cm$ 8. $242.83cm^3$

Exercise 3.6

1. (i) 6, 188.57, 113.14, 301.701 (ii) 5, 47.14, 28.28, 75.42 (iii) 23.32, 707.14, 254.57, 961.701
(iv) 7, 7.14, 10, 220 2. (i) $37.7cm^3$ (ii) $513.33cm^3$ (iii) $125.7cm^3$ (iv) $201.35cm^3$
3. $83.79cm^3$(approx.) 4. $75.43cm^3$(approx.) 5. $134.75cm^3$ 6. 44

Exercise 3.7

1. (a) $363.62ft^3$, $1620ft^2$ (b) $127cm^3$, $480cm^2$ 2. (a) $13.6cm^3$ (b) $109.44ft^3$ 3. (a) $2590cm^2$
(b) $11775cm^3$ 4. (a) $50.29cm^2$ (b) $10.48cm^3$

Review Exercise 3

1. (i) b (ii) a (iii) c (iv) a (v) b (vi) d (vii) b (viii) a (ix) c
(x) a (xi) b (xii) d (xiii) a (xiv) d (xv) c
3. (i) $137.31cm^3$ (ii) $37.71cm^3$ (iii) $1cm$ (iv) (a) $5.23cm$, $13.08cm^2$ (b) $11.78cm$, $44.16cm^2$ (v) 87.8^o
4. $4m^3$ 5. $6cm$ 6. $7.5cm$ 7. $595cm^3$ 8. $894348ft^2$ 9. Rs.27130 10. $509805891km^2$ 11. $2.43cm$
12. $0.2355m^2$ 13. $44.2cm$ 14. $20.93m^3$ 15. $1256kg$

Exercise 4.1

1. Congruent shapes: C and F ; P and Q ; M and R ; B and L.
Similar shapes: S and U ; A and Q ; E and J; D and T ; K and N.
2. Congruent sides: $\overline{AB}$ and $\overline{AD}$; $\overline{BC}$ and $\overline{CD}$.
Congruent angles: $\angle BAC$ and $\angle DAC$; $\angle ABC$ and $\angle ADC$; $\angle ACB$ and $\angle ACD$
3. ΔAOD and ΔBOC ; ΔAOB and ΔDOC 4. $\overline{AC} \cong \overline{LN}; \overline{BC} \cong \overline{MN}$
$\overline{OD} \cong \overline{OB}; \overline{AD} \cong \overline{BC}; \overline{OA} \cong \overline{OC}$, $\overline{AB} \cong \overline{CD}; \overline{OB} \cong \overline{OD}; \overline{OA} \cong \overline{OC}$

Review Exercise 4

1. (i) b (ii) d (iii) b (iv) c (v) b (vi) c (vii) b (viii) d

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Exercise 5.2

1.

Class limits	Tally marks	f	c.f
31 – 40	𝍸	3	3
41 – 50	𝍸	2	2 + 3 = 5
51 – 60	𝍸 𝍸	8	8 + 5 = 13
61 – 70	𝍸 𝍸 𝍸	11	11 + 13 = 24
71 – 80	𝍸	5	5 + 24 = 29
81 – 90	𝍸 𝍸	7	7 + 29 = 36
91 – 100	𝍸	4	4 + 36 = 40
Total		$\Sigma f = 40$	

2.

Class limits	Tally marks	f	c.f
41 – 50	𝍸	2	2
51 – 60	𝍸 𝍸	7	7 + 2 = 9
61 – 70	𝍸 𝍸	6	6 + 9 = 15
71 – 80	𝍸	4	4 + 15 = 19
81 – 90	𝍸 𝍸	6	6 + 19 = 25
91 – 100	𝍸 𝍸	10	10 + 25 = 35
101 – 110	𝍸	4	4 + 35 = 39
111 – 120	𝍸	4	4 + 39 = 43
121 – 130	𝍸	3	3 + 43 = 46
131 – 140	𝍸	2	2 + 46 = 48
141 – 150		0	0 + 48 = 48
151 – 160	𝍸	2	2 + 48 = 50
Total		$\Sigma f = 40$	

3.

Class Limits	Tally Marks	f	c.f	Class Boundaries
40 – 44	𝍸	1	1	39.5 – 44.5
45 – 49	𝍸	1	1 + 1 = 2	44.5 – 49.5
50 – 54	𝍸	1	1 + 2 = 3	49.5 – 54.5
55 – 59		0	0 + 3 = 3	54.5 – 59.5
60 – 64	𝍸	2	2 + 3 = 5	59.5 – 64.5
65 – 69	𝍸	3	3 + 5 = 8	64.5 – 69.5
70 – 74	𝍸	2	2 + 8 = 10	69.5 – 74.5
75 – 79	𝍸 𝍸	10	10 + 10 = 20	74.5 – 79.5
80 – 84	𝍸 𝍸	8	8 + 20 = 28	79.5 – 84.5
85 – 89	𝍸	4	4 + 28 = 32	84.5 – 89.5
90 – 94	𝍸	3	3 + 32 = 35	89.5 – 94.5
95 – 99	𝍸	1	1 + 35 = 36	94.5 – 99.5
Total		$\Sigma f = 36$		

4.

Class Limits	Tally Marks	f	Class Boundaries
01 – 10	𝍸	1	0.5 – 10.5
11 – 20	𝍸	5	10.5 – 20.5
21 – 30	𝍸 𝍸	8	20.5 – 30.5
31 – 40	𝍸 𝍸 𝍸	15	30.5 – 40.5
41 – 50	𝍸 𝍸 𝍸	11	40.5 – 50.5
51 – 60	𝍸 𝍸	7	50.5 – 60.5
61 – 70	𝍸	3	60.5 – 70.5
Total		$\Sigma f = 50$	

5.

Class Limits	Tally Marks	f	c.f	Class Boundaries
01 – 07	𝍸 𝍸	10	10	0.5 – 7.5
08 – 14	𝍸 𝍸	10	10 + 10 = 20	7.5 – 14.5
15 – 21	𝍸 𝍸	8	8 + 20 = 28	14.5 – 21.5
22 – 28	𝍸	1	1 + 28 = 29	21.5 – 28.5
29 – 35	𝍸	2	2 + 29 = 31	28.5 – 35.5
36 – 42	𝍸	2	2 + 31 = 33	35.5 – 42.5
43 – 49	𝍸	2	2 + 33 = 35	42.5 – 49.5
50 – 56		0	0 + 35 = 35	49.5 – 56.5
57 – 63		0	0 + 35 = 35	56.5 – 63.5
64 – 70	𝍸	1	1 + 35 = 36	63.5 – 70.5
Total		$\Sigma f = 36$		

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Exercise 5.3

2. (i) $\overline{X} = 5.4$ (ii) $\overline{X} = 4.64$ (iii) $\overline{X} = 4.56$ (iv) $\overline{X} = 7.27$ (v) $\overline{X} = -0.91$
3. (i) $\overline{X} = 190$ (ii) $\overline{X} = -70$ (iii) $\overline{X} = 133.6$ 4. (i) $\Sigma X = 3750$ (ii) $z = 17.8$
5. $\overline{X} = 21.048$ 6. The mass of 10^{th} students is $33.1kg$ 7. The total of height is $15m$
8. The length of 5^{th} ribbon is $18.5m$. 10. (i) $\overline{X} = 76.2, \tilde{X} = 75;\ \hat{X} =$ No mode (ii) $\overline{X} = 72.9, \tilde{X} = 76.5;\ \hat{X} = 80$
   (iii) $\overline{X} = 3.15, \tilde{X} = 3;\ \hat{X} = 3$ (iv) $\overline{X} = 105,\ \tilde{X} = 106;\ \hat{X} = 106, 107, 108$
11. $\overline{X} = 11.175;\ \tilde{X} = 10;\ \hat{X} = 1, 5\ and\ 10$ 13. Medium class = 3(Size of family) ; Medium class = 2(Size of family)
14. $\overline{X} = 7.04;\ \tilde{X} = 6.86;\ \hat{X} = 6.7$ 15. $\overline{X} = 105.81;\ \tilde{X} = 111.75;\ \hat{X} = 127.11$
16. $\overline{X} = 1954 marks;\ \tilde{X} = 19.55 marks;\ \hat{X} = 22.5 marks$
   (ii) Mean number of children per family is 2. 17. (i) Modal number of children per family is 2.
   (iii) Median number of children per family is 2.
18. $\overline{X} = 17.625;\ \tilde{X} = 17.56;\ \hat{X} = 17.44$ 19. $\overline{X}B > \overline{X}A$ (So, the class–B is better on average)
   $58.312 > 56.47$
20. $a = 17$

Exercise 5.4

2. (i) Range = 9 , Var = 8.76, S.D = 2.96 (ii) Range = 15, Var = 23.5, S.D = 4.85
   (iii) Range = 127, Var = 2080.27, S.D = 45.61 (iv) Range = 9.5, Var = 6.323, S.D = 2.515
3. Var = 1980, S.D = 44.5 4. $\Sigma X = 41.41$
5. $\overline{X}A = 34; \overline{X}B = 41$, Var(A) = 110.8; Var(B) = 11.6, S.D(A) = 10.53; S.D(B) = 3.41,
   (i) Student "B" is better on average (ii) Student "B" is more consistent in his performance

Exercise 5.5

1.

Sr.No.	$n(A)$	$n(S)$	$P(A)$	$P'(A)$
(i)	5	15	$\frac{5}{15} = \frac{1}{3}$	$\frac{2}{3}$
(ii)	17	25	$\frac{17}{25}$	$\frac{8}{25}$
(iii)	1	6	$\frac{1}{6}$	$\frac{5}{6}$

2. $P(4) = \frac{1}{6}; P'(4) = \frac{5}{6}$ 3. $P(red) = \frac{1}{4}; P'(red) = \frac{3}{4}$ 4. (i) $P(22) = \frac{1}{8}$ (ii) $P(26) = \frac{1}{8}$ (iii) $P'(25) = \frac{7}{8}$
5. (i) $P(5) = \frac{1}{6}$ (ii) $P'(5) = \frac{5}{6}$ (iii) $P(3) = \frac{1}{6}$ 6. P(not win the match) = 0.14
7. P(temperature will not be 0.45) = 0.55 8. (i) $P(red) = \frac{20}{100}$ (ii) $P(blue) = \frac{22}{100}$ (iii) $P(red\ or\ orange) = \frac{43}{100}$
   (iv) $P'(blue) = \frac{78}{100}$ (v) $P'(yellow) = \frac{87}{100}$ 9. (i) $P(boy) = \frac{7}{15}$ (ii) $P(girl) = \frac{8}{15}$

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(iii) $P'(boy) = \frac{8}{15}$ (iv) $P'(girl) = \frac{7}{15}$ 10. (i) $P(red) = \frac{5}{10}$ (ii) $P(blue) = \frac{2}{10}$ (iii) $P(not\ blue) = \frac{8}{10}$

(iv) $P(not\ green) = \frac{7}{10}$ (v) $P(not\ red) = \frac{5}{10}$ 11. (i) $P(B) = \frac{5}{12}$ (ii) $P(not\ B) = \frac{7}{12}$ (iii) $P(D) = \frac{1}{12}$

(iv) $P(A) = \frac{3}{12}$ (v) $P(not\ D) = \frac{11}{12}$ (vi) $P(not\ C) = \frac{9}{12}$ 12. $P(A \cup C) = 0.25 + 0.55 = 0.80; P(C \cup B) = 0.83$

13. $P(winning\ the\ game) = \frac{4}{6}$ 14. $P(4 \text{ or } 5) = \frac{2}{6}$ 15. (i) $P(red\ pin\ or\ blue\ pin) = \frac{5}{15}$

(ii) $P(green\ hairpin\ and\ black\ hairpin) = \frac{25}{225}$ 16. $P(head\ and\ digit\ 4) = \frac{1}{12}$

17. (i) $P(green\ and\ blue) = \frac{12}{169}$ (ii) $P(yellow\ and\ green\ marble) = \frac{20}{169}$ (iii) $P(different\ colours) = \frac{104}{169}$

18. (i) $P(red\ and\ black\ colour\ shirt) = \frac{2}{25}$ (ii) $P(green\ and\ blue\ colour\ shirt) = \frac{3}{50}$

(iii) $P(red\ and\ green\ colour\ shirt) = \frac{6}{25}$

19. (i) S = Sample space

	1	2	3	4	5	6
1	1,1	1,2	1,3	1,4	1,5	1,6
2	2,1	2,2	2,3	2,4	2,5	2,6
3	3,1	3,2	3,3	3,4	3,5	3,6
4	4,1	4,2	4,3	4,4	4,5	4,6
5	5,1	5,2	5,3	5,4	5,5	5,6
6	6,1	6,2	6,3	6,4	6,5	6,6

(ii) (a) $P(exactly\ two\ 6's) = \frac{1}{36}$ (b) $P(exactly\ two\ 4's) = \frac{1}{36}$ (c) $P(At\ least\ once\ times\ 4) = \frac{11}{36}$

20. (i)

Die/colour	1	2	3	4	5	6
Red (R)	(R,1)	(R,2)	(R,3)	(R,4)	(R,5)	(R,6)
Green (G)	(G,1)	(G,2)	(G,3)	(G,4)	(G,5)	(G,6)
Blue (B)	(B,1)	(B,2)	(B,3)	(B,4)	(B,5)	(B,6)
Yellow (Y)	(Y,1)	(Y,2)	(Y,3)	(Y,4)	(Y,5)	(Y,6)

(ii) (a) $P(green\ on\ the\ spinner\ and\ the\ digit\ '2'\ on\ die) = \frac{1}{24}$

(b) $P(green\ or\ yellow\ on\ the\ spinner\ and\ the\ digit\ '6'\ on\ die) = \frac{2}{24}$

(c) $P(red\ or\ blue\ on\ the\ spinner\ and\ 2,\ 3\ or\ 4\ on\ the\ die) = \frac{6}{24}$

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21. (i) S = Sample space

Die/colour	Small(S)	Medium(M)	Large(L)
Mango(M)	(M,S)	(M,M)	(M,L)
Strawberry(S)	(S,S)	(S,M)	(S,L)
Pistachio(P)	(P,S)	(P,M)	(P,L)

(ii) (a) *P(mango flavour with medium size)* $= \frac{1}{9}$ (b) *P(strawberry flavour with small or large size)* $= \frac{2}{9}$

(c) *P(pistachio flavour with medium or large)* $= \frac{2}{9}$

Review Exercise 5

1. (i) b (ii) c (iii) b (iv) b (v) a (vi) a (vii) c (viii) b (ix) c (x) b (xi) a (xii) c (xiii) c (xiv) a

3. $\overline{X}(Average\,height) = 56.8 inches$

4. (i) $\overline{X} = 55$ (ii) $\overline{X} = 244$

5. Average price of drinking water = $\overline{X} = 77$

6. (i) $\overline{X} = 5$, $\tilde{X} = 4$; $\hat{X} = 3$ (ii) $\overline{X} = 11$, $\tilde{X} = 11$; $\hat{X} = 12$

(iii) $\overline{X} = 5.5$, $\tilde{X} = 5$; $\hat{X} = 5$ (iv) $\overline{X} = 7$; $\tilde{X} = 7.5$; $\hat{X} = 8$ (v) $\overline{X} = 154.34$; $\tilde{X} = 154.3$; $\hat{X} = 152.3$

7. $\overline{X} = 15.7, \tilde{X} = 16.11$; $\hat{X} = 17.25$

8. Range = 27; Var = 51.29; S.D = 7.16

9. (i) Company "B" is better based on average. (ii) Company "A" is more consistent in his performance.

10. $P(1) = \frac{1}{6}; P'(1) = \frac{5}{6}$

11. (i) S = Sample Space = {1, 2, 4, 5, 5, 2, 3, 4, 2, 5} (ii) *P*(Even numbers) $= \frac{5}{10}$

(iii) $P(2) = \frac{3}{10}; P(1) = \frac{1}{10}; P(5) = \frac{3}{10}$

12. *P*(do not use glasses) $= \frac{18}{30}$; *P*(who uses glasses) $= \frac{12}{30}$

13. (i) $P(12) = \frac{1}{20}$ (ii) $P(20) = \frac{1}{20}$ (iii) $P(not\,20) = \frac{19}{20}$

14. (i) S = Sample space

Coin \ Die	1	2	3	4	5	6
H	(H,1)	(H_1,2)	(H_1,3)	(H_1,4)	(H_1,5)	(H_1,6)
T	(T,1)	(T_1,2)	(T_1,3)	(T_1,4)	(T_1,5)	(T_1,6)

(ii) (a) *P*(5 and tail) $= \frac{1}{12}$ (b) *P*(2, 4 or 6 and head) $= \frac{3}{12}$ (c) *P*(odd numbers and tail) $= \frac{3}{12}$

15. $P(A \cup B) = 0.85$

16. (i) $P(A \cap B)$ = *P*(both of them will be alive) = 0.697 (ii) *P*(both of them will be die) = 0.303

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GLOSSARY

Absolute value (or modulus): The absolute value (or modulus) of a real number x (represented by $|x|$) is the non-negative value of x without keeping in view its sign.

Additive identity: If 'a' is a real number, there is a unique number zero (0) called additive identity such that

$$a + 0 = 0 + a = a$$

Approximation error: It is numerical value, which tells us how far is the approximated value from the actual / accurate value.

s.f.: stands for significant figure.

d.p.: stands for decimal place.

Arc length (L): Arc length of a sector of circle $= \frac{x}{360^\circ} \times$ circumference

Arc length: It is the distance along the curved line that makes up the arc.

Arc: A continuous part of the boundary/circumference of a circle is called an arc.

Area of a Sector of Circle: Area of a sector of circle $= \frac{x}{360^\circ} \times$ area of circle

Arithmetic sequence: A sequence of numbers such that the difference 'd' between each consecutive term is a constant is called arithmetic sequence.

$a_1, a_1 + d, a_1 + 2d, a_1 + 3d, \ldots$ Where a_1 is the first term of the sequence.

Cartesian plane: It is formed by two perpendicular number lines and is used to plot ordered pairs on it.

Centre of rotation: A rotation turns each point on a preimage around a fixed point, called the centre of rotation, a given angle measure.

Chord of the circle: A line segment joining two points on a circle is called a chord of the circle.

Circle & Centre of the Circle: A circle is a plane figure bounded by one curved line, and such that all straight lines drawn from a certain point within it to the bounding line, are equal. That point is called the Centre of the Circle.

Closure property: If we take two real numbers, then their sum and product is also a real number. It is called closure property.

Coefficient: In a term the number multiplied by the variable is called the coefficient of the variable as well as constant.

Complement of a set: It is defined as a set that contains the elements present in the universal set but not in set A.

Compound proportion: The relationship between two or more proportions is known as compound proportion.

Continued proportion: If three quantities a, b and c are written as: $a : b :: b : c$ then these quantities are in continued proportion and b is called the mean proportional.

Cube of a number: means to multiply the number by itself three times.

Cube root of a number: It is denoted by $\sqrt[3]{\ }$, is a number such that $a^3 = x$. i.e., $a = \sqrt[3]{x}$

Cumulative frequency: It is calculated by adding each frequency from a frequency distribution to the sum of its predecessors.

Data: The collection of information in the form of facts and figures is called data

Difference of two sets: We can write A-B as A\B for difference of two sets.

Direct proportion: Direct proportion is a relation in which increases / decreases one quantity causes proportional increases / decreases in the other quantity.

Enlargement: Enlarge a figure with the given scale factor (positive or negative).

- The enlargement of a figure with the given scale factor (positive or negative).
- Locating the centre and scale factor of enlargement given the original figure and its enlargement.

Equal sets: Two sets are equal if they contain the same elements.

Equation: A sentence that shows equality (=) between two expressions is called an equation. The equation has fixed and singular solution.

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Equivalent sets: Equivalent sets are the sets with an equal number of elements. These sets do not have exactly the same elements, just these sets have the same number of elements.

Event: The result of an experiment is called an event.

Experimental probability: It is obtained by performing an experiment.

Factorization: Expressing polynomials as product of two or more polynomials that cannot be further expressed as product of factors is called factorization.

Foreign currency exchange rate: It is a price that represents how much it costs to buy the currency of one country using the currency of another country.

Fourth proportional: If four quantities a, b, c and d are written as: $a : b :: c : d$ then a is called the first, b is called the second, c is called the third and d is called the fourth proportional.

Frequency distribution: To write a data in the form of a table in such a way that the frequency of each class can be observed at once is called frequency distribution.

Frequency: The number of times a value occurs in a data is called the frequency of that value.

Geometric sequence: It is a sequence in which each term is obtained by multiplying a non-zero constant 'r' to the previous term.

$a_1, a_1r, a_1r^2, a_1r^3, \ldots$ Where 'a_1' is he first term and 'r' the common ratio.

Grouped data: Data which is arranged in systematic order/groups or classes in called grouped data.

Identity Congruence: Every triangle is congruent to itself (in the correspondence in which its sides and angles correspond to themselves). Such a congruence is called "Identity Congruence".

Inequality: It is a statement that contains one of the symbols: $<, >, \leq$ *or* $\geq$. The inequality has infinite answers.

Inheritance: When a person dies, then the assets left by him are called inheritance and it is distributed among his legal inheritors according to Islamic Shariah Law.

Inverse proportion: Inverse proportion is a relation in which if one quantity increases / decreases then other quantity decreases / increases proportionally.

Irrational numbers: The numbers which are not rational are called irrational numbers. The set of irrational numbers is represented by Q′.

Kinds of proportion: (i) Direct proportion (ii) Inverse proportion

Life insurance: It is an agreement between the policy owner and the insurance company for an agreed time period.

Linear equation: If a, b and c are real numbers and a and b are not both zero, then $ax + by = c$ is called linear equation in two variables.

Loss percentage: $$\text{Loss percentage} = \frac{\text{loss}}{\text{cost price}} \times 100$$

Loss: If the cost price (C.P.) is higher than the selling price (S.P.), then loss occurs.

Markup: When we borrow money from bank to run a business, the bank in return receives some extra amount along with the actual money given. This extra money which the bank receives is known as markup.

Mean: It is calculated by adding all the values and then dividing by the number of values.

Median of the data: If a data is arranged in ascending or descending order then median of the data is:

- The middle value of the data, if it consists of odd number of values.
- The mean of the two middle values of the data, if the number of values in data is even.

Members or elements: The objects of a set are called its members or elements.

Mode: It is the value that occurs most frequently in a data.

Multiplicative identity: '1', is called multiplicative identity, such that $a.1 = a = 1.a$

Number sequence: is a list of numbers arranged in an order.

Open sentence: It is neither true nor false until the variables or unknowns have been replaced by specific values.

Ordered pair: It is a composition of two elements that are separated by a comma and written inside the parentheses.

Partnership: A business in which two or more persons run the business and they are responsible for the profit and loss is called the partnership.

Percentage discount: $$\text{Percentage discount} = \frac{\text{discount}}{\text{marked price}} \times 100$$

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Perfect cube: is a number that is the result of multiplying an integer by itself three times. In other words, it is an integer to the third power of another integer.

Period: The time for which a particular amount is invested in a business is known as period.

Pie chart: It is used for comparison of values of different items by making the sectors of a circle. Angles are used to represent the sectors

Polynomial expression or simply a polynomial: is an algebraic expression consisting of one or more terms in each of which the exponent of the variable is zero or a positive integer.

Polynomials: In which the highest exponent or sum of exponents is always 2.

Position to term rule: defines the value of each term with respect to its position.

Principal amount: The amount we borrow or deposit in the bank is called principal amount.

Probability: It tells us how likely it is that an event will occur.

Profit / markup rate: The rate at which the bank gives share to its account holders is known as profit / markup rate. It is expressed in percentage.

Profit / markup: Profit / markup = Principal × Time × Rate

Profit percentage: Profit percentage $= \frac{\text{profit}}{\text{cost price}} \times 100$

Profit: If the selling price (S.P.) is higher than the cost price (C.P.) then profit occurs.

Properties of real numbers: It holds the following properties:

(i) Closure property (ii) Commutative property (iii) Associative property (iv) Identity

(v) Inverse (vi) If we cannot find the number whose square is $\sqrt{x}$, then x is an irrational number.

Proportion: The relation of equality of two ratios is called proportion, It is denoted by "::".

Pyramid: A 3D shape with a flat polygon base and three or more triangular sides converging at the top, is called Pyramid.

Radical sign: Square root and square are inverse of each other. The symbol used for square root is " $\sqrt{\ }$ ". It is called radical sign.

Ratio: It is a comparison of two quantities of same kind. Ratio is denoted by ":" e.g., ratio "a" is to "b" is written as $a:b$, ratio "c" is to "d" is written as $c:d$.

Rational numbers: The members which can be written in the form of $\frac{p}{q}, q \neq 0, p, q \in z$ are called rational numbers.

Real numbers: All the numbers (N, W, Z, all rational and irrational numbers) are real numbers. It is represented by $\mathbb{R}$.

Recurring decimal numbers: The decimal numbers in which one digit or group of digits repeat again and again in its decimal part are called recurring decimal numbers.

Rotation: Rotate an object and find the centre of rotation by construction

Sample space: The set of all possible outcomes is called sample space. It is denoted by "S".

Scientific Notation: It is a way of expressing very large and very small numbers conveniently.

Secant line: A straight line that intersects a circle in two points is called a secant line.

Sector of a circle: It is defined as the portion of a circle that is enclosed between its two radii and the arc adjoining them.

Sequence: It goes on forever is called an infinite sequence. Otherwise, it is called finite sequence.

Set expression: A set can be expressed in three ways.

(a) Tabular Form or Roster Form (b) Descriptive Form (c) Set Builder Notation

Similar Shapes: Two shapes are said to be mathematically similar if all of the angles in the shapes are equal, but the shapes are not necessarily of the same size.

Square of the number: When a number is multiplied by itself then the product is known as the square of the number i.e., the square of x is x^2

Surface area of a hemisphere: Surface area of a hemisphere $= \frac{1}{2} \times$ surface area of sphere + area of base

Symbol of cube root: It is $\sqrt[3]{\ }$ remember that 3 is the part of symbol.

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Term to term rule: It is not helpful if you want to find a term that is far away from the ones that are known. Position to term rule in more powerful.

Terminating decimal numbers: These are the decimals which have finite digits in its decimal parts.

Theoretical probability: It is obtained without performing any experiment.

Total surface area of a cone: Total surface area of a cone = Base area + curved surface area

$$= \pi r^2 + \pi r \ell$$
$$= \pi r(r + \ell)$$

Types of Rational numbers: These are two:

(i) Terminating decimals (ii) Recurring

Universal set: A set that consists of all the elements of the sets under consideration, including its own elements is called universal set. It is denoted by the symbol U.

Vehicle insurance: It provides a protection against risks to the vehicle. The amount of policy in this case depends upon the actual value of the vehicle.

Venn diagram: It is an illustration that uses circles to show the relationships among sets in a perspective way. In Venn diagram, a universal set is usually represented by a rectangle and its subsets are represented by closed figures inside the rectangle.

Volume of a cone: Volume of a cone = $\frac{1}{3}\pi r^2 h$

Volume of hemi-sphere: Volume of hemi-sphere = $\frac{\text{Volume of sphere}}{2}$

Volume of Pyramid: Volume of Pyramid = $\frac{1}{3} \times$ volume of prism

Symbols / Notations

Symbol	Stands for	Symbol	Stands for
$<$	is less than	:	ratio
$>$	is greater than	: :	is proportional to
$\leq$	is less than or equal to	|	tally mark
$\geq$	is greater than or equal to	$\sum$	summation
$=$	is equal to	$\overline{AB}$	line segment *AB*
$\neq$	is not equal to	$\overrightarrow{AB}$	ray *AB*
$\not<$	is not less than	$\overleftrightarrow{AB}$	line
$\not>$	is not greater than	$\angle$	angle
$\in$	belongs to	Δ	triangle
$\notin$	not belongs to	$\sim$	is similar to
$\forall$	for all	$\cong$	is congruent to
$\sqrt{\ }$	square root	$\approx$	is approximately equal to
$\Longrightarrow$	implies that	$\parallel$	is parallel to
$\cup$	union	$\overset{\frown}{AB}$	arc *AB*
$\cap$	intersection	$\leftrightarrow$	correspondence
$\because$	because / as	%	percentage
$\therefore$	therefore / so	ϕ or $\{\}$	the empty set / the null set
U	universal set	|	such that

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