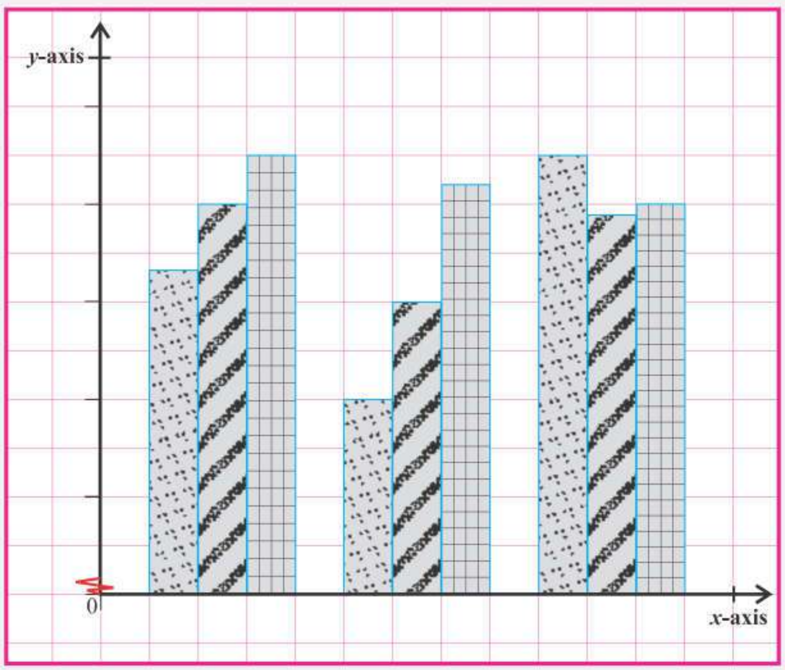
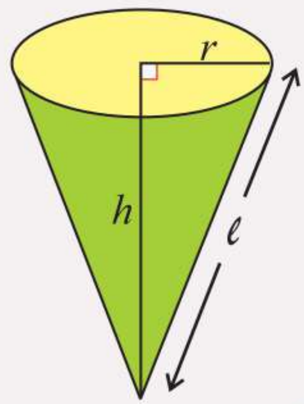
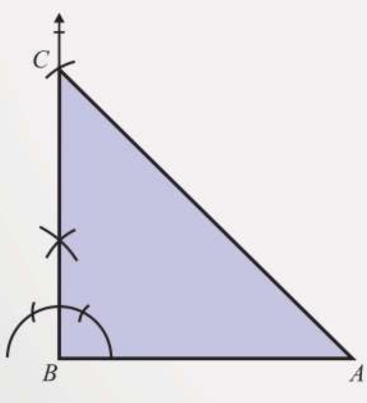
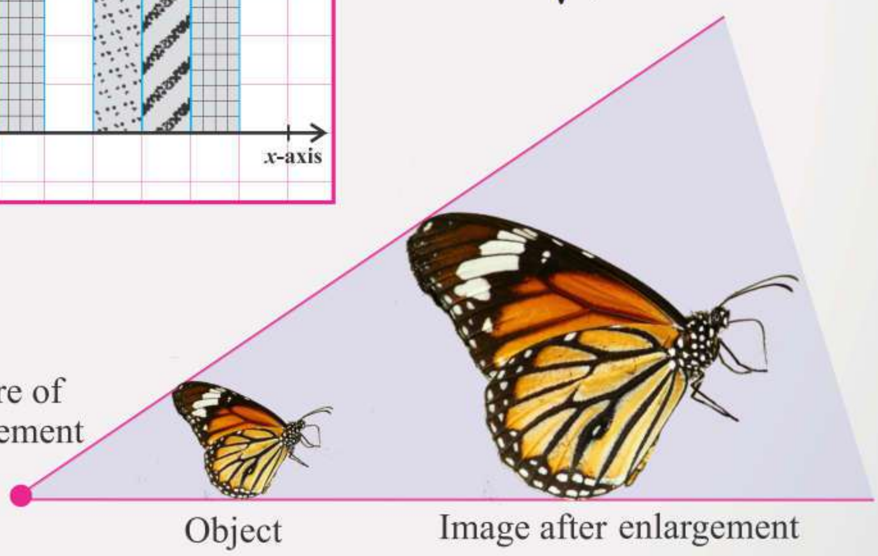
MATHEMATICS 8

Based on Single National Curriculum 2022









Punjab Curriculum and Textbook Board, Lahore

بِسْمِ اللّٰهِ الرَّحْمٰنِ الرَّحِيْمِ

(In the Name of Allah, the Most Compassionate, the Most Merciful)

MATHEMATICS

8

Based on Single National Curriculum 2022

ONE NATION, ONE CURRICULUM

PUNJAB CURRICULUM AND TEXTBOOK BOARD, LAHORE

This textbook is based on Single National Curriculum 2022 and has been approved by the Board.

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Experimental Edition

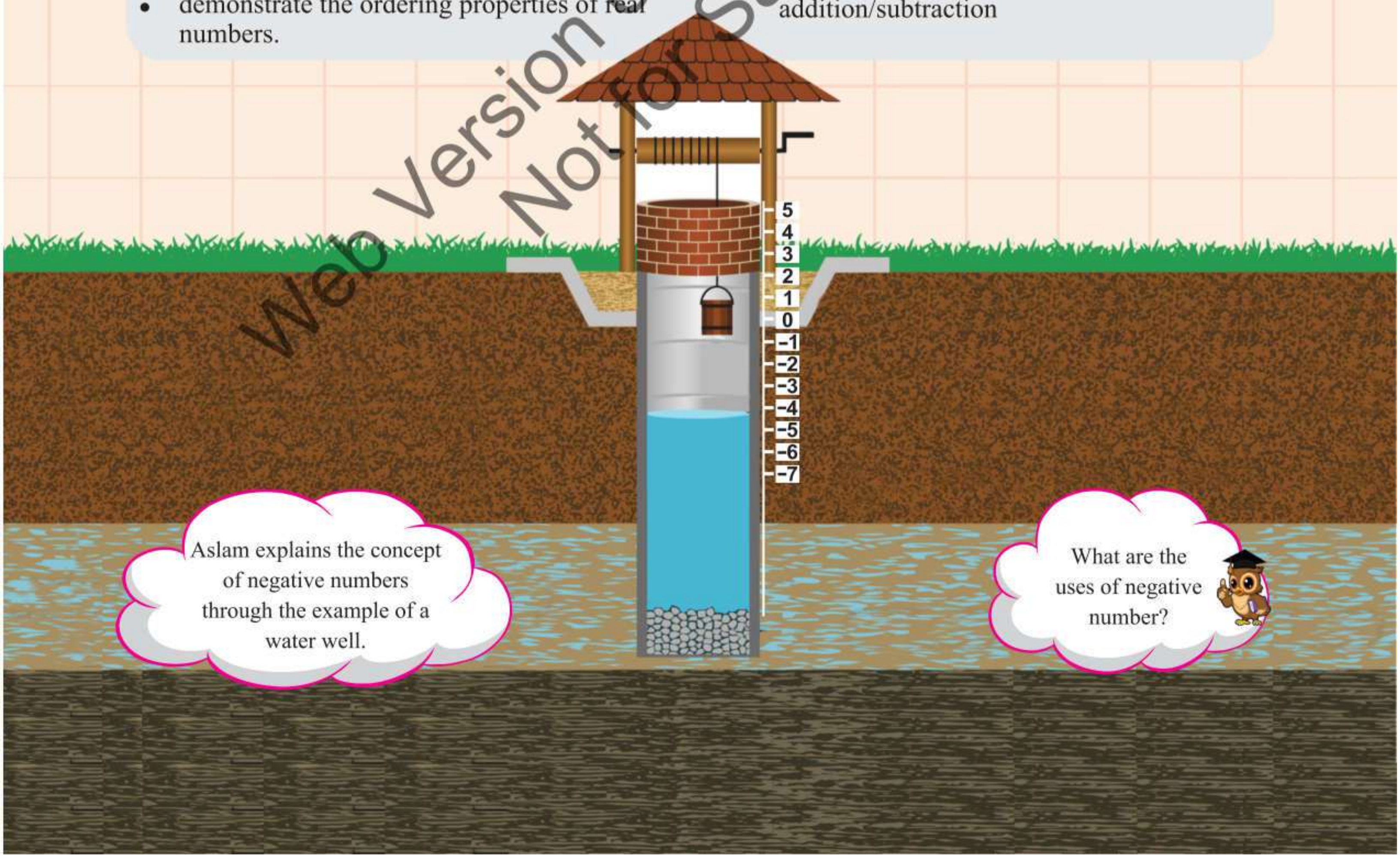
Domain 1 Numbers and Operations

Sub-Domain (i): Real Numbers

Students' Learning Outcomes

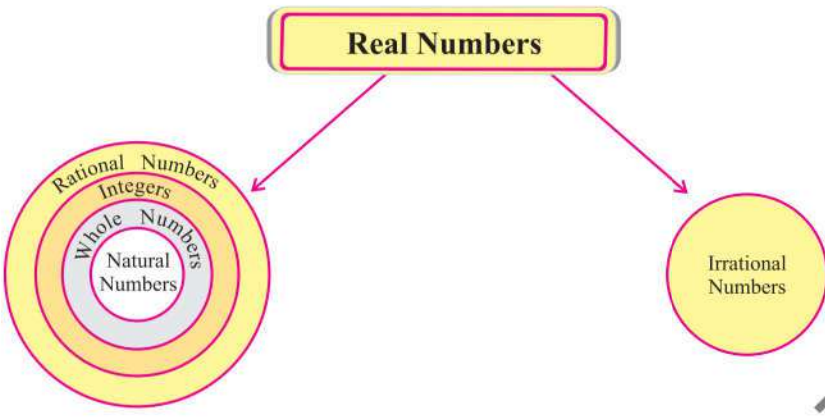
After completing this sub-domain, the students will be able to:

- differentiate rational and irrational numbers
- represent real numbers on a number line
- demonstrate decimal numbers as terminating, non-terminating, recurring and non-recurring
- solve real life situations/word problems involving calculation with decimals and fractions
- identify the absolute value of a real number
- demonstrate the ordering properties of real numbers.
- demonstrate the properties of real numbers and their subsets with respect to addition and multiplication:
  - ❖ closure property
  - ❖ associative property
  - ❖ existence of identity element
  - ❖ existence of inverses
  - ❖ commutative property
- distributive property of multiplication over addition/subtraction



1.1.1 Real Numbers ($\mathbb{R}$)

Numbers are the roots of Mathematics. We use these numbers in our daily life. Before discussing real numbers, we discuss some other numbers.



Need to know!

All the numbers (N, W, Z, all rational and irrational numbers) are real numbers. The set of real numbers is represented by $\mathbb{R}$.

(i) Natural Numbers (N)

Natural numbers start from digit 1 and the set of natural numbers is represented by N. i.e.,

N = {1, 2, 3, 4, …}

(ii) Whole Numbers (W)

If we include zero (0) in natural numbers, then these numbers become whole numbers and the set of whole numbers is represented by W. i.e., W = {0, 1, 2, 3, …}

(iii) Integers (Z)

If we include negative numbers in whole numbers, then these numbers become integers and the set of integers is represented by Z. i.e., Z = {0, ±1, ±2, ±3, ±4, …}

(iv) Rational Numbers (Q)

The numbers which can be written in the form of $\frac{p}{q}$, where $p, q \in Z$, and $q \neq 0$, are called rational numbers. For example, $\frac{7}{9}, \frac{-5}{3}, \frac{0}{11}, 28, \frac{12}{17}, \frac{6}{23}, \frac{18}{26}, 15$ etc. are all rational numbers. The set of rational numbers is represented by Q. Rational numbers can also be represented in decimal form which has two types.

Key fact!

Every rational number can also be written into decimal form.

(a) Terminating Decimal Numbers

Terminating decimal numbers are the decimals which have finite number of digits in its decimal part. For example: 0.4, 3.14, 0.375 etc. All terminating decimal numbers can also be written in fraction. For example: $0.4 = \frac{2}{5}$, $3.14 = \frac{157}{50}$, and $0.375 = \frac{3}{8}$ etc.

(b) Non-Terminating and Recurring Decimal Numbers

The decimal numbers in which one digit or group of digits repeat again and again infinite times in its decimal part are called recurring decimal numbers. It can also be written in fraction. For example:

$\frac{2}{9} = 0.22222,...$; $\frac{4}{11} = 0.363636,...$; $\frac{43}{99} = 0.434343,...$; $\frac{156}{9} = 17.33333,...$; $\frac{22}{7} = 3.142857142857,...$

(v) Irrational Numbers (Q′)

The numbers which are not rational are called irrational numbers. The set of irrational numbers is represented by Q′.

For example: $\sqrt{2} = 1.41421356,...;\quad \pi = 3.141592654,...;$

All the non-terminating and non-recurring decimal numbers are irrational numbers.

Important Note!

Value of π and $\frac{22}{7}$ are not equal. $\frac{22}{7}$ is a rational number but value of π and $\frac{22}{7}$ are very very close but we cannot write them equal. It can be written as $\pi \approx \frac{22}{7}$.

Real Numbers
Real numbers are the union of rational and irrational numbers.

- Rational Numbers
  - Terminating Decimal Numbers: 0.45, 7.215
  - Non-Terminating and Recurring Decimal Numbers: 0.8181…, 0.373737…
- Irrational Numbers
  - Non-Terminating and Non-Recurring Decimal Numbers: 5.23456789… , 0.0289537…

Key facts!

1. Every integer is a rational number.
2. All the rational numbers can be written in decimal fraction but all decimal fractions are not rational numbers.

Example 1: Separate rational and irrational numbers.

(i)	$\sqrt{17}$	(ii)	8.04	(iii)	π	(iv)	$\frac{22}{7}$
(v)	$\sqrt{2}$	(vi)	63	(vii)	0.33333…	(viii)	0

Solution:

(i) Since 17 is not perfect square and $\sqrt{17}$ is non-terminating non-recurring, it is irrational.

(ii) Since 8.04 is terminating decimal number. So, it is a rational number.

(iii) Since the value of π is non-terminating and non-recurring, it is an irrational number.

(iv) $\frac{22}{7}$ is in the form of rational number, it is a rational number.

(v) $\sqrt{2}$ is an irrational number.

(vi) 63 can be written as $\frac{63}{1}$ which is in $\frac{p}{q}$ form so, it is a rational number.

(vii) 0.3333… is recurring decimal number so, it can be written as $\frac{1}{3}$. Therefore, it is a rational number.

(viii) 0 can be written as $\frac{0}{1}$, so it is a rational number.

1.1.2 Representation of Real Numbers on Number Line

To represent the real number $\frac{\ell}{m}$ on the number line, we divide each unit into 'm' equal parts.

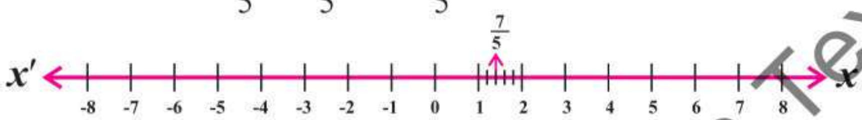
For positive number e.g., $\frac{\ell}{m}$, the "ℓth" point of division is represented to the right of the origin.

For negative number e.g., $-\frac{\ell}{m}$, the "ℓth" point of division is represented to the left side of the origin.

Example 2: Represent the following numbers on the number line.

(i) $\frac{7}{5}$ (ii) $-\frac{1}{3}$

Solution: (i) $\frac{7}{5} = 1\frac{2}{5} = 1 + \frac{2}{5}$

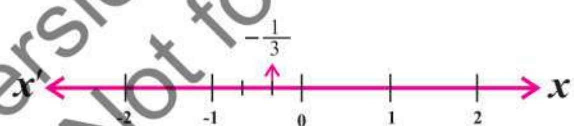


To represent the rational number $\frac{7}{5}$ on the number line, divide the distance between 1 and 2 into five equal parts. Take two parts to the right from 1.

(ii) $-\frac{1}{3}$

To represent the rational number $-\frac{1}{3}$ on the number line, divide the distance between 0 and -1 into three equal parts.

Take 1 part to the left from 0



Keep in mind!

A real number line always goes on both directions from zero(0) i.e., left and right directions.

Keep in mind!

$\frac{7}{5}$ lies between 1 and 2.

1.1.3 Absolute Value of Real Number

The absolute value (or modulus) of a real number x (represented by $|x|$) is the non-negative value of x. For example, the absolute value of 4 is 4 and absolute value of -4 is also 4. We can write it as:

$$|4| = 4$$

or $$|-4| = 4$$

The absolute value of a real number may be thought of its distance from zero along real number line. Furthermore, the absolute value of difference of two real numbers is the distance between them. For example, the absolute difference between -3 and 2 is:

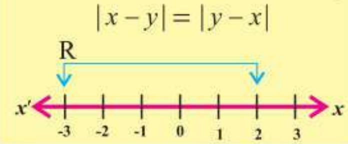
$$|-3-2| = |-5| = 5$$

or $$|2-(-3)| = |2+3| = |5| = 5$$

So, -3 and 2 are at a distance of 5 units apart.

Showing the absolute difference between two real numbers x and y.

$|x-y| = |y-x|$



Try yourself!

The temperature of city "A" is 33°C and city "B" is 40°C. Can you find out the absolute difference between them?

1.1.4 Real Life Situations Involving Fractions and Decimals

Example 3: How many pieces of $\frac{1}{4}$ metre of ribbon can be cut from a ribbon which is $\frac{15}{2}$ metres long?

Solution: The length of the ribbon $= \frac{15}{2}$ metres

The required length of each ribbon $= \frac{1}{4}$ metre

To find out the number of pieces of ribbon of required length, we will have to divide $\frac{15}{2}$ by $\frac{1}{4}$.

$$= \frac{15}{2} \div \frac{1}{4}$$

Number of pieces of ribbon $= \frac{15}{2} \times 4 = \frac{60}{2} = 30$

Thus, the total length of ribbon can be cut into 30 equal parts of length $\frac{1}{4}$ metre.

Example 4: Azhar ran 325.650 *km* on a jogging track in the first month and 410.350 *km* in the second month. Find the total distance he covered in both the months. Also tell how many kilometres did he cover per day in the first month?

Solution: To find the total distance, we will have to add both the distances.

$325.650 + 410.350 = 736$ *km*

Per day distance (*km*) covered in the first month $= \frac{325.650}{30} = 10.855\,km$

So, Azhar covered total 736 km in both months

and 10.855 kilometres covered per day in the first month.

Exercise 1.1

1. Identify which of the following fractions are terminating and which are recurring decimal fractions?

(i) $\frac{4}{6}$ (ii) $\frac{11}{12}$ (iii) $\frac{1}{3}$ (iv) $\frac{2}{5}$ (v) $\frac{8}{11}$ (vi) $\frac{5}{12}$

(vii) $\frac{5}{7}$ (viii) $\frac{6}{10}$ (ix) $\frac{16}{20}$ (x) $\frac{9}{10}$ (xi) $\frac{1}{8}$ (xii) $\frac{14}{20}$

(xiii) $\frac{2}{3}$ (xiv) $\frac{4}{5}$ (xv) $\frac{1}{4}$ (xvi) $\frac{7}{9}$ (xvii) $\frac{8}{10}$ (xviii) $\frac{3}{5}$

2. Separate rational and irrational numbers from the following:

(i) $\sqrt{5}$ (ii) $\frac{1}{7}$ (iii) 2.152 (iv) 0.818281828182….. (v) 2.751

(vi) $\frac{22}{7}$ (vii) $\sqrt{14}$ (viii) $\sqrt{25}$

3. Represent the following numbers on the number line.

(i) $\frac{21}{4}$ (ii) $-\frac{53}{3}$ (iii) $\sqrt{11}$ (iv) $\frac{25}{2}$ (v) $-\frac{5}{7}$ (vi) $\sqrt{31}$

4. Find the absolute value of each of the following:

(i) 15 (ii) -21 (iii) -36 (iv) -9 (v) -100 (vi) 88

5. Fine the absolute difference between each of the following:

(i) -7 and -2 (ii) 8 and -3 (iii) -5 and 1 (iv) 2 and 3

6. $50\frac{3}{4}$ kilograms of rice is to be packed equally in $5\frac{1}{2}$ kilograms packets. Find:

(a) how many packets of rice will be packed?

(b) how much rice will be there in 8 packets of mass $5\frac{1}{2}$ kilograms?

7. To decorate a birthday party, Zara used ribbons of two colours. The length of blue ribbon is $15\frac{2}{3}$ metres. The length of green ribbon is $\frac{4}{5}$ times of the length of blue ribbon.

(a) What is the length of green ribbon?

(b) Find the total length of both the ribbons.

8. If 230.75 kilograms of jaggery powder is to be packed equally in the packets, each packet contains 2.5 kg of jaggery powder. Find the number of required packets. Also find how much jaggery powder will be there in 15 packets of mass 2.5 kg.

9. Zeeshan wants to fill 137.5 litres of petrol in bottles. Find the number of bottles if:

(a) The capacity of each bottle is 5.5 litres.

(b) The capacity of each bottle is 6.25 litres.

(c) Also tell how much petrol will be there in 18 bottles of capacity 6.25 litres each.

1.1.5 Properties of Real numbers

The following properties are held in real numbers:

Properties	Examples
(i) Closure Property w.r.t Addition ◎ If we take any two real numbers, then their sum is also a real number. It is called closure property w.r.t addition. $\forall\ a, b \in \mathbb{R} \Rightarrow a + b \in \mathbb{R}$	◎ -5 and $11 \in \mathbb{R}$, $-5 + 11 = 6 \in \mathbb{R}$
(ii) Closure Property w.r.t Multiplication ◎ If we take any two real numbers, then their product is also a real number. It is called closure property w.r.t multiplication. $\forall\ a, b \in \mathbb{R} \Rightarrow a \,.\, b \in \mathbb{R}$	◎ -5 and $11 \in \mathbb{R}$, $(-5) \times (11) = -55 \in \mathbb{R}$

Properties	Examples
(iii) Associative Property w.r.t Addition ◎ If a , b and c are any three real numbers, then $(a+b)+c=a+(b+c)$	◎ $(7+4)+5=7+(4+5)$
(iv) Associative Property w.r.t Multiplication ◎ If a , b and c are any three real numbers, then $(a\cdot b)\cdot c = a\cdot(b\cdot c)$	◎ $(7\cdot 4)\cdot 5 = 7\cdot(4\cdot 5)$
(v) Additive Identity Property ◎ If a is real number, there is a unique number zero (0) called additive identity such that: $a+0=0+a=a$	◎ $2+0=0+2$
(vi) Multiplicative Identity Property ◎ For any real number a, there is a unique number one (1) called multiplicative identity such that: $a\cdot 1=1\cdot a=a$	◎ $2\cdot 1=1\cdot 2$
(vii) Additive Inverse Property ◎ For any real number a, there exists a unique real number $-a$ called additive inverse of a such that: $a+(-a)=0=(-a)+a$	◎ Additive inverse of 8 is -8 Since $8+(-8)=0=-8+8$
(viii) Multiplicative Inverse Property ◎ If $a\neq 0$ is a real number, there exists a unique real number $\frac{1}{a}$ called multiplicative inverse of a such that: $a\cdot\frac{1}{a}=1=\frac{1}{a}\cdot a$ **Remember!** $0\in\mathbb{R}$ has no multiplicative inverse	◎ 7 and $\frac{1}{7}$ are multiplicative inverse of each other i.e., $7\cdot\frac{1}{7} = 1 = \frac{1}{7}\cdot 7$ or $\frac{3}{5}\cdot\frac{5}{3} = 1 = \frac{5}{3}\cdot\frac{3}{5}$ $\frac{3}{5}$ and $\frac{5}{3}$ are multiplicative inverse of each other .
(ix) Commutative Property w.r.t Addition ◎ If a and b are any two real numbers, then $a+b=b+a$	◎ $13+7=7+13$; $\sqrt{3}+\sqrt{5}=\sqrt{5}+\sqrt{3}$
(x) Commutative Property w.r.t Multiplication ◎ If a and b are any two real numbers, then $a\cdot b=b\cdot a$	◎ $13\times 7=7\times 13$; $\sqrt{3}\times\sqrt{5}=\sqrt{5}\times\sqrt{3}$

Properties	Examples
(xi) Distributive Property of Multiplication over addition ◎ If a, b and c are real numbers, then $a(b+c) = a \cdot b + a \cdot c$ or $(a+b)c = a \cdot c + b \cdot c$	◎ If $a=2$, $b=3$, $c=4$ then $2(3+4) = 2 \cdot 3 + 2 \cdot 4$ $14 = 14$ or $(2+3) \cdot 4 = 2 \cdot 4 + 3 \cdot 4$ $20 = 20$

1.1.6 Order (Inequalities) of Real Numbers

Properties	Examples
(i) Trichotomy Property $\forall\ a, b \in \mathbb{R}$ Exactly one of the following holds: ◎ $a < b$ or $a > b$ or $a = b$	**Keep in mind!** $\forall$ stands for "for all" ◎ $7 < 10$, but $7 \not> 10$ and $7 \neq 10$
(ii) Transitive Property $\forall\ a, b, c \in \mathbb{R}$ ◎ If $a > b$ and $b > c$, then $a > c$ or ◎ If $a < b$ and $b < c$, then $a < c$	◎ $8 > 5$ and $5 > 4 \Rightarrow 8 > 4$ or ◎ $2 < 5$ and $5 < 6 \Rightarrow 2 < 6$
(iii) Addition Property $\forall\ a, b, c \in \mathbb{R}$ ◎ If $a < b$, then $a + c < b + c$ or ◎ If $a > b$, then $a + c > b + c$	◎ $5 < 7$ and $c = 2 \Rightarrow 5 + 2 < 7 + 2$ i.e., $7 < 9$ or ◎ $10 > 8$ and $c = 2 \Rightarrow 10 + 2 > 8 + 2$ i.e., $12 > 10$
(iv) Subtraction Property $\forall\ a, b, c \in \mathbb{R}$ ◎ If $a < b$, then $a - c < b - c$ or ◎ If $a > b$, then $a - c > b - c$	◎ $10 < 11$ and $c = 2 \Rightarrow 10 - 2 < 11 - 2$ i.e., $8 < 9$ or ◎ $13 > 9$ and $c = 2 \Rightarrow 13 - 2 > 9 - 2$ i.e., $11 > 7$
(v) Multiplication Property ◎ If $a < b$ and $c > 0$, then $a \cdot c < b \cdot c$ or ◎ If $a < b$ and $c < 0$, then $a \cdot c > b \cdot c$	◎ $5 < 8$ and $c = 2 \Rightarrow 5 \cdot 2 < 8 \cdot 2$ i.e., $10 < 16$ or ◎ $5 < 8$ and $c = -2 \Rightarrow 5(-2) > 8(-2)$ i.e., $-10 > -16$

Try yourself!

Write the multiplicative property for a > b

Remember!

If we multiply or divide an inequality by a negative number, then the inequality sign is always changed.

Properties	Examples
(vi) Division Property	
◎ If $a < b$ and $c > 0$, then $\frac{a}{c} < \frac{b}{c}$	◎ $6 < 9$ and $c = 3 \Rightarrow \frac{6}{3} < \frac{9}{3}$ i.e., $2 < 3$
or	or
◎ If $a < b$, and $c < 0$, then $\frac{a}{c} > \frac{b}{c}$	◎ $6 < 9$ and $c = -3 \Rightarrow \frac{6}{-3} > \frac{9}{-3}$ i.e., $-2 > -3$
Try yourself! Write the division property for a > b.	
(vii)	
◎ If a and b both are positive (+) or both are negative (–), then $a < b \Leftrightarrow \frac{1}{a} > \frac{1}{b}$ or $a > b \Leftrightarrow \frac{1}{a} < \frac{1}{b}$	◎ $2 < 4 \Rightarrow \frac{1}{2} > \frac{1}{4}$ or $-3 < -2 \Rightarrow -\frac{1}{3} > -\frac{1}{2}$

Exercise 1.2

1. Identify which property of real numbers is used in the following:

(i) $11 + 0 = 11$ (ii) $x \cdot (y \cdot z) = (x \cdot y) \cdot z$

(iii) $12 + (-12) = 0$ (iv) $14 + 20 = 20 + 14$

(v) $\sqrt{5} \cdot \sqrt{5} \in \mathbb{R}$ (vi) $2 \times \frac{1}{2} = 1$

(vii) $22 \times 1 = 22$ (viii) $\left(-\frac{6}{10}\right) \cdot \left(-\frac{10}{6}\right) = 1$

(ix) $(19 + 12) + 8 = 19 + (12 + 8)$ (x) $-3 + (23 + 48) = (-3 + 23) + 48$

(xi) $89 \times 1 = 89$ (xii) $12 \times 46 = 46 \times 12$

(xiii) $\sqrt{27} + 0 = \sqrt{27}$ (xiv) $xy = yx$

(xv) $\pi + (-\pi) = 0$

2. Identify which order property is used in the following:

(i) 15 (ii) -27 (iii) $\frac{7}{9}$ (iv) $-\frac{9}{16}$ (v) $-\frac{20}{23}$

(vi) 1 (vii) $-\frac{2}{3}$ (viii) $\frac{7}{2}$ (ix) -7 (x) 12

3. Identify which order property is used in the following:

(i) $x < y$ or $x > y$ or $x = y$ (ii) $a > b \Rightarrow \frac{a}{c} < \frac{b}{c}$, $c < 0$

(iii) $a > b \Rightarrow a - c > b - c$ (iv) $5 > 2 \Rightarrow 5 \cdot c < 2 \cdot c$, $c < 0$

(v) $7 > z$ and $z > 5 \Rightarrow 7 > 5$ (vi) $10 < 15 \Rightarrow 10 \cdot c > 15 \cdot c$, $c < 0$

(vii) $x < y \Rightarrow x - w < y - w$ (viii) $x < z \Rightarrow \frac{x}{\ell} < \frac{z}{\ell}$, $\ell > 0$

SUMMARY

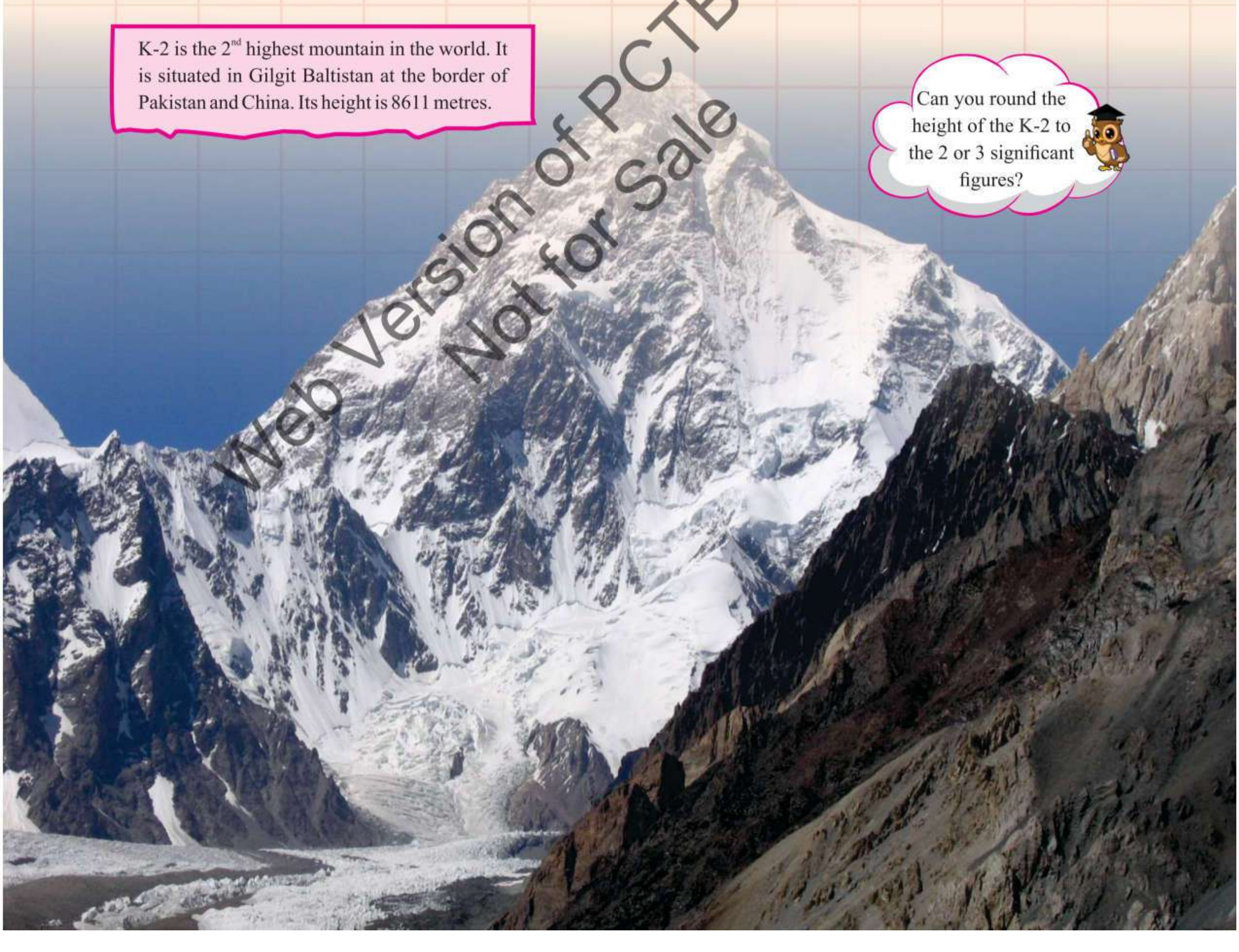
- The numbers which can be written in the form of $\frac{p}{q}$, where $p, q \in Z$ and $q \neq 0$ are called rational numbers.
- Decimal representations of rational numbers are of two types:
  (i) Terminating decimals (ii) Recurring decimals
- Terminating decimal numbers are the decimals which have finite number of digits in its decimal part.
- The decimal numbers in which one digit or group of digits repeat again and again infinite times in its decimal part are called recurring decimal numbers.
- The numbers which are not rational are called irrational numbers. The set of irrational numbers is represented by Q'.
- All the numbers (N, W, Z, all rational and irrational numbers) are real numbers. The set of real numbers is represented by $\mathbb{R}$.
- The absolute value (or modulus) of a real number x (represented by $|x|$) is the non-negative value of x without keeping in view its sign.
- If we take any two real numbers, then their sum and product is also a real number. It is called closure property.
- If a is a real number, there is a unique number zero (0) called additive identity such that
  $$a + 0 = 0 + a = a$$
- 1 is called multiplicative identity, such that $a \cdot 1 = a = 1 \cdot a$
- The real numbers hold the following properties:
  (i) Closure property (ii) Associative property
  (iii) Additive identity property (iv) Multiliplicative identity property
  (v) Additive inverse property (vi) Multiplicative inverse property
  (vii) Commutative property (viii) Distributive property

Sub-Domain (ii): Estimation and Approximation

Students' Learning Outcomes

After completing this sub-domain, the students will be able to:

- round whole numbers, integers, rational numbers and decimal numbers to a required degree of accuracy
- analyze approximation error when numbers or quantities are rounded
- solve real life situations involving approximation



K-2 is the 2nd highest mountain in the world. It is situated in Gilgit Baltistan at the border of Pakistan and China. Its height is 8611 metres.

Can you round the height of the K-2 to the 2 or 3 significant figures?

1.2.1 Round Whole Numbers

Sometimes, in our daily life, we are unable to find out the actual value of any item, then we have to estimate/round the values to the required degree of accuracy. For example, if the cost of a biscuit packet is Rs. 19.75, in such situation, we are unable to pay amount in decimal form. That's why, we have to pay Rs. 20 after rounding.

Estimation
A process in which we round the number to the actual value or required degree of accuracy.

Noor purchased 343 metres cloth.

Ayesha also purchased the cloth having the same length.

Noor and Ayesha want to round the given length of cloth.

Noor does round the length of the cloth to the 2-significant figures i.e. 340

Ayesha does round the length of the cloth to the 1-significant figure i.e. 300 metres

Can you tell that who is more accurate?

Noor is more accurate because he rounded the length of the cloth to the 2-significant figures.

Remember!
When a number is rounded to a greater number of significant figures, then that number is called more accurate.

Significant Figure
The first non-zero digit of the number from the left is called the first significant figure. All zeros between non-zero digits are also significant.

Remember!
- s.f. stands for significant figure.
- d.p. stands for decimal place.

Example 1: Round 548735 to the given degree of accuracy.

(i) 1 significant figure.
(ii) 3 significant figures.
(iii) 5 significant figures.

Solution:
(i) 548735 = 500000 (1 s.f.)
(ii) 548735 = 549000 (3 s.f.)
(iii) 548735 = 548740 (5 s.f.)

Key fact!
To round whole number to the required degree of accuracy.
- Look at the digit next to the required degree of accuracy.
- If the digit is 5 or greater than 5, we will add 1 to the digit of the required degree of accuracy.
- If the digit is smaller than 5 we will leave the digit as it is given on the required degree of accuracy.

Ahmad says the cost of one litre petrol is Rs. 150.40

Azra says the cost of one litre petrol is Rs. 150.4

Can you tell that who is more accurate?

Ahmad is more accurate because he rounded the cost of one litre of petrol to the maximum degree of accuracy (5-significant figures) as compare to Azra.

Keep in mind!

In decimal part of a decimal number, all zeros after a non-zero digit are significant and all zeros before a non-zero digit are not significant.

All non-zero digits are significant and all zeros between non-zero digits are significant.

Try yourself!

How many significant figures does 0.00295 have?

1.2.2 Round Decimal Numbers

Example 2:

Round 87.5376 to the given degree of accuracy.

(i) 1 decimal place

(ii) 4 significant figures.

(iii) 3 decimal places.

Solution:

(i) 87.5376 = 87.5 (1 d.p.)

(ii) 87.5376 = 87.54 (4 s.f.)

(iii) 87.5376 = 87.538 (3 d. p.)

The digit next to 2-decimal places i.e., "7" is greater than 5. So, round the digit 3 up to 4 and remove all the next digits towards its right.

Try yourself!

Round 54285 to the 4 significant figures.

Example 3:

Round 0.0785229 to the given degree of accuracy.

(i) 3 significant figures

(ii) 5 significant figures

Solution:

(i) 0.0785229 = 0.0785 (3 s.f.)

(ii) 0.0785229 = 0.078523 (5 s.f.)

The first significant figure is 7, the second is 8 and third is 5. The digit to the right of 5 is 2 and it is less than 5. So, 5 stays the same and remove all the next digits towards its right.

1.2.3 Round Negative Integers

The method to round negative integers is different from the method used to round natural numbers.

Remember!
In negative integers, the number which is close to the zero (0) is greater number. While the number which is far from the zero (0) is smaller number.

Example 4: Round -234576 to the given degree of accuracy.

(i) 3 significant figures

(ii) 5 significance figures

Solution:

(i) $-234576 = -234000$ (3 s.f.) If we want to round the number towards zero (0).

$= -235000$ (3 s.f.) If we want to round the number away from zero (0).

(ii) $-234576 = -234570$(5 s.f.) If we want to round the number towards zero (0).

$= -234580$ If we want to round the number away from zero (0).

Example 5: Round -48.3567 to the given degree of accuracy.

(i) 3 decimal places (ii) 5 significant figures

Solution:

(i) $-48.3567 = -48.357$ (3 d.p.) If we round the number away from zero (0).

$-48.3567 = -48.356$ (3 d.p.) If we round the number towards zero (0).

(ii) $-48.3567 = -48.356$ (5 s.f.) If we round the number towards zero (0).

$-48.3567 = -48.357$ (5 s.f.) If we round the number away from zero (0).

1.2.4 Round Rational Numbers

Example 6: Round $\frac{17}{3}$ to the given degree of accuracy.

(i) 2 significant figures

(ii) 5 significant figures

Solution:

(i) $\frac{17}{3} = 5.66666 = 5.7$ (2 s.f.)

(ii) $\frac{17}{3} = 5.66666 = 5.6667$ (5 s.f.)

Example 7: Round $\frac{235}{28}$ to the given degree of accuracy.

(i) 2 decimal places

(ii) 3 decimal places

Solution:

(i) $\frac{235}{28} = 8.392857 = 8.39$ (2 d.p.)

(ii) $\frac{235}{28} = 8.392857 = 8.393$ (3 d.p.)

1.2.5 Analysis of Approximation Error

Example 8: (i) Work out an approximation of $4.53 + 786$,

(ii) Calculate its accurate value,

(iii) Compare your approximate and accurate value and find the approximation error, where $4.53 \approx 5$ and $786 \approx 800$

Keep in mind!

Approximation error is also known as bias. The symbol ≈ is used for approximation.

Solution: (i) $5 + 800 = 805$

Approximated Value = 805

(ii) $4.53 + 786 = 790.53$

Accurate Value = 790.53

(iii) Approximation error = Approximated value − Accurate value

$= 805 - 790.53$

Approximation error = 14.47

Example 9: Calculate approximated value, accurate value and approximation error of $\frac{1.45 \times 560}{23.5}$.

Solution: Approximated value $= 1.45 \approx 1$; $560 \approx 600$; $23.5 \approx 24$

$$= \frac{1 \times 600}{24} = 25$$

Approximated value = 25

Accurate value $= \frac{1.45 \times 560}{23.5} = 34.55$

Keep in mind!

Approximation error is numerical value, which tells us how far is the approximated value from the actual / accurate value.

Approximation error = Accurate value − Approximated value

$= 34.55 - 25$

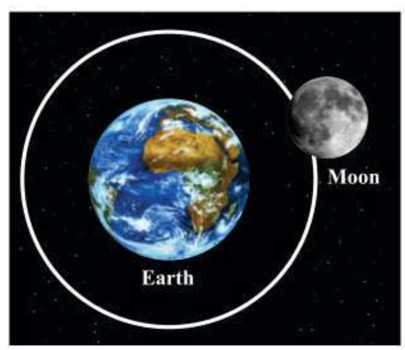
Approximation error = 9.45

1.2.6 Real Life Situation Involving Approximation

Example 10: The minimum distance between the earth and the moon is about 363104 km. Round the number according to the given degree of accuracy.

(i) 3 significant figures.

(ii) 5 significant figures.



Solution: (i) $363104 \approx 363000$ (3 s.f.)

(ii) $363104 \approx 363100$ (5 s.f.)

Exercise 1.3

1. Round the following numbers to the given degree of accuracy.

(i) 4763 (2 s.f.) (ii) 785029 (3 s.f.) (iii) 8095321 (4 s.f.)

(iv) 0.0289357 (5 s.f.) (v) 2345987 (5 s.f.) (vi) 0.780247 (4 s.f.)

2. Round the given decimal numbers to the stated number of decimal places or significant figures.
   (i) 2.785 (2 d.p.) (ii) 88.9856 (4 s.f.)
   (iii) 15.3456 (2 d.p.) (iv) 10.4579 (3 s.f.)

3. Round the integers away from zero (0) as given.
   (i) − 23.456 (1 d.p.) (ii) − 45325 (3 s.f.)
   (iii) − 17.7898 (4 s.f.) (iv) 1172859 (5 s.f.)

4. Round the rational numbers to the stated number of decimal places or significant figures.
   (i) $\frac{23}{3}$ (3 d.p.) (ii) $\frac{55}{3}$ (2 d.p.) (iii) $\frac{47}{7}$ (2 d.p.)
   (iv) $\frac{22}{6}$ (3 d.p.) (v) $\frac{25}{29}$ (5 s.f.) (vi) $\frac{5}{521}$ (4 s.f.)

5. Write the number of significant figures in each of the following:
   (i) 0.0028 (ii) 25901 (iii) 0.1080 (iv) 0.000235

6. Compare your approximated value with the accurate value and also find an approximation error.
   (i) $3.78 \div 1.98$ (ii) $\frac{330 \times 7.62}{2.53}$ (iii) $\frac{28.35+30.15}{4593-3690}$ (iv) $\frac{133+8.95}{5.36}$

7. The total length of the Great Wall of China is 21196.18 km. Round the number to 1 decimal place and 5-significant figures.

8. The diameter of earth is 12742 km. Round the number to 3 and 4 significant figures.

SUMMARY

- To round whole number to the required degree of accuracy:
  - Look at the digit next to the degree of accuracy.
  - If the digit is 5 or greater than 5, we will add 1 to the digit of the required degree of accuracy.
  - If the digit is smaller than 5, we will leave the digit as it is given on the required degree of accuracy.
- s.f. stands for significant figure.
- d.p. stands for decimal place.
- The method to round negative integers is different from the method used to round natural numbers.
- In negative integer, the number which is close to the zero (0) is greater number. While the number which is far from the zero (0) is smaller number.
- Approximation error is numerical value, which tells us how far is the approximated value from the actual / accurate value.

Sub-Domain (iii): Square Roots and Cube Roots

Students' Learning Outcomes

After completing this sub-domain, the students will be able to:

- recall squares and cubes of natural numbers up to 3- digits

Square Roots

compute square root of:

- a natural number
- a common fraction
- a decimal
- given in perfect square form by prime factorization method up to 5- digits
- calculate square roots up to 4 digits with maximum 2-decimal places which is not a perfect square
- apply square and square roots in real life situations

Cubes and Cube Roots

- calculate cube roots of a number up to 5-dgits which are perfect cubes by prime factorization method
- apply cubes and cube roots in real life situations/word problems

Advanced/ Additional

compute square root of:

- a natural number
- a common fraction
- a decimal
- given in perfect square form by division method up to 5-digits

Nida wants to plant 12 rose plants in rows and 12 rose plants in columns in her home garden as shown in figure.

Can you tell how many number of rose plants Nida can grow?

1.3.1 Squares of Natural Numbers

When a number is multiplied by itself, then the product is known as the square of the number i.e, the square of x is $x \times x = x^2$

For Example: $3 \times 3 = 3^2 = 9$

Read as square of 3 is 9

Similarly,

$5 \times 5 = 5^2 = 25$ (square of 5 is 25)

Finding perfect square of a number

A natural number is called a perfect square, if it is the square of another natural number. e.g., the number 4 is a perfect square because $4 = 2^2$.

Similarly, 25 is a perfect square because $25 = 5^2$ and so on.

Now, we learn to find a perfect square of a number:

Try yourself!

Can you find out the perfect square of -4?

Example 1: Find the perfect square of 13

Solution: The perfect square of 13 is

$$13^2 = 13 \times 13 = 169$$

Establish patterns for the squares of natural numbers

We know that $4^2 = 4 \times 4 = 16$

We can also write the square of 4 in a pattern form as

$4^2 = 1+2+3+4+3+2+1 = 16$

Similarly $5^2 = 1+2+3+4+5+4+3+2+1 = 25$

And $6^2 = 1+2+3+4+5+6+5+4+3+2+1 = 36$

So, we observed that the square of any natural number can be found with the help of summation of above patterns.

1^2	1	$= 1$
2^2	$1 + 2 + 1$	$= 4$
3^2	$1 + 2 + 3 + 2 + 1$	$= 9$
4^2	$1 + 2 + 3 + 4 + 3 + 2 + 1$	$= 16$
5^2	$1 + 2 + 3 + 4 + 5 + 4 + 3 + 2 + 1$	$= 25$
6^2	$1 + 2 + 3 + 4 + 5 + 6 + 5 + 4 + 3 + 2 + 1$	$= 36$
7^2	$1 + 2 + 3 + 4 + 5 + 6 + 7 + 6 + 5 + 4 + 3 + 2 + 1$	$= 49$
8^2	$1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1$	$= 64$
9^2	$1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1$	$= 81$
10^2	$1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 + 9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1$	$=100$

In the above pattern, we notice that:

(i) Each row starts and ends with digit 1.
(ii) The digits increase up to the number whose square is required and then decrease.
(iii) The number of digits in each row increases by two digits.
(iv) The difference of any two consecutive squares is an odd number.
(v) The number of digits in a particular row is the addition of the number and the previous consecutive numbers whose squares are to be found.

Consider another pattern of squares of natural numbers.

		Sum of odd numbers
1^2	1	$= 1$
2^2	$1 + 3$	$= 4$
3^2	$1 + 3 + 5$	$= 9$
4^2	$1 + 3 + 5 + 7$	$= 16$
5^2	$1 + 3 + 5 + 7 + 9$	$= 25$
6^2	$1 + 3 + 5 + 7 + 9 + 11$	$= 36$
7^2	$1 + 3 + 5 + 7 + 9 + 11 + 13$	$= 49$
8^2	$1 + 3 + 5 + 7 + 9 + 11 + 13 + 15$	$= 64$
9^2	$1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17$	$= 81$
10^2	$1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19$	$=100$

In the above pattern, we notice that:

(i) The summation is in ascending order.
(ii) The square of each number is written as the sum of odd numbers only.
(iii) Each row of the pattern starts from an odd number 1.
(iv) The number of odd numbers in each row is equal to the number whose square is to be found.
(v) The sum of each row is equal to the required square.
(vi) The last odd number in each row is one less than the double of the given number.

Exercise 1.4

1. Find the square of the following numbers:

(i) 7 (ii) 11 (iii) 19 (iv) 25 (v) 37 (vi) 75

2. Write the summation patterns for the following squares.

(i) 6^2 (ii) 7^2 (iii) 4^2 (iv) 5^2 (v) 3^2 (vi) 8^2

1.3.2 Square Roots

Finding the square root of (a) a natural number (b) a common fraction (c) a decimal given in perfect square form, by prime factorization and division method

(a) Finding square root of a natural number

- **By Prime Factorization Method**

First of all, find prime factors, then make pairs of these factors. Choose one prime number from each pair and then find the product of all those prime factors, which will be the square root of the given number.

Example 2: Find the square root of 12100

Solution: $12100 = 2\times 2\times 5\times 5\times 11\times 11$

$$\sqrt{12100} = \sqrt{\overline{2\times 2}\times \overline{5\times 5}\times \overline{11\times 11}}$$

$$= \sqrt{2^2\times 5^2\times 11^2}$$

$$= 2\times 5\times 11$$

$$\therefore \sqrt{12100} = 110$$

2	12100
2	6050
5	3025
5	605
11	121
11	11
	1

Key fact!

Square root and square are inverse of each other. The symbol used for square root is " $\sqrt{\ }$ ". It is called radical sign.

Example 3: Find the square root of 225

Solution: $225 = 3\times 3\times 5\times 5$

$$\sqrt{225} = \sqrt{\overline{3\times 3}\times \overline{5\times 5}}$$

$$= 3\times 5$$

$$= 15$$

$$\therefore \sqrt{225} = 15$$

3	225
3	75
5	25
5	5
	1

Think!

Is the index of each prime factor should be even to find out square root? Why?

Example 4: Find the square root of 1600

Solution: $1600 = 2\times 2\times 2\times 2\times 2\times 2\times 5\times 5$

$$\sqrt{1600} = \sqrt{\overline{2\times 2}\times \overline{2\times 2}\times \overline{2\times 2}\times \overline{5\times 5}}$$

$$= 2\times 2\times 2\times 5$$

$$= 40$$

$$\therefore \sqrt{1600} = 40$$

2	1600
2	800
2	400
2	200
2	100
2	50
5	25
5	5
	1

Try yourself!

Can you solve?

$\sqrt{-81} = \boxed{?}$

- **By Division Method**

To find the square root of natural numbers by division method, we will proceed as under:

(i) Make pairs of digits from right to left. If the number of digits is even, we have to complete pairs. If the number of digits is odd, the last digit on extreme left will remain single.

(ii) Look for the numbers whose square is equal to or less than the number of extreme left, which may be a single digit or a pair. This number will be divisor as well as quotient.

(iii) Subtract the product. Bring down the next pair to the right of the remainder.

(iv) Double the quotient and write as divisor as ten's digit.

(v) Look for the number whose square will be equal to or less than the dividend. Write that number with the right side of the quotient as well as with divisor at unit place.

Example 5: Find the square root of 1024

Solution:

$$\begin{array}{r|l} & \quad 32 \\ \hline 3 & \overline{10}\ \overline{24} \\ & -9 \downarrow \\ \hline 62 & \quad 124 \\ & -124 \\ \hline & \quad\ 0 \end{array}$$

Example 6: Find the square root of 15129

Solution:

$$\begin{array}{r|l} & \quad 123 \\ \hline 1 & 1\overline{51}\ \overline{29} \\ & -1\downarrow \\ \hline 22 & \quad 51 \\ & -44 \downarrow \\ \hline 243 & \quad 729 \\ & -729 \\ \hline & \quad\ 0 \end{array}$$

Exercise 1.5

1. Find the square root of the following by prime factorization method:

(i)	784	(ii)	1225	(iii)	2500	(iv)	4225	(v)	5184	(vi)	7744
(vii)	1296	(viii)	1764	(ix)	29241	(x)	51529	(xi)	418609	(xii)	240100

2. Find the square root of the following by division method:

(i) 1369 (ii) 13689 (iii) 50625 (iv) 103041

(b) Finding square root of a common fraction

We know that in fraction $\frac{4}{9}$, 4 is numerator and 9 is denominator. The square root of a fraction is equal to the square root of the numerator divided by the square root of the denominator. This is illustrated with the help of following examples:

Example 7: Find the square root of $1\frac{11}{25}$

Solution:

$$1\frac{11}{25} = \frac{36}{25} = \frac{2\times 2\times 3\times 3}{5\times 5}$$

$$\sqrt{1\frac{11}{25}} = \sqrt{\frac{36}{25}}$$

$$= \frac{\sqrt{\overline{2\times 2}\times \overline{3\times 3}}}{\sqrt{\overline{5\times 5}}} = \frac{2\times 3}{5} = \frac{6}{5} = 1\frac{1}{5}$$

Example 8: Find the square root of $9\frac{67}{121}$

Solution: $$9\frac{67}{121} = \frac{1156}{121}$$

Now $$\sqrt{9\frac{67}{121}} = \sqrt{\frac{1156}{121}}$$

$$= \frac{\sqrt{1156}}{\sqrt{121}}$$

$$= \frac{34}{11}$$

$$= 3\frac{1}{11}$$

$$\begin{array}{r|l} & \quad 3\,4 \\ \hline 3 & \overline{11}\,\overline{56} \\ & -9 \downarrow \\ \hline 64 & \;\; 256 \\ & -256 \\ \hline & \quad 0 \end{array} \qquad \begin{array}{r|l} & \quad 1\,1 \\ \hline 1 & 1\,\overline{21} \\ & -1 \downarrow \\ \hline 21 & \;\; 21 \\ & -21 \\ \hline & \quad 0 \end{array}$$

$\therefore$ $$\sqrt{9\frac{67}{121}} = 3\frac{1}{11}$$

Exercise 1.6

1. Find the square root of the following fractions by prime factorization:

(i) $\frac{49}{64}$ (ii) $\frac{121}{625}$ (iii) $\frac{196}{441}$ (iv) $1\frac{13}{36}$ (v) $\frac{676}{729}$ (vi) $12\frac{24}{25}$

2. Find the square root of the following fractions by division method:

(i) $\frac{144}{225}$ (ii) $\frac{169}{256}$ (iii) $5\frac{41}{64}$

(c) Finding square root of a decimal

- **By Prime Factorization**

We convert the decimal to common fraction and then find square root.

Example 9: Find the square root of decimal 2.25

Solution: $$2.25 = \frac{225}{100}$$

$$\sqrt{\frac{225}{100}} = \frac{\sqrt{225}}{\sqrt{100}} = \frac{\sqrt{3\times 3\times 5\times 5}}{\sqrt{2\times 2\times 5\times 5}}$$

$$= \frac{\sqrt{\overline{3\times 3}\times \overline{5\times 5}}}{\sqrt{\overline{2\times 2}\times \overline{5\times 5}}}$$

$$= \frac{3\times 5}{2\times 5} = \frac{15}{10}$$

$$= 1.5$$

$$\therefore \quad \sqrt{2.25} = 1.5$$

Example 10: Find the square root of 1239.04

Solution:

$$1239.04 = \frac{123904}{100}$$

$$\sqrt{\frac{123904}{100}} = \sqrt{\frac{2\times 2\times 2\times 2\times 2\times 2\times 2\times 2\times 2\times 2\times 11\times 11}{2\times 2\times 5\times 5}}$$

$$= \sqrt{\frac{\overline{2\times 2}\times \overline{2\times 2}\times \overline{2\times 2}\times \overline{2\times 2}\times \overline{2\times 2}\times \overline{11\times 11}}{\overline{2\times 2}\times \overline{5\times 5}}}$$

$$= \frac{2\times 2\times 2\times 2\times 2\times 11}{2\times 5} = \frac{352}{10}$$

$$\therefore \quad \sqrt{1239.04} = 35.2$$

- **By Division Method**

For using this method, the following steps will be taken:

(i) Make pairs of digits on the left side of the decimal point from right to left.
(ii) Make pairs of digits on the right side of the decimal point from left to right.
(iii) If the number of digits is odd on the right side of the decimal point, then write 0 along with the last digit while making pairs from left to right.
(iv) Place the decimal point in the quotient while bringing down the pair after the decimal point.
(v) While bringing down two pairs at a time, place a zero in the quotient.

This method is illustrated with the help of following examples:

Example 11: Find the square root of 180.9025

Solution:

$$\begin{array}{r|l}
 & \quad 1\,3\,.\,4\,5 \\ \hline
1 & 1\,\overline{80}.\overline{90}\,\overline{25} \\
 & -1 \\ \hline
23 & \quad 80 \\
 & -69 \\ \hline
264 & \quad 1190 \\
 & -1056 \\ \hline
2685 & \quad\quad 13425 \\
 & \quad -13425 \\ \hline
 & \quad\quad\quad 0
\end{array}$$

$1 \times 21 = 21$
$2 \times 22 = 44$
$3 \times 23 = 69$
$4 \times 24 = 96$

$1 \times 261 = 261$
$2 \times 262 = 524$
$3 \times 263 = 789$
$4 \times 264 = 1056$
$5 \times 265 = 1325$

$1 \times 2681 = 2681$
$2 \times 2682 = 5364$
$3 \times 2683 = 8049$
$4 \times 2684 = 10736$
$5 \times 2685 = 13425$

$$\therefore \quad \sqrt{180.9025} = 13.45$$

Example 12: Find the square root of 0.053361

Solution:

$$\begin{array}{r|r}
 & 0.231 \\ \hline
2 & 0.\overline{05}\,\overline{33}\,\overline{61} \\
 & -04 \\ \hline
43 & 133 \\
 & -129 \\ \hline
461 & 461 \\
 & -461 \\ \hline
 & 0
\end{array}$$

$\therefore \quad \sqrt{0.053361} = 0.231$

Exercise 1.7

1. Find the square root of the following decimals by prime factorization:

(i)	1.21	(ii)	0.64	(iii)	7.29
(iv)	1.44	(v)	1.69	(vi)	12.25

2. Find the square root of the following decimals by division method:

(i)	0.3249	(ii)	0.5184	(iii)	10.24
(iv)	20.5209	(v)	3856.41	(vi)	1866.24
(vii)	0.680625	(viii)	0.797449	(ix)	1281.64

1.3.3 Find Square Root of a Number Which is Not a Perfect Square

Example 13: Find the square root of 2 up to 2 decimal places.

Solution:

$$\begin{array}{r|l}
 & 1.414 \\ \hline
1 & 2.00\,00\,00 \\
 & -1 \\ \hline
24 & 1\,00 \\
 & -96 \\ \hline
281 & 400 \\
 & -281 \\ \hline
2824 & 11900 \\
 & -11296 \\ \hline
 & 604 \\
 & \ldots\ldots
\end{array}$$

$\therefore \ \sqrt{2} \approx 1.414$, rounded to 1.41 (2 d.p.)

Key fact!

If we cannot find out a rational number whose square is x, then $\sqrt{x}$ is an irrational number.

We observe that:

The process is non-terminating, so we cannot get zero as remainder. In the quotient after the decimal point, there is no group of integers which is repeating itself, as in the case of rational numbers.

$$\frac{2}{3} = 0.666\ldots \quad \text{and} \quad \frac{7}{9} = 0.777\ldots$$

Example 14: Find the square root of 2.5 up to 2 decimal places.

Solution:

$$\begin{array}{r|r} & 1.581 \\ \hline 1 & 2.\overline{50}\ \overline{00}\ \overline{00} \\ & -1 \\ \hline 25 & 1\,50 \\ & -1\,25 \\ \hline 308 & 25\,00 \\ & -24\,64 \\ \hline 3161 & 36\,00 \\ & -31\,61 \\ \hline & 4\,39 \\ & \ldots\ldots \end{array}$$

$\therefore \quad \sqrt{2.5} \approx 1.581$, rounded to 1.58 (2 d.p.)

In such cases, we restrict the process after some decimal places. Here we shall restrict it up to 2 decimal places.

Example 15: Find the square root of 2723.56 up to 3 decimal places.

Solution:

$$\begin{array}{r|r} & 52.1877 \\ \hline 5 & \overline{27}\,\overline{23}.\overline{56}\,\overline{00} \\ & -25 \\ \hline 102 & 223 \\ & -204 \\ \hline 1041 & 1956 \\ & -1041 \\ \hline 10428 & 91500 \\ & -83424 \\ \hline 104367 & 807600 \\ & -730569 \\ \hline & 77031 \\ & \ldots\ldots \end{array}$$

$\therefore \quad \sqrt{2723.56} \approx 52.1877$, rounded to 52.188 (3 d.p.)

Exercise 1.8

1. Find the square root of the following up to 3 decimal places:

   (i) 2 (ii) 3 (iii) 5 (iv) 7 (v) 11 (vi) 21

2. Find the square root of the following up to 2 decimal places:

   (i) 3.6 (ii) 6.4 (iii) 28.9 (iv) 63.34 (v) 921.25 (vi) 5921.25

1.3.4 Use the Rule to Determine the Number of Digits in the Square Root of a Perfect Square

Rule: Let n be the number of digits in the perfect square, then its square root contains:

(i) $\frac{n}{2}$ digits if n is even (ii) $\frac{n+1}{2}$ digits if n is odd

Now, we apply the above rule for finding the number of digits in the square root of a perfect square with the help of following examples:

Example 16: Find the number of digits in the square root of 49729

Solution: Number of digits of the given number = 5

$n = 5$ is odd, so above mentioned rule (ii) will be applied.

Thus, the number of digits in the square root will be $\frac{n+1}{2} = \frac{5+1}{2} = \frac{6}{2} = 3$

To check the answer, we proceed as under:

$$\begin{array}{r|l}
 & 223 \\
\hline
2 & \overline{4}\ \overline{97}\ \overline{29} \\
 & -4 \\
\hline
42 & 97 \\
 & -84 \\
\hline
443 & 1329 \\
 & -1329 \\
\hline
 & 0
\end{array}$$

$\therefore \quad \sqrt{49729} = 223$

Example 17: Find the number of digits in the square root of 10329796.

Solution: Number of digits $(n) = 8$

Now $n = 8$ is even, so part (i) of the rule will be applied.

The number of digits in the square root $= \frac{n}{2} = \frac{8}{2} = 4$

Now, we can verify it

$$\begin{array}{r|l}
 & 3214 \\
\hline
3 & \overline{10}\ \overline{32}\ \overline{97}\ \overline{96} \\
 & 9 \\
\hline
62 & 132 \\
 & 124 \\
\hline
641 & 897 \\
 & 641 \\
\hline
6424 & 25696 \\
 & 25696 \\
\hline
 & 0
\end{array}$$

$\therefore \quad \sqrt{10329796} = 3214$

Exercise 1.9

1. Find the number of digits in the square root of the following perfect square and verify it by finding square root:

(i)	63504	(ii)	66564	(iii)	50625	(iv)	837225
(v)	839056	(vi)	1054729	(vii)	1577536	(viii)	2119936
(ix)	3283344	(x)	614656	(xi)	7778521	(xii)	12880921

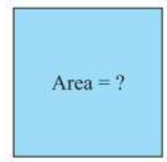
1.3.5 Real Life Problems Involving Square Roots

Example 18: The length of side of a square is $55m$. Find the area of the square.

Solution: Let 'x' be the area of the square

As we know that

$$\begin{aligned} Area &= (\text{length of a side})^2 \\ x &= (55m)^2 = 55m \times 55m \\ x &= 3025m^2 \end{aligned}$$



Length = $55m$

The area of the square is $3025m^2$

Example 19: 1225 students stand in rows in such a way that the number of rows is equal to the number of students in a row. How many students are there in each row?

Solution: Since the number of students in a row is the same as the number of rows. So, square root of 1225 will be found.

$$\begin{array}{r|l} & \;\;35 \\ 3 & \overline{12}\,\overline{25} \\ & \;\;9 \\ \hline 65 & \;\;325 \\ & \;\;325 \\ \hline & \;\;\;\;0 \end{array}$$

Thus, there are 35 students in each row.

Example 20: Find the least number which, when subtracted from 58780, the answer is a complete square.

Solution: To find which number is subtracted from the given number, we find the square root of 58780 and the remainder will be the required number.

$$\begin{array}{r|l} & 2\;4\;2 \\ 2 & 5\;\overline{8\;7}\;\overline{8\;0} \\ & 4 \\ \hline 44 & 1\;8\;7 \\ & 1\;7\;6 \\ \hline 482 & \;\;1\;1\;8\;0 \\ & \;\;\;\;\;9\;6\;4 \\ \hline & \;\;\;\;\;2\;1\;6 \end{array}$$

Remaining Number = Given number – Remainder = 58780 – 216 = 58564

Thus, if 216 is subtracted from 58780, the remaining number 58564 will be a complete square.

Exercise 1.10

1. The length of a squared shaped box is 324*cm*. Find the area of the box.
2. The length of a square shaped field is 36*m*. Find the area of the square field.
3. The length of a square shaped lawn is 250 metres. What will be the area of the lawn and also tell what will be its cost of growing grass at the rate of Rs. 75 per sq. metres?
4. The area of a square shaped field is 14400 sq. metres. Find the length of the side of the square.
5. The area of a square shaped field is 422500 sq. metres. How much string is required for fixing along the sides as a fence?
6. A gardener wants to plant 122500 trees in his field in such a way that the number of trees in a row is equal to the number of rows. How many trees will he plant in each row?
7. The area of a rectangular field is 10092 sq. metres. Its length is three times as long as its width. Find its perimeter.
8. A rectangular field has an area 28800 sq. metres. Its length is twice as long as its width. What is the length of its sides?
9. Find that least number which, when subtracted from 109087, the answer is a complete square.
10. The cost of levelling the ground of a circular region at a rate of Rs.2 per square metre is Rs.4928. Find the radius of the ground.
11. The cost of ploughing in a square field is Rs.2450 at the rate of Rs.2 per 100 sq. metres. Find the length of the side of the square.
12. The area of a square shaped lawn is 62500 sq. metres. A wooden fence is to be laid around the lawn. What is the length of a wooden fence required? What will be its cost at the rate of Rs.50 per metre?

1.3.6 Cubes and Cube Roots

Recognition of cubes and perfect cubes

- **Cubes**

Cube of a number means to multiply the number by itself three times.

Let x be any number,

then, $x \times x \times x = x^3$ is the cube of x.

For example, $2 \times 2 \times 2 = 2^3$

$3 \times 3 \times 3 = 3^3$

$4 \times 4 \times 4 = 4^3$ and so on.

Try yourself!

Can you find out the perfect cube of –5?

- **Perfect cubes**

Perfect cube is a number that is the result of multiplying an integer by itself three times. In other words it is an integer to the third power of another integer.

Example 21: Show that 8 and 27 are perfect cubes.

Solution: $8 = 2 \times 2 \times 2 = 2^3$ $\therefore$ 8 is a perfect cube of 2

$27 = 3 \times 3 \times 3 = 3^3$ $\therefore$ 27 is a perfect cube of 3

Try yourself!

.... is a perfect cube of 4.

Example 22: Find cube of 1.2

Solution:
$$(1.2)^3 = (1.2) \times (1.2) \times (1.2)$$
$$= (1.44) \times (1.2)$$
$$= 1.728$$

Try yourself!

Can you solve?

$\sqrt[3]{-64} = \boxed{?}$

Finding cube roots of numbers which are perfect cubes

In Mathematics, a cube root of a number, denoted by $x^{\frac{1}{3}}$, is a number 'a' such that $a^3 = x$. i.e. $a = x^{\frac{1}{3}}$ e.g., 216, 64 and 512 are perfect cubes of 6, 4 and 8.

Remember!

Symbol of cube root is $\sqrt[3]{}$ 3 is a part of the symbol.

Example 23: Find the cube root of 125

Solution:

$$125 = 5 \times 5 \times 5 = 5^3$$

$$\sqrt[3]{125} = \sqrt[3]{5 \times 5 \times 5}$$
$$= (5^3)^{1/3}$$
$$= 5$$

5	125
5	25
5	5
	1

Example 24: Find the cube root of 9261

Solution:

$$9261 = 3 \times 3 \times 3 \times 7 \times 7 \times 7$$
$$= 3^3 \times 7^3$$
$$\therefore \quad \sqrt[3]{9261} = \sqrt[3]{3^3 \times 7^3}$$
$$= (3^3 \times 7^3)^{1/3}$$
$$= (3^3)^{1/3} \times (7^3)^{1/3}$$
$$= 3 \times 7$$
$$= 21$$

3	9261
3	3087
3	1029
7	343
7	49
7	7
	1

Think!

Is the index of each prime factor should be a multiple of 3 to find out the cube root? Why?

Recognition of properties of cubes of numbers

(i) Cube of a positive number is positive. e.g., $3^3 = 27$

(ii) Cube of a negative number is negative. e.g.,$(-4)^3 = -64$

(iii) Cube of an even number is even, e.g., $6^3 = 216$

(iv) Cube of an odd number is odd. e.g., $7^3 = 343$

(v) Properties under (a) multiplication and (b) division

(a) $(a \times b)^3 = a^3 \times b^3$ (b) $\left(\frac{a}{b}\right)^3 = \frac{a^3}{b^3}$

(vi) Cube root of the perfect cubes

$6^3 = 216$, $4^3 = 64$, $8^3 = 512$

$\sqrt[3]{216} = 6$ $\sqrt[3]{64} = 4$ $\sqrt[3]{512} = 8$

$$
\begin{aligned}
216 &= 2 \times 2 \times 2 \times 3 \times 3 \times 3 \\
&= 2^3 \times 3^3 \\
&= (2 \times 3)^3 \\
&= 6^3
\end{aligned}
$$

$\therefore$ 216 is a perfect cube of 6

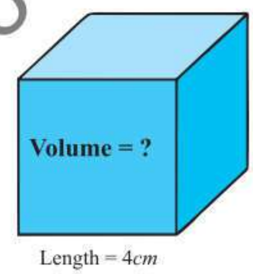
1.3.7 Real Life Problems Involving Cube and Cube Roots

Example 25: Find the volume of cube shaped gift box having the length of a side $4cm$.

Solution:

Let "ℓ" is the length of the cube shaped gift box.

$$\ell = 4cm$$



$$
\begin{aligned}
\text{Volume of the cube shaped gift box} &= \ell \times \ell \times \ell \\
&= 4cm \times 4cm \times 4cm \\
&= 64\ cm^3
\end{aligned}
$$

Thus, the volume of the cube shaped gift box is $64\ cm^3$

Example 26: Find the length of a cube shaped jewelry box if the volume of the jewelry box is $216cm^3$.

Solution:

Let "ℓ" is the length of the jewelry box.

As we know that

Volume of the cube $= \ell \times \ell \times \ell$

Volume of the cube $= \ell^3$

$216cm^3 = \ell^3$

To find out the length of one side of the jewelry box, we will take cube root on both sides.

$$\ell^3 = 216cm^3$$

$$\sqrt[3]{\ell^3} = \sqrt[3]{216}$$

$$\ell^{\cancel{3}\times\frac{1}{\cancel{3}}} = (2 \times 2 \times 2 \times 3 \times 3 \times 3)^{\frac{1}{3}}$$

$$\ell = (2^3 \times 3^3)^{\frac{1}{3}}$$

$$\ell = 2^{\cancel{3}\times\frac{1}{\cancel{3}}} \times 3^{\cancel{3}\times\frac{1}{\cancel{3}}}$$

$$\ell = 6cm$$

2	216
2	108
2	54
3	27
3	9
3	3
	1

Thus, the length of one side of the cube shaped jewelry box is $6cm$.

Exercise 1.11

1. Which numbers are perfect cubes?
   (i) 512 (ii) 1100 (iii) 6859 (iv) 729 (v) $\frac{1000}{125}$
2. Find the cube roots of the following:
   (i) 729 (ii) 15625 (iii) 13824
3. Find the cubes of the following:
   (i) 1.4 (ii) 0.4 (iii) 0.8
4. Find the cube roots of the following:
   (i) $\frac{27}{216}$ (ii) 35937 (iii) 3375
5. If a cube has length of $6cm$, then what will be the volume of the cube?
6. Find the volume of a cube if the length of one side is $8cm$.
7. If the volume of a cube shape is $3375cm^3$, then what is the measure of the edge of the cube?
8. The volume of a cube is $729m^3$. What will be the length of a side of the cube?
9. How many cubical boxes of dimensions $5cm \times 5cm \times 5cm$ can be packed in a large cubical case having volume $875cm^3$?

SUMMARY

- When a number is multiplied by itself, then the product is known as the square of that number i.e., the square of x is $x \times x = x^2$
- Square root and square are inverse of each other. The symbol used for square root is "$\sqrt{\ }$". It is called radical sign.
- If n be the number of digits in the perfect square, then its square root contains:
  (i) $\frac{n}{2}$ digits if n is even (ii) $\frac{n+1}{2}$ digits if n is odd
- Cube of a number means to multiply the number by itself three times.
- Perfect cube is a number that is the result of multiplying an integer by itself three times. In other words, it is an integer to the third power of another integer.
- A cube root of a number, denoted by $x^{\frac{1}{3}}$, is a number such that $a^3 = x$. i.e., $a = x^{\frac{1}{3}}$
- Symbol of cube root is $\sqrt[3]{\ }$. Remember that 3 is the part of symbol.
- Cube of an even number is even, e.g., $6^3 = 216$
- Cube of an odd number is odd. e.g., $7^3 = 343$

Review Exercise 1 (a)

1. Four options are given against each statement. Encircle the correct one.

i. Real number is:
- (a) Difference of rational number and irrational numbers
- (b) Intersection of rational numbers and irrational numbers
- (c) Union of rational number and irrational numbers
- (d) Complete set of natural numbers.

ii. Which of the following is true about $\sqrt{8}$?
- (a) Natural number (b) Whole number
- (c) Rational number (d) Irrational number

iii. Irrational numbers are non terminating and ____________ numbers.
- (a) rational number (b) non-recurring
- (c) recurring (d) decimal

iv. Round 0.0234589 to the 4 significant figures:
- (a) 0.02346 (b) 0.02345 (c) 2346 (d) 0.0235

v. The difference between approximated value and actual accurate value is called:
- (a) round (b) significant figure
- (c) approximation error (d) estimated value.

vi. Which one of the following is perfect square?
- (a) 25.6 (b) 0.256 (c) 2.56 (d) 2560

vii. Square of 0.9 is:
- (a) 0.081 (b) 8.10 (c) 0.81 (d) 81.027

viii. $\left(\frac{7}{9}\right)^2 = ?$
- (a) $\frac{49}{6}$ (b) $\frac{7}{81}$ (c) $\frac{49}{81}$ (d) $\frac{7}{3}$

ix. 1+2+3+4+5+6+7+8+7+6+5+4+3+2+1=?
- (a) 8^2 (b) 9^2 (c) 65 (d) 81

x. If the side length of a square is 0.5m, then its area is:
- (a) 0.50 m^2 (b) 2.5 m^2 (c) 0.25 m^2 (d) 25 m^2

xi. $\sqrt{0.04} = ?$
- (a) 0.02 (b) 2.0 (c) 0.2 (d) 20

xii. $\sqrt{1^2 \times 4^2} = ?$
- (a) 4 (b) 14 (c) 41 (d) 2

xiii. $\sqrt{\frac{4}{9}} = ?$

(a) $\frac{2}{3}$ (b) $\frac{2}{9}$ (c) $\frac{4}{3}$ (d) $\frac{3}{2}$

xiv. $\sqrt{\frac{a}{b}} = ?$

(a) $\frac{a}{b}$ (b) ab (c) $\sqrt{\frac{b}{a}}$ (d) $\frac{\sqrt{a}}{\sqrt{b}}$

2. Identify terminating, non-terminating and recurring decimal fractions from the following:

(i) $\frac{17}{5}$ (ii) $\frac{23}{3}$ (iii) $\frac{49}{8}$ (iv) $\frac{8}{15}$

3. Separate rational and irrational numbers.

(i) $\frac{25}{6}$ (ii) $\frac{55}{7}$ (iii) 8.111111 (iv) $\sqrt{21}$

4. Identify the property of real numbers used in the following:

(i) $25+(7+5)=(25+7)+5$ (ii) $17+(-17)=0$ (iii) $\sqrt{z}+0=\sqrt{z}$ (iv) $25\times 5=5\times 25$

5. Round the following to the given degree of accuracy:

(i) 598235 (4 s.f.) (ii) 0.002859 (3 s.f.) (iii) 45.35946 (2 d.p.)

(iv) $\frac{25}{8}$ (2 d.p.) (v) $\frac{15}{7}$ (2 d.p.) (vi) 0.078952 (4 s.f.)

6. Javeria bought 25.75 metres of cloth and Salma bought 78.28 metres of cloth. Find the total length of cloth. Also round the answer too.

(i) 3 s.f. (ii) 1 d.p.

7. The cost of 10 kg rice is Rs. 1557 and the cost of 15 kg sugar is Rs. 1278. What will be cost of both the items. Also round the answer up to 2 significant figures.

8. Find the number of digits in the square root of the following numbers. Also find the square root.

(i) 9604 (ii) 30625 (iii) 12544 (iv) 422500

9. Find the square root of the following:

(i) $28\frac{4}{9}$ (ii) $17\frac{128}{289}$ (iii) $101\frac{92}{169}$ (iv) 0.053361

(v) 0.204304 (vi) 152.7696 (vii) 0.316406 (viii) 38.01 (2 d.p.)

(ix) 6422.53 (2 d.p.)

10. If the area of a square field is 161604 m^2. Find the length of its one side.

11. Saeeda has 196 marbles that she is using to make a square formation. How many marbles should be in each row?

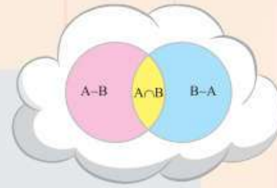
12. Find the cube root of the following numbers:

(i) 1728 (ii) 3375 (iii) $\frac{216}{125}$

13. The volume of a cube is 512 mm^3. What will be the length of a side of the cube?

14. What will be the volume of cube, whose side is 10 cm long?

Sub-Domain (iv): Sets



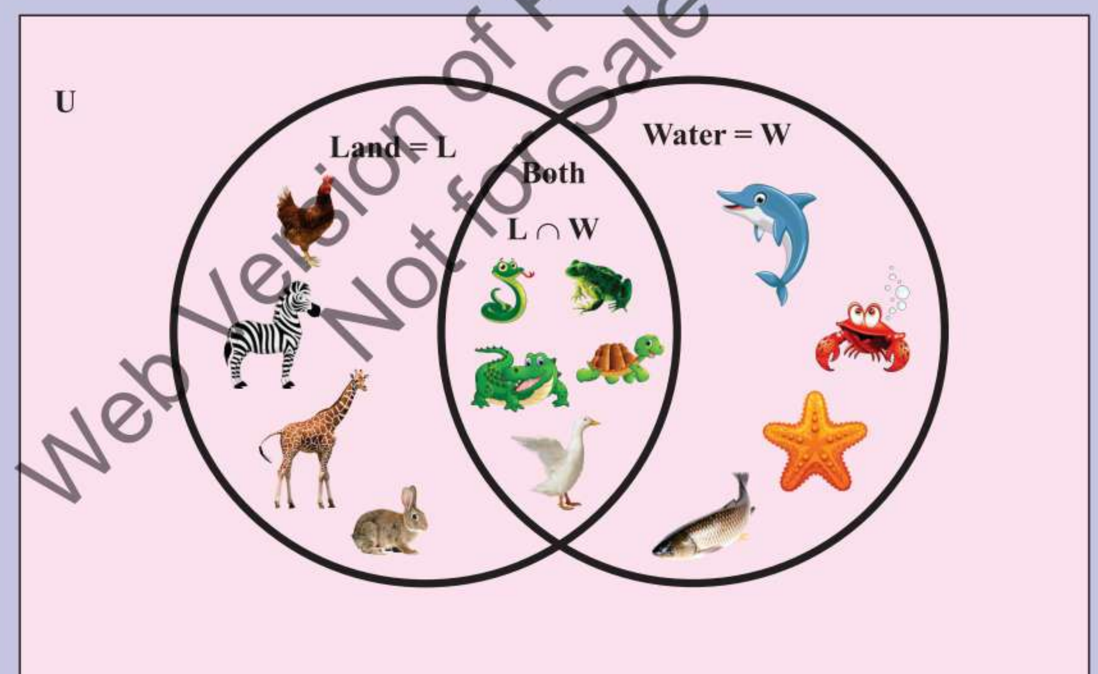
Students' Learning Outcomes

After completing this sub-domain, the students will be able to:

- discover sets in natural numbers
- express sets using tabular, descriptive and set-builder notations
- differentiate equivalent, and equal sets
- write subsets
- write power set P(A) of a set A, where A has up to four elements
- describe operations on sets tabular form:
  - ❖ union of two sets
  - ❖ intersection of two sets
  - ❖ difference of two sets
  - ❖ complement of a set
  - ❖ apply sets in real life situations
- describe operations on sets by using Venn diagram
  - ❖ union of two sets
  - ❖ intersection of two sets
  - ❖ difference of two sets
  - ❖ complement of two sets
- `Verify commutative, associative and distributive laws w.r.t. union and intersection.

Advanced/ Additional

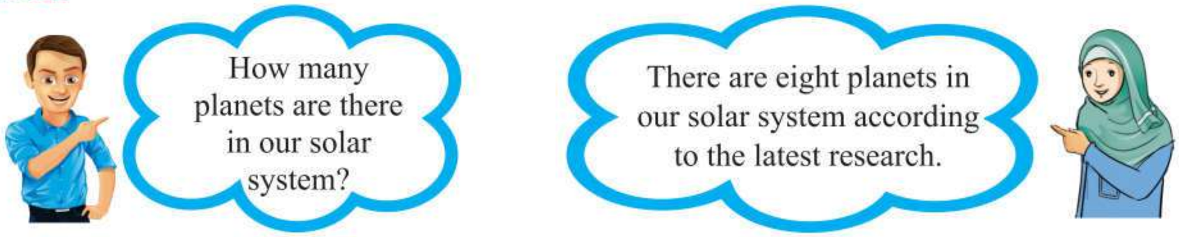
- use Venn diagram to demonstrate union and intersection of two sets (subsets, overlapping sets and disjoint sets)
- describe operations on sets by using Venn diagram:
- verify De Morgans's Laws and represent through Venn diagram.



Do you know?
Frog is an amphibian. Amphibians are cold-blooded vertebrates (Vertebrates have backbone). Amphibian comes from a Greek word means "both lives". Frogs start their lives in the water and then live on land.

Can you write the set of the names of animals which live in water and on land?

1.4.1 Sets



The set of eight planets revolve around the sun. The names of these planets are: Mercury, Venus, Earth, Mars, Jupiter, Saturn, Uranus and Neptune.

A set is a collection of well-defined distinct objects or symbols.

Some important sets and their notations

Set of natural numbers	$N = \{1, 2, 3, 4, \ldots\}$
Set of whole numbers	$W = \{0, 1, 2, 3, \ldots\}$
Set of integers	$Z = \{0, \pm1, \pm2, \pm3, \ldots\}$
Set of prime numbers	$P = \{2, 3, 5, 7, \ldots\ldots\}$
Set of odd integers	$O = \{\pm1, \pm3, \pm5, \ldots\}$
Set of even integers	$E = \{0, \pm2, \pm4, \pm6, \ldots\}$
Set of rational numbers	$Q = \{\frac{p}{q} \mid p, q \in Z \wedge q \neq 0\}$

Key fact!

The objects of a set are called its members or elements.

Expression of a set

A set can be expressed in three ways.

(a) Tabular Form or Roster Form

(b) Descriptive Form

(c) Set Builder Notation

(a) Tabular or Roster Form:

In this form, all the elements of a set are expressed within braces "{}" and each element of the set is separated by comma ",".

For Example:

(i) $A = \{2, 4, 6, 8, 10\}$

(ii) $B = \{1, 3, 5, 7, 9\}$

(iii) $C = \{0, \pm1, \pm2, \pm3, \ldots\}$

(b) Descriptive Form:

In this form, a set is described by a given statement (sentence). The statement indicates all the elements of that set.

For Example:

(i) L = Set of five odd numbers

(ii) N = Set of natural numbers

(iii) S = Set of provinces of Pakistan

(c) Set-Builder Notation

In this form, all the members of a set are expressed by an alphabet showing all the properties of the members of that set.

For Example:

(i) $A = \{1, 2, 3, 4, \ldots, 8\}$ is expressed in set builder notation as:
$A = \{x \mid x \in N \wedge x \leq 8\}$

(ii) $B = \{\pm 1, \pm 3, \pm 5, \ldots\}$ is written in set builder notation as:
$B = \{x \mid x \in O\}$

(iii) $C = \{0, 1, 2, 3, \ldots, 10\}$
$C = \{y \mid y \in W \wedge y \leq 10\}$

Remember!

- The symbol ":" or "|" is read as "such that".
- The symbol "$\wedge$" is used for "and".
- The symbol "$\vee$" is used for "or".
- The symbol "$\in$" is used for "belongs to".

1.4.2 Difference between equivalent and equal sets

Equivalent sets

Equivalent sets are the sets with an equal number of elements. These sets do not have exactly the same elements, just these sets have the same number of elements.

For Example:

$D = \{1, 2, 3, 4, 5\}$
$E = \{L, M, N, O, P\}$

Although set D and set E have completely different elements but they have same number of elements. So, these are equivalent sets and also written as $D \sim E$.

Remember!

Equal sets are always equivalent but equivalent sets are not always equal sets.

Do you know?

"$\sim$" indicates equivalent sign.

Equal sets

When two sets have the same elements, these sets are called equal sets. It does not matter what order the elements are in.

For Example:

$A = \{2, 4, 6, 8\}$
$B = \{8, 6, 4, 2\}$

These are equal sets i.e., $A = B$

Remember!

- Two sets are equal if they contain the same elements.
- Two sets are equivalent if these sets have the same number of elements.

Do you know?

$\{a, b, c\} = \{c, b, a\}$

Exercise 1.12

1. Write the following sets in descriptive form:

(i) $A = \{1, 3, 5, 7, 9\}$ (ii) $B = \{2, 4, 6, 8, 10\}$
(iii) $C = \{0, \pm 1, \pm 2, \ldots, \pm 10\}$ (iv) $D = \{a, e, i, o, u\}$
(v) $E = \{x \mid x \in N \wedge x \leq 100\}$ (vi) $F = \{y \mid y \in P\}$

2. Write the following sets in tabular form:
   (i) A = Set of whole numbers.
   (ii) B = Letters of the word "football".
   (iii) C = Set of integers.
   (iv) D = Set of solar months start with letter "J".
   (v) E = Set of English alphabet.
   (vi) F = Numbers less than 30 and divisible by 5.
3. Write the following sets in set builder notation:
   (i) $A = \{0, 1, 2, 3, \ldots, 10\}$
   (ii) B = Set of odd numbers
   (iii) $C = \{3, 6, 9, 12, 15\}$
   (iv) $D = \{a, b, c, \ldots, z\}$
   (v) E = Set of days in a week.
   (vi) F = Set of prime numbers between 10 and 30.
4. Choose equivalent sets from the following:
   (i) $A = \{a, b, c, d\}$, $B = \{2, 4, 6\}$
   (ii) $C = \{1, 3, 5, 7\}$, $D = \{0, 1, 2, 3\}$
   (iii) E = Set of vowel letters , $F = \{L, M, N, O, P\}$
   (iv) $G = \{1, 2, 3, 4, 5\}$, $H = \{0, 1, 2, 3, 4, 5\}$
5. Choose equal sets from the following:
   (i) $A = \{3, 6, 9, 12\}$, $B = \{9, 12, 3, 6\}$
   (ii) $C = \{a, e, i, o, u\}$, $D = \{a, e, i, o\}$
   (iii) $E = \{2, 3, 5, 7\}$, F = Set of first 4 prime numbers
   (iv) $G = \{a, b, c, d\}$, $H = \{2, 4, 6, 8\}$

1.4.3 Subset

Consider that A and B are any two sets, if all elements of set A are also elements of set B, then set A is called subset of set B.

For Example:

$A = \{2, 4, 6\}$, $B = \{2, 4, 6, 8\}$

From these sets it is clear that all the elements of set A are the elements of set B. So, set A is subset of set B. Mathematically, it can be written as $A \subseteq B$.

1.4.4 Power Set

A set consisting of all possible subsets of a given set A is called the power set of A and is denoted by P(A).

For Example,

If $A = \{a, b\}$, then all its subsets are: $\phi, \{a\}, \{b\}, \{a, b\}$

So, power set of A, $P(A) = \{\phi, \{a\}, \{b\}, \{a, b\}\}$

Example 1: Write the power set of $B = \{3, 6, 9, 12\}$

Solution: $P(B) = \{\phi, \{3\}, \{6\}, \{9\}, \{12\}, \{3,6\}, \{3,9\}, \{3,12\}, \{6,9\}, \{6,12\}, \{9,12\}, \{3,6,9\}, \{3,6,12\}, \{3,9,12\}, \{6,9,12\}, \{3, 6, 9, 12\}\}$

Can you tell?

If a set A consists of 4 elements, then how many elements are in $P(A)$?

Remember!

If a set contains 'n' elements, then the number of all its subsets will be 2^n:
For example, if $X = \{1, 2, 3\}$, then all its subsets are ϕ, $\{1\}$, $\{2\}$, $\{3\}$, $\{1,2\}$, $\{2,3\}$, $\{1,3\}$, $\{1,2,3\}$ which are 8 in number and $2^3 = 8$

Note that:

- The members of P(A) are all subsets of set A i.e., $\{a\} \in P(A)$ but $a \notin P(A)$
- The power set of ϕ is not empty as number of subsets of ϕ is $2^0 = 1$
  i.e. $P(\phi) = \{\phi\}$ or $\{\{\}\}$

Exercise 1.13

1. Write all subsets of the following sets:
   (i) $\{\}$ (ii) $\{1\}$ (iii) $\{a, b\}$
2. Write the power set of the following sets:
   (i) $\{-1, 1\}$ (ii) $\{a, b, c\}$ (iii) $\{+, -, \times, \div\}$

1.4.5 Describe Operations on Sets in Tabular Form

Union of two sets

The union of two sets is a set containing all elements of both the sets and is denoted by the symbol $\cup$.

For Example:

If $A = \{1, 2, 3\}, B = \{4, 5, 6\}$

then $A \cup B = \{1, 2, 3\} \cup \{4, 5, 6\}$
$= \{1, 2, 3, 4, 5, 6\}$

Intersection of two sets

The intersection of two sets is the set of elements that are common in both the sets and is denoted by the symbol $\cap$.

For Example:

If $X = \{1, 2, 3, 4\}, \quad Y = \{2, 4, 6, 8\}$

then $X \cap Y = \{1, 2, 3, 4\} \cap \{2, 4, 6, 8\}$
$= \{2, 4\}$

Difference of two sets

If A and B are any two sets, then their difference is given by A–B or B–A. A–B means the set of those elements of A which are not in B.

For Example:

If $A = \{a, b, c, d\}, B = \{a, e, i, o\}$

then $A - B = \{a, b, c, d\} - \{a, e, i, o\}$
$= \{b, c, d\}$

and $B - A = \{a, e, i, o\} - \{a, b, c, d\}$
$= \{e, i, o\}$

Remember!

We can write $A-B$ as $A \backslash B$ for difference of two sets.

Universal set

A set that consists of all the elements of the sets under consideration, including its own elements, is denoted by the symbol U.

Let us consider an example with two sets A and B.

Here $A = \{a, b, c, d\}, B = \{e, f, g, h\}$

Thus, the universal set U of A and B may be given by

$U = \{a, b, c, d, e, f, g, h\}$

Complement of a set

The complement of a set A is defined as a set that contains the elements present in the universal set but not in set A.

For Example:

If $U = \{1, 2, 3, 4, \ldots, 10\}$ and $A = \{2, 4, 6, 8\}$, then the complement of set A,

$A' = U - A$

$= \{1, 2, 3, 4, \ldots\ldots, 10\} - \{2, 4, 6, 8\}$

$A' = \{1, 3, 5, 7, 9, 10\}$

The complement of a set A is denoted by A^c or A'.

Remember!

We can write A' as A^c for complement of a set A.

1.4.6 De Morgan's Laws

If A and B are the subsets of a universal set U, then

(a) $(A \cup B)^c = A^c \cap B^c$ (b) $(A \cap B)^c = A^c \cup B^c$

Augustus De Morgan (1806-1871), a British Mathematician who formulated De Morgan's laws.

Example 2: Verify De Morgan's Laws if:

$U = \{1, 2, 3, \ldots, 10\}$, $A = \{2, 4, 6\}$ and $B = \{1, 2, 3, 4, 5, 6, 7\}$

Solution: **(a)** L.H.S $= (A \cup B)^c$

$A \cup B = \{2, 4, 6\} \cup \{1, 2, 3, 4, 5, 6, 7\}$

$= \{1, 2, 3, 4, 5, 6, 7\}$

$\therefore \quad (A \cup B)^c = U - (A \cup B) = \{8, 9, 10\}$ .... (i)

R.H.S $= A^c \cup B^c$

$A^c = U - A$

$= \{1, 2, 3, \ldots, 10\} - \{2, 4, 6\}$

$= \{1, 3, 5, 7, 8, 9, 10\}$

$B^c = U - B$

$B^c = \{1, 2, 3, \ldots, 10\} - \{1, 2, 3, 4, 5, 6, 7\}$

$B^c = \{8, 9, 10\}$

$A^c \cap B^c = \{1, 3, 5, 7, 8, 9, 10\} \cap \{8, 9, 10\}$

$= \{8, 9, 10\}$ .... (ii)

Thus, from (i) and (ii) we have $(A \cup B)^c = A^c \cap B^c$

(b) L.H.S $= (A \cap B)^c$

$A \cap B = \{2, 4, 6\} \cap \{1, 2, 3, 4, 5, 6, 7\}$

$= \{2, 4, 6\}$

$(A \cap B)^c = U - (A \cap B)$

$= \{1, 2, 3, \ldots, 10\} - \{2, 4, 6\}$

$= \{1, 3, 5, 7, 8, 9, 10\}$ .... (iii)

R.H.S $= A^c = U - A = \{1, 2, 3, \ldots, 10\} - \{2, 4, 6\}$

$= \{1, 3, 5, 7, 8, 9, 10\}$

and $B^c = U - B = \{1, 2, 3, \ldots, 10\} - \{1, 2, 3, 4, 5, 6, 7\}$

$$= \{8, 9, 10\}$$

$$A^c \cup B^c = \{1, 3, 5, 7, 8, 9, 10\} \cup \{8, 9, 10\}$$

$$= \{1, 3, 5, 7, 8, 9, 10\} \quad \text{.................} \quad \text{(iv)}$$

Thus, from (iii) and (iv), we have $(A \cap B)^c = A^c \cup B^c$

1.4.7 Venn Diagram

A Venn diagram is an illustration that uses circles or ovals to show the relationships among sets in a perspective way. In Venn diagram, a universal set is usually represented by a rectangle and its subsets are represented by closed figures inside the rectangle, e.g.;

$$U = \{1, 3, 5, 7, 9\},\ A = \{1, 5, 9\} \text{ and } B = \{5, 9, 11, 13\}$$

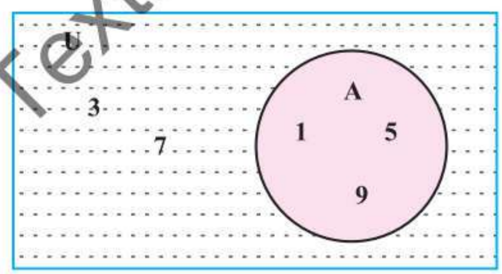
Do you know?

An English logician and the Mathematician John Venn (1834 - 1883A.D) used Venn Diagram first time.

The rectangular region shown in the figure represents the universal set U and the region enclosed by a closed circle inside the rectangular region represents the set A.

The dotted region of U outside set A represents complement of A i.e. A' or A^c

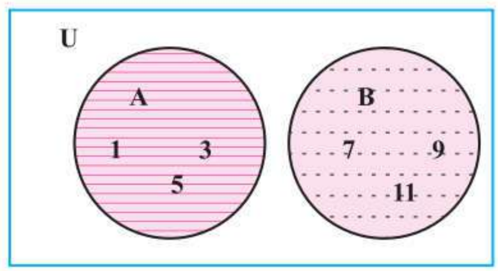
Thus, $A' = \{3, 7\}$



Disjoint sets

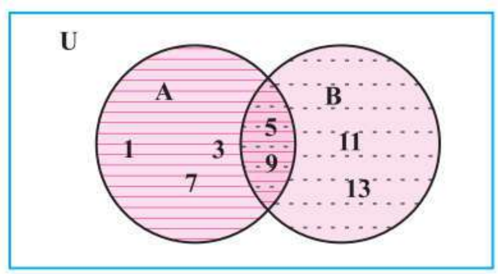
In the given figure, no element of set A and set B is common. Therefore, these sets are disjoint sets. So, the lined portion and the dotted portion represents $A \cup B$. In the same way the dotted portion and the lined portion represents $B \cup A$.

For disjoint sets, $A \cap B = \phi$



Overlapping sets

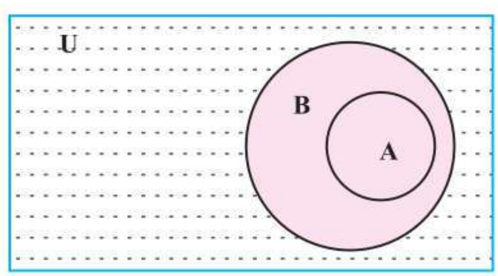
In the given figure, only a small portion is common in both the sets A and B. So, the dotted and the lined common portion between set A and set B is called overlapping sets. This portion is also called $A \cap B$ or $B \cap A$.



Subsets of a set

In the given figure, the rectangular region represents U (Universal set) and set A and B represent its subsets.

Here $A \subset B$ means all the elements of set A are present in set B.

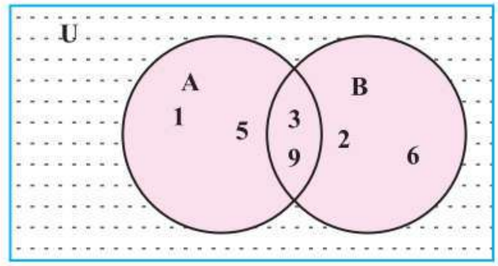


Union of two sets

In the given figure, the total region bounded by set A and set B represents $A \cup B$.

Therefore, $$A \cup B = \{1, 3, 5, 9\} \cup \{2, 3, 6, 9\}$$
$$= \{1, 2, 3, 5, 6, 9\}$$

The shaded part represents $A \cup B$.

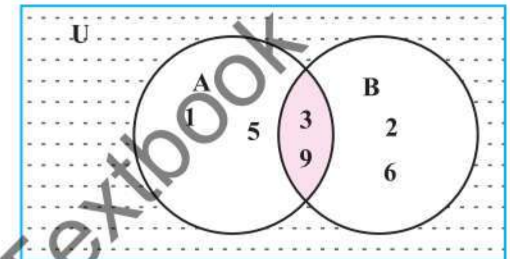


Intersection of two sets

In the given figure, the common region between the two sets A and B represents $A \cap B$.

Therefore, $$A \cap B = \{1, 3, 5, 9\} \cap \{2, 3, 6, 9\}$$
$$= \{3, 9\}$$

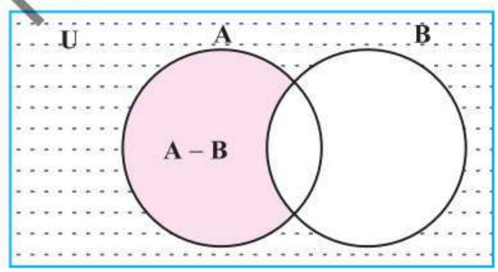
The shaded part represents $A \cap B$.



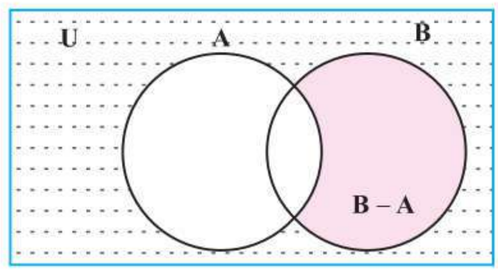
Difference of two sets

The set of those elements of A which are not in B, is called difference of two sets A and B. It is denoted by $A - B$.

In the given figure, the shaded portion represents $A - B$.



The set of those elements of B which are not in set A is denoted by $B - A$. In the given figure, the shaded portion represents $B - A$.

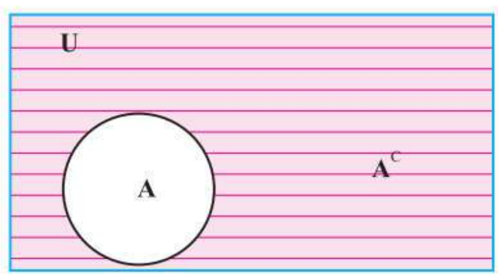


Complement of a set

The given figure represents complement of set A.

$$A^c = U - A$$

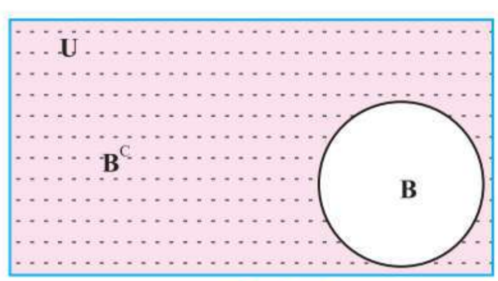
The shaded part represents A^c.



The given figure represents complement of set B.

$$B^c = U - B.$$

The shaded part represents B^c.



1.4.8 Operations on Sets

Verification of Commutative and Associative Laws with respect to Union and Intersection

- **Commutative Laws of Union and Intersection on Sets**

If A and B are any two sets, then the commutative laws with respect to union and intersection are written as:

(i) $A \cup B = B \cup A$ (Commutative law over union)

(ii) $A \cap B = B \cap A$ (Commutative law over intersection)

Remember!

To find union / intersection of three sets, first we find the union / intersection of any two of them and then the union / intersection of the third set with the resultant set.

Example 3: If $A = \{1, 2, 3, ..., 10\}$ and $B = \{3, 5, 7, 9\}$

(i) Verify the commutative law of union

(ii) Verify the commutative law of intersection

Solution: $A = \{1, 2, 3, ..., 10\}$, $B = \{3, 5, 7, 9\}$

(I) $A \cup B = \{1, 2, 3 ... 10\} \cup \{3, 5, 7, 9\}$

$= \{1, 2, 3, ..., 10\}$

$B \cup A = \{3, 5, 7, 9\} \cup \{1, 2, 3, ..., 10\}$

$= \{1, 2, 3, ..., 10\}$

Therefore, $A \cup B = B \cup A$

(ii) $A \cap B = \{1, 2, 3 ... 10\} \cap \{3, 5, 7, 9\}$

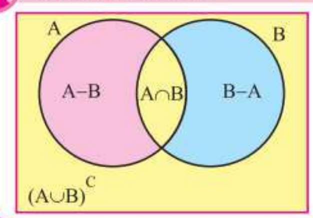
$= \{3, 5, 7, 9\}$

$B \cap A = \{3, 5, 7, 9\} \cap \{1, 2, 3, ..., 10\}$

$= \{3, 5, 7, 9\}$

Therefore, $A \cap B = B \cap A$

Remember!



- **Associative Laws of Union and Intersection**

If A, B and C are any three sets, then the associative laws with respect to union and intersection are written respectively as:

(I) $A \cup (B \cup C) = (A \cup B) \cup C$

(II) $A \cap (B \cap C) = (A \cap B) \cap C$

Example 4: Verify the associative law of union

$A \cup (B \cup C) = (A \cup B) \cup C$

where $A = \{1, 2, 3, 4\}$, $B = \{3, 4, 5, 6, 7, 8\}$ and $C = \{6, 7, 8, 9, 10\}$

Solution: L.H.S $= A \cup (B \cup C) = \{1, 2, 3, 4\} \cup (\{3, 4, 5, 6, 7, 8\} \cup \{6, 7, 8, 9, 10\})$

$= \{1, 2, 3, 4\} \cup \{3, 4, 5, 6, 7, 8, 9, 10\}$

$= \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ ………………… (i)

R.H.S $= (A \cup B) \cup C = (\{1, 2, 3, 4\} \cup \{3, 4, 5, 6, 7, 8\}) \cup \{6, 7, 8, 9, 10\}$

$= \{1, 2, 3, 4, 5, 6, 7, 8\} \cup \{6, 7, 8, 9, 10\}$

$= \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ ………………… (ii)

Thus, from (i) and (ii), we conclude that $A \cup (B \cup C) = (A \cup B) \cup C$

Example 5: Verify the associative law of intersection
$A \cap (B \cap C) = (A \cap B) \cap C$ for sets given in example (4).

Solution: L.H.S $= A \cap (B \cap C) = \{1, 2, 3, 4\} \cap (\{3, 4, 5, 6, 7, 8\} \cap \{6, 7, 8, 9, 10\})$
$= \{1, 2, 3, 4\} \cap \{6, 7, 8\} = \phi$ .... (i)
R.H.S $= (A \cap B) \cap C = (\{1, 2, 3, 4\} \cap \{3, 4, 5, 6, 7, 8\}) \cap \{6, 7, 8, 9, 10\}$
$= \{3, 4\} \cap \{6, 7, 8, 9, 10\} = \phi$ .... (ii)

Thus, from (i) and (ii), we conclude that $A \cap (B \cap C) = (A \cap B) \cap C$

Verification of Distributive Laws

If A, B and C are any three sets, then $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ is called the distributive law of union over intersection.

If A, B and C are three sets, then $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ is called the distributive law of intersection over union.

Example 6: Verify:

(I) Distributive law of union over intersection
(II) Distributive law of intersection over union

where $A = \{1, 2, 3,..., 20\}$, $B = \{5, 10, 15,..., 30\}$ and $C = \{3, 9, 15, 21, 27, 33\}$

Solution: (I) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
L.H.S $= A \cup (B \cap C) = \{1, 2, 3,..., 20\} \cup (\{5, 10, 15,..., 30\} \cap \{3, 9, 15, 21, 27, 33\})$
$= \{1, 2, 3, ..., 20\} \cup \{15\}$
$A \cup (B \cap C) = \{1, 2, 3,..., 20\}$ .... (i)
R.H.S $= (A \cup B) \cap (A \cup C)$

$A \cup B = \{1, 2, 3, ..., 20\} \cup \{5, 10, 15,..., 30\}$
$= \{1, 2, 3,..., 20, 25, 30\}$
and $A \cup C = \{1, 2, 3,...., 20\} \cup \{3, 9, 15, 21, 27, 33\}$
$= \{1, 2, 3,..., 20, 21, 27, 33\}$
$(A \cup B) \cap (A \cup C) = \{1, 2, 3, 4,..., 20, 25, 30\} \cap \{1, 2, 3 ,..., 20, 21, 27, 33\}$
$(A \cup B) \cap (A \cup C) = \{1, 2, 3,...., 20\}$ .... (ii)

Thus, from (i) and (ii), we conclude that $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

(II) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

L.H.S $= A \cap (B \cup C) = \{1,2, 3,..., 20\} \cap (\{5, 10, 15,... , 30\} \cup (3, 9, 15, 21, 27, 33\})$
$= \{1, 2, 3,..., 20\} \cap \{3, 5, 9, 10,15, 20, 21, 25, 27, 30, 33\}$
$A \cap (B \cup C) = \{3, 5, 9, 10, 15, 20\}$ .... (i)

R.H.S $= (A \cap B) \cup (A \cap C)$
$A \cap B = \{1, 2, 3,..., 20\} \cap \{5, 10, 15,..., 30\}$
$= \{5, 10, 15, 20\}$

and $\quad A \cap C = \{1, 2, 3, ..., 20\} \cap \{3, 9, 15, 27, 33\}$

$\quad\quad = \{3, 9, 15\}$

$(A \cap B) \cup (A \cap C) = \{3, 5, 9, 10, 15, 20\} \cup (3, 9, 15\}$

$(A \cap B) \cup (A \cap C) = \{3, 5, 9, 10, 15, 20\}$ ………………… (ii)

Thus, from (i) and (ii), we conclude that $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

Exercise 1.14

1. Verify:

   (a) $A \cup B = B \cup A$ and (b) $A \cap B = B \cap A$, when

   (i) $A = \{1, 2, 3, ..., 10\}$, $B = \{7, 8, 9, 10, 11, 12\}$

   (ii) $A = \{1, 2, 3, ..., 15\}$, $B = \{6, 8, 10, \ldots, 20\}$

2. Verify:

   (a) $X \cup (Y \cup Z) = (X \cup Y) \cup Z$ and (b) $X \cap (Y \cap Z) = (X \cap Y) \cap Z$, when

   (i) $X = \{a, b, c, d\}$, $Y = \{b, d, c, f\}$ and $Z = \{c, f, g, h\}$

   (ii) $X = \{1, 2, 3, ..., 10\}$, $Y = \{2, 4, 6, 7, 8\}$ and $Z = \{5, 6, 7, 8\}$

   (iii) $X = \{-1, 0, 2, 4, 5\}$, $Y = \{1, 2, 3, 4, 7\}$ and $Z = \{4, 6, 8, 10\}$

   (iv) $X = \{1, 2, 3, ..., 14\}$, $Y = \{6, 8, 10, ..., 20\}$ and $Z = \{1, 3, 5, 7\}$

3. If $A = \{a, b, c\}$, $B = \{b, d, f\}$ and $C = \{a, f, c\}$, then show that:

   $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

4. If $A = \{0\}$, $B = \{0, 1\}$ and $C = \{\,\}$, then show that:

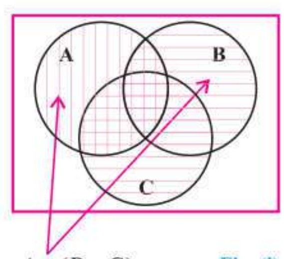
   $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

5. Verify De Morgan's Laws if:

   $U = N$, $A = \phi$ and $B = P$

1.4.9 Demonstration of Union and Intersection of three overlapping sets through Venn diagram

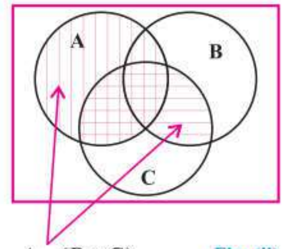
(i) $A \cup (B \cup C)$



$A \cup (B \cup C)$ Fig. (i)

In fig. (i) set $B \cup C$ is represented by horizontal lines and set A is represented by vertical lines. Thus, $A \cup (B \cup C)$ is represented by (horizontal, vertical) lines and squares.

(ii) $A \cup (B \cap C)$

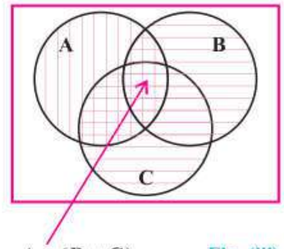


$A \cup (B \cap C)$ Fig. (ii)

In fig. (ii) set $B \cap C$ is represented by horizontal lines and set A is represented by vertical lines. Thus, $A \cup (B \cap C)$ is represented by (horizontal, vertical) lines and squares.

(iii) $A\cap(B\cup C)$

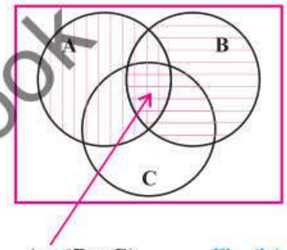
In fig. (iii) set $B \cup C$ is represented by horizontal lines and set A is represented by vertical lines. Thus, $A \cap (B \cup C)$ is represented only by squares i.e, small boxes.



$A\cap(B\cup C)$ Fig. (iii)

(iv) $A\cap(B\cap C)$

In fig. (iv) set $B \cap C$ is represented by horizontal lines and set A is represented by vertical lines. Thus, $A \cap (B \cap C)$ is represented only by squares i.e, small boxes.



$A\cap(B\cap C)$ Fig. (iv)

Verify associative and distributive laws through Venn diagram

- **Associative Laws**

(a) Associative Law of Union

$A\cup(B\cup C)=(A\cup B)\cup C$

Let $A = \{1, 3, 5, 7, 9, 10\}$, $B = (2, 4, 6, 8, 9, 10\}$ and
$C = \{2, 3, 5, 7, 11, 13\}$

L.H.S $= A\cup(B\cup C)$

$B \cup C = \{2, 4, 6, 8, 9, 10\} \cup \{2, 3, 5, 7, 11, 13\}$
$= \{2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 13\}$

$A\cup(B\cup C) = \{1, 3, 5, 7, 9, 10\} \cup \{2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 13\}$
$=\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 13\}$

R.H.S $= (A\cup B)\cup C$

$A \cup B = \{1, 3, 5, 7, 9, 10\} \cup \{2, 4, 6, 8, 9, 10\}$
$= \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

$(A\cup B)\cup C = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\} \cup \{2, 3, 5, 7, 11, 13\}$
$= \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 13\}$

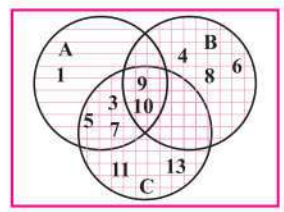


Fig. (v)

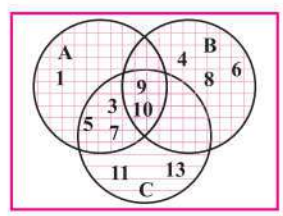


Fig. (vi)

From fig. (v) and (vi), it is clear that:

$A\cup(B\cup C)=(A\cup B)\cup C$

(b) Associative Law of Intersection

$A\cap(B\cap C)=(A\cap B)\cap C$

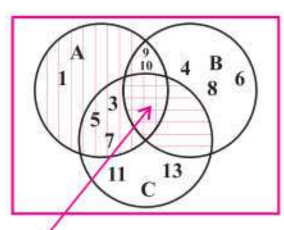
$A = \{1, 3, 5, 7, 9, 10\}$, $B = \{2, 4, 6, 8, 9, 10\}$ and $C = \{2, 3, 5, 7, 11, 13\}$

L.H.S $= A\cap(B\cap C)$

$B\cap C = \{2, 4, 6, 8, 9, 10\} \cap \{2, 3, 5, 7, 11, 13\} = \{2\}$

$A\cap(B\cap C) = \{1, 3, 5, 7, 9, 10\} \cap \{2\} = \{\ \}$

Horizontal lines represent $B\cap C$ and vertical lines represent A, then squares (boxes) represent $A\cap(B\cap C)$. Thus, $A\cap(B\cap C) = \{\ \}$.



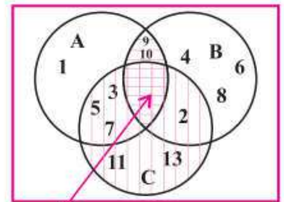
$A\cap(B\cap C)$ Fig. (vii)

R.H.S $= (A \cap B) \cap C$

$A \cap B = \{1, 3, 5, 7, 9, 10\} \cap \{2, 4, 6, 8, 9, 10\} = \{9, 10\}$

$(A \cap B) \cap C \quad = \{9, 10\} \cap \{2, 3, 5, 7, 11, 13\} = \{\ \}$

Horizontal lines represent $A \cap B$ and vertical lines represent C, then squares represent $(A \cap B) \cap C$. Thus, $(A \cap B) \cap C = \{\ \}$



$(A \cap B) \cap C$ Fig. (viii)

From fig. (vii) and (viii), it is clear that $A \cap (B \cap C) = (A \cap B) \cap C$

- **Distributive Laws**

(a) Distributive Law of Intersection over Union

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

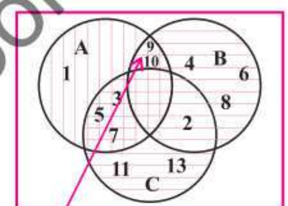
Let $A = \{1, 3, 5, 7, 9, 10\}$, $B = \{2, 4, 6, 8, 9, 10\}$ and
$C = \{2, 3, 5, 7, 11, 13\}$

L.H.S $= A \cap (B \cup C)$

$B \cup C = \{2, 4, 6, 8, 9, 10\} \cup \{2, 3, 5, 7, 11, 13\}$

$B \cup C \quad = (2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 13\}$

$A \cap (B \cup C) \quad = \{1, 3, 5, 7, 9, 10\} \cap \{2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 13\} = \{3, 5, 7, 9, 10\}$



$A \cap (B \cup C)$ Fig. (ix)

Horizontal lines represent $B \cup C$ and vertical lines represent A. Then the squares represent $A \cap (B \cup C)$.

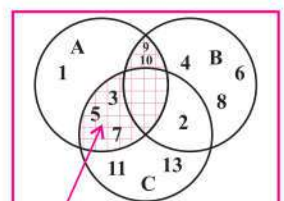
Thus, $A \cap (B \cup C)$ is represented by squares (both horizontal and vertical lines).

R.H.S $= (A \cap B) \cup (A \cap C)$

$A \cap B = \{1, 3, 5, 7, 9, 10\} \cap \{2, 4, 6, 8, 9, 10\}$
$= \{9, 10\}$

$A \cap C = \{1, 3, 5, 7, 9, 10\} \cap \{2, 3, 5, 7, 11, 13\}$
$= \{3, 5, 7\}$

$(A \cap B) \cup (A \cap C) = \{9, 10\} \cup \{3, 5, 7\} = \{3, 5, 7, 9, 10\}$



$(A \cap B) \cup (A \cap C)$ Fig. (x)

Horizontal lines represent $A \cap B$, vertical lines represent $A \cap C$ and squares (both horizontal and vertical lines) represent $(A \cap B) \cup (A \cap C)$.

From fig. (ix) and (x), it is clear that $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.
Hence distributive law of intersection over union holds.

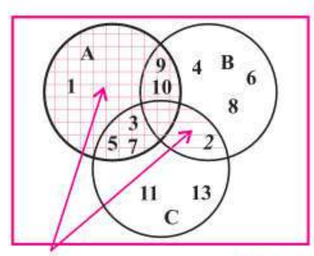
(b) Distributive Law of Union over Intersection

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

$A = \{1, 3, 5, 7, 9, 10\}$, $B = \{2, 4, 6, 8, 9, 10\}$ and
$C = \{2, 3, 5, 7, 11, 13\}$

L.H.S $= A \cup (B \cap C)$

$B \cap C = \{2, 4, 6, 8, 10\} \cap \{2, 3, 5, 7, 11, 13\} = \{2\}$



$A \cup (B \cap C)$ Fig. (xi)

Horizontal lines represent $B \cap C$, vertical lines represent A. Then the squares (horizontal and vertical lines) represent $A \cup (B \cap C)$.

$A \cup (B \cap C) = \{1, 3, 5, 7, 9, 10\} \cup \{2\}$

$= \{1, 2, 3, 5, 7, 9, 10\}$

R.H.S $= (A \cup B) \cap (A \cup C)$

$A \cup B = \{1, 3, 5, 7, 9, 10\} \cup \{2, 4, 6, 8, 9, 10\}$

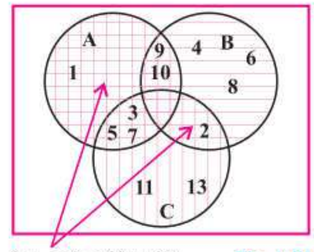
$= \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

$A \cup C = \{1, 3, 5, 7, 9, 10\} \cup \{2, 3, 5, 7, 11, 13\}$

$= \{1, 2, 3, 5, 7, 9, 10, 11, 13\}$

$(A \cup B) \cap (A \cup C) = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\} \cap \{1, 2, 3, 5, 7, 9, 10, 11, 13\}$

$= \{1, 2, 3, 5, 7, 9, 10\}$



$(A \cup B) \cap (A \cup C)$ Fig. (xii)

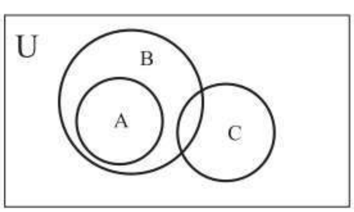
Horizontal lines represent $A \cup B$, vertical lines represent $A \cup C$ and squares represent $(A \cup B) \cap (A \cup C)$. From fig. (xi) and (xii), it is clear that $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$. Hence, distributive law of union over intersection holds.

Exercise 1.15

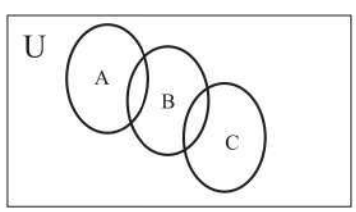
1. Verify the commutative law of union and intersection of the following sets through Venn diagrams.
   - (i) $A = \{3, 5, 7, 9, 11, 13\}$
   - (ii) The sets N and Z.
   - (iii) $C = \{x \mid x \in N \wedge 8 \leq x \leq 18\}$, $D = \{y \mid y \in N \wedge 9 \leq y \leq 19\}$
   - (iv) The sets E and O.

2. For the given sets, verify the following laws through Venn Diagram.
   - (i) Associative law of Union of sets.
   - (ii) Associative law of Intersection of sets.
   - (iii) Distributive law of Union over Intersection of sets.
   - (iv) Distributive law of Intersection over Union of sets.
     - (a) $A = \{2, 4, 6, 8, 10, 12\}$, $B = \{1, 3, 5, 7, 9, 11\}$ and $C = \{3, 6, 9, 12, 15\}$
     - (b) $A = \{x \mid x \in Z \wedge 8 \leq x \leq 25\}$ and $B = \{y \mid y \in Z \wedge -2 < y < 6\}$ and $C = \{z \mid z \in Z \wedge Z \leq 8\}$

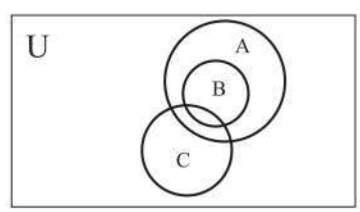
3. Copy the following Venn diagrams and shade according to the operation, given below each diagram.



$(A \cap B) \cup C$



$(A \cup B) \cap C$



$(A \cap B) \cup C$

1.4.10 De Morgans's Laws through Venn diagram

Following are the De Morgans's Laws:

(i) $(A \cup B)' = A' \cap B'$ (ii) $(A \cap B)' = A' \cup B'$

(i) $(A \cup B)' = A' \cap B'$

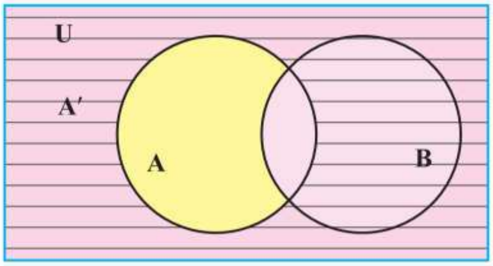


Fig. (i): A' is shown by horizontal line segments

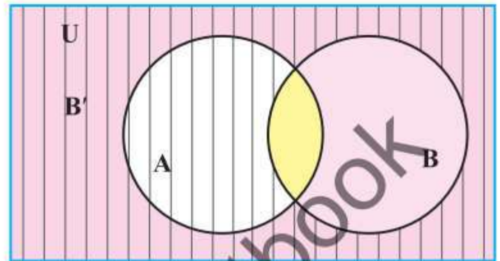


Fig. (ii): B' is shown by vertical line segments

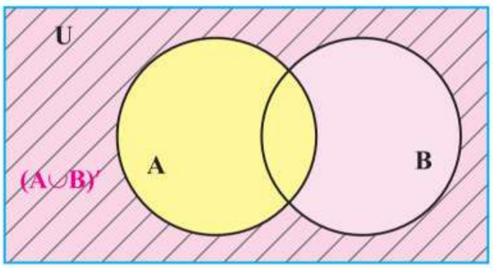


Fig. (iii): $(A \cup B)'$ is shown by slanting line segments

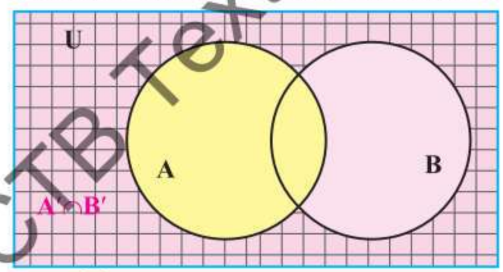


Fig. (iv): $A' \cap B'$ is shown by squares

From Fig. (iii) and (iv), it is clear that $(A \cup B)' = A' \cap B'$.

(ii) $(A \cap B)' = A' \cup B'$

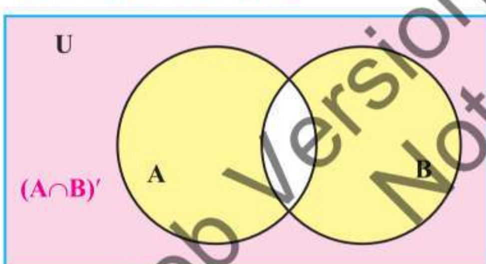


Fig. (v): $U - (A \cap B) = (A \cap B)'$ is shown by shading

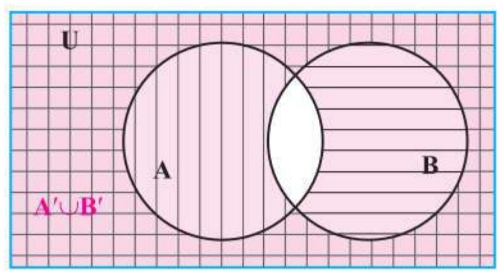


Fig. (vi): $A' \cup B'$ is shown by squares, horizontal and vertical line segments

From Fig. (v) and (vi), it is clear that $(A \cap B)' = A' \cup B'$.

1.4.11 Sets in Real Life Situations

Example: In a class of 50 students, 35 students like mangoes, 25 like bananas and 5 students like neither. Find the number of students who like both.

Solution:

Let U = Total number of students, i.e., $n(U) = 50$

M = Students who like mangoes. $n(M) = 35$

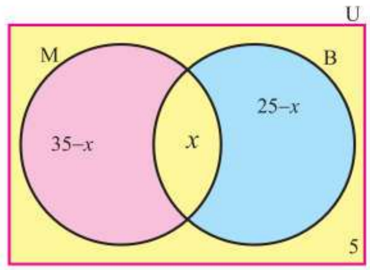
B = Students who like bananas. $n(B) = 25$

x = The number of students who like both mangoes and bananas.

Total number of students $= 50$

$$n(U) = 50$$
$$35 - x + x + 25 - x + 5 = 50$$
$$65 - x = 50$$
$$x = 65 - 50$$
$$x = 15$$

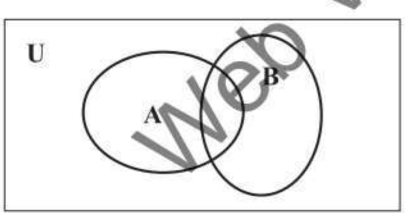
So, 15 students like both mangoes and bananas.



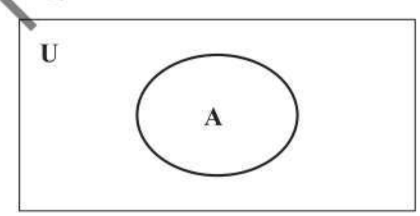
Note:
$n(M \cup B) = 15$
$n(M \backslash B) = 35 - x$
$= 35 - 15 = 20$
$n(M \backslash B) = 25 - x$
$= 25 - 15 = 10$
$n(M \cup B)' = 5$

Exercise 1.16

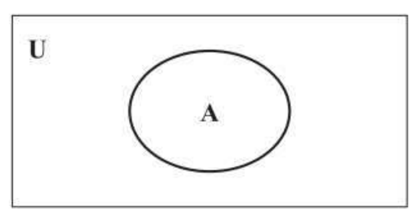
1. Find the Union of the following sets:
   (i) $A = \{1, 3, 5, 7\}$, $B = \{2, 4, 6, 8\}$ (ii) $C = \{a, b, c\}$, $D = \{x, y, z\}$
   (iii) $E = \{3, 6, 9, 12\}$, $F = \{4, 8, 12\}$
2. Find the Intersection of the following sets:
   (i) $A = \{0, \pm1, \pm2, \pm3\}$, $B = \{0, 1, 2, 3\}$
   (ii) $C = \{3, 6, 9, 12, 15\}$, $D = \{1, 3, 5, 7, 9\}$
   (iii) $E = \{a, b, c, d\}$, $F = \{c, d, e, f\}$
3. If $U = \{0, 1, 2, 3, ..., 10\}$, $A = \{2, 4, 6, 8\ 10\}$, $B = \{1, 3, 5, 7, 9\}$,
   $C = \{0, 3, 6, 9\}$ and $D = \{5, 6, 7, 8, 9, 10\}$, then find:
   (i) A^C (ii) B^C (iii) C^C (iv) D^C
4. If $A = \{1, 2, 3, 4, 5, 6\}$ and $B = \{2, 4, 6, 8\}$ then find:
   (i) $A - B$ (ii) $B - A$
5. Verify De-Morgan's Laws if $U = \{1, 2, 3, ..., 15\}$, $A = \{1, 3, 5, 7, 9, 11, 13\}$ and $B = \{2, 4, 6, 8, 10, 12, 14\}$. Verify through Venn Diagrams also.
6. Copy the following figures and shade according to the operation mentioned below each:



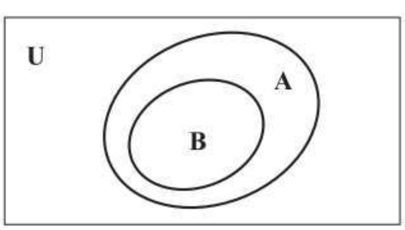
(i) $A \cup B$



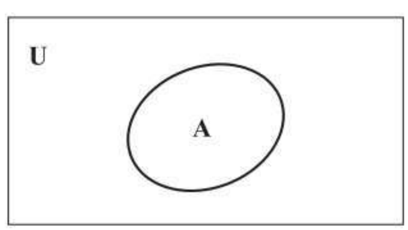
(ii) $U \cup A$



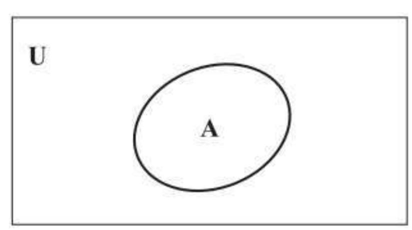
(iii) $A \cup A$



(iv) $A \cap B$



(v) $U \cap A$



(vi) $A \cap A$

7. 65 customers choose either red colour shirt or blue colour (or both). If 34 customers choose red colour and 41 customers choose blue colour. How many chose both colours?
8. In a class of 50 students, 22 students have pencils, 32 students have pens and 8 have both. How many don't have both of them?

SUMMARY

- A set can be expressed in three ways.
  (a) Tabular Form or Roster Form (b) Descriptive Form (c) Set Builder Notation
- The objects of a set are called its members or elements.
- Equivalent sets are the sets with an equal number of elements. These sets do not have exactly the same elements, just these sets have the same number of elements.
- Two sets are equal if the sets contain the same elements.
- A set that consists of all the elements of the sets under consideration, including its own elements is called universal set. It is denoted by the symbol U.
- We can write $A-B$ as $A\backslash B$ for difference of two sets.
- The complement of a set A is defined as a set that contains the elements present in the universal set U but not in set A.
- A Venn diagram is an illustration that uses circles to show the relationships among sets in a perspective way. In Venn diagram, a universal set is usually represented by a rectangle and its subsets are represented by closed figures inside the rectangle.
- Intersection of two sets, $A\cap B$, is a set which consists of only the common elements of both A and B.
- Union of two sets, $A\cup B$ is a set which consists of elements of both A and B with common elements represented only once.
- If A and B are any two sets, then
  (i) $A\cup B = B\cup A$ (Commutative law over union)
  (ii) $A\cap B = B\cap A$ (Commutative law over intersection)
- Let A, B and C be any three sets, then
  (i) $A\cup(B\cup C) = (A\cup B)\cup C$ (Associative law of union over sets)
  (ii) $A\cap(B\cap C) = (A\cap B)\cap C$ (Associative law of intersection over sets)
- Let A, B and C be any three sets, then distributive laws are given below.
  (i) $A\cup(B\cap C) = (A\cup B)\cap(A\cup C)$ (Distributive law of union over intersection)
  (ii) $A\cap(B\cup C) = (A\cap B)\cup(A\cap C)$ (Distributive law of intersection over union)
- Let A and B be any two sets, then according to the De Morhan's laws.
  (i) $(A\cup B)^c = A^c \cap B^c$
  (ii) $(A\cap B)^c = A^c \cup B^c$

Sub-Domain (v): Ratio, Rate and Proportion

Students' Learning Outcomes

After completing this sub-domain, the students will be able to:

- recall the difference between direct and inverse proportion
- solve problems involving direct proportion of two quantities using:
  - table
  - equation
  - graph
- solve real life situations/word problems involving compound proportion

Advanced / Additional

- solve problems involving inverse proportion of two quantities using:
  - table
  - equation
  - graph

Ratio is written as:

5 is to 3 or 5 : 3 or $\frac{5}{3}$

Equally to the ratios is proportion

$$\frac{\text{Rs.}10}{\text{Rs.}40} = \frac{2 \text{ chocolates}}{x \text{ chocolates}}$$

1.5.1 Proportion

The relation of equality of two ratios is called proportion, it is denoted by "::".

If two ratios *a:b* and *c:d* are proportional to each other, then we can write these as:

Extremes

$$a \quad : \quad b \quad :: \quad c \quad : \quad d$$

Means

> **Remember!**
> Ratio is a comparison of two quantities of same kind. Ratio is denoted by : e.g., ratio *a* is to *b* is written as *a:b*, ratio *c* is to *d* is written as *c:d*.

> **Remember!**
> In fraction form the ratio "a:b" is written as $\frac{a}{b}$.
> $$a : b \neq b : a$$
> $$\frac{a}{b} \neq \frac{b}{a}$$

If four quantities *a*, *b*, *c* and *d* are written as:

$$a : b :: c : d$$

then *a* is called the first, *b* is the second, *c* is the third and *d* is the fourth proportional.

To simplify proportion, we write the above as:

$$a \cdot d = b \cdot c$$

Product of Extremes = Product of Means

Example 1: The mass of 72 books is 9 kg. What will be the mass of 80 such books?

Solution: Lets x be the mass of books.

Ratio of mass of books :: Ratio of number of books.

$$9 \quad : \quad x \quad :: \quad 72 \quad : \quad 80$$

$$\text{Product of means} = \text{Product of extremes}$$

$$(72)(x) = (9)(80)$$

$$x = \frac{9 \times 80}{72}$$

$$x = 10 \text{ kg.}$$

Thus, the mass of 80 books will be 10 kg.

> **Remember!**
> If three quantities *a*, *b* and *c* are written as:
> $$a : b :: b : c$$
> then these quantities are in continued proportion and *b* is called the mean proportional.

There are two kinds of proportion.

Proportion

Direct Proportion | Inverse Proportion

Direct Proportion

It is a relation in which increases / decreases one quantity causes proportional increases / decreases in the other quantity, e.g., if we increase the number of eggs, then the cost of eggs will increase.

Direct Proportion by Using Table

Example 2: The cost of 2 pencils is Rs. 10 and 4 pencils is Rs. 20, what will be the cost of 6 and more pencils?

No. of pencils	x	2	4	6	8	10
Cost in (Rs.)	y	10	20			

Solution: The number of pencils is directly proportional to the cost of the pencil (Rs.). In other words, x is directly proportional to y.

To complete the table, we will use the pair of values x as 2 and y as 10 will be used.

We multiply 2 by 2 to obtain 4. As these quantities are in direct proportion, which means that x is double, y will also be double.

×2

No. of pencils	x	2	4	6	8	10
Cost in (Rs.)	y	10	20	30		

×2

Step 1: We multiply 2 by 3 to get 6. Which means, we will multiply 10 by 3 to get 30.

×3

No. of pencils	x	2	4	6	8	10
Cost in (Rs.)	y	10	20	30		

×3

Remember!
In direct proportion, we apply the same operation on both the quantities.

Step-2: We multiply 2 by 4 to get 8, which means, we will multiply 10 by 4 to get 40.

No. of pencils	x	2	4	6	8	10
Cost (Rs.)	y	10	20	30	40	

×4

×4

Step-3: We multiply 2 by 5 to get 10, which means we will multiply 10 by 5 to get 50.

No. of pencils	x	2	4	6	8	10
Cost (Rs.)	y	10	20	30	40	50

×5

×5

Direct Proportion by Using Equation

Example 3: A washerman irons 3 shirts in 15 minutes. How many shirts can be ironed in 45 minutes?

Solution: Let x be the number of shirts which can be ironed in 45 minutes.

We can see that when the time is increased the number of ironed shirts is also increased. So, it is a direct proportion.

So,

Ratio in shirts :: Ratio in time

$$3 : x :: 15 : 45$$

$$\frac{3}{x} = \frac{15}{45}$$

$$15 \times x = 3 \times 45$$

$$x = \frac{3 \times 45}{15}$$

$$x = 9 \text{ shirts}$$

Thus, 9 shirts can be ironed in 45 minutes.

In vertical form, it can be solved as:

Shirts	:	Time (min)
↑ 3	:	↑ 15
x	:	45

$$\frac{x}{3} = \frac{45}{15}$$

$$x \times 15 = 3 \times 45$$

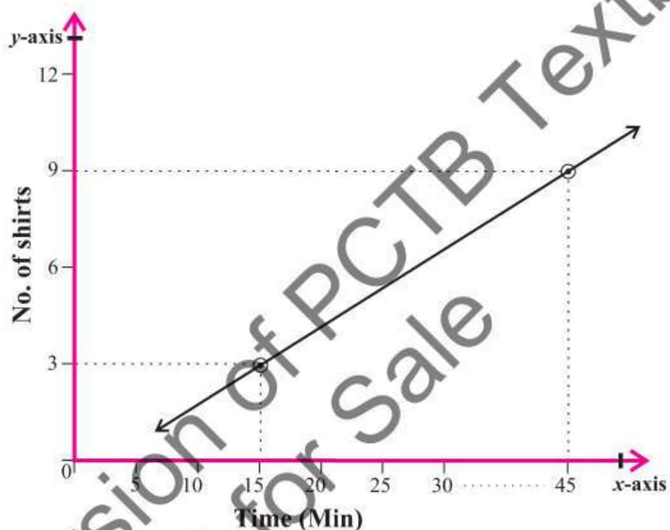
$$x = \frac{3 \times 45}{15}$$

$$x = 9 \text{ shirts}$$

Thus, 9 shirts can be ironed in 45 minutes.

1.5.2 Graphical Representation of Direct Proportion

The given example can also be represented by using graph.



Inverse Proportion

It is a relation in which if one quantity increases / decreases then other quantity decreases / increases proportionally. For example, if we increase the speed of the car, then less time will be required to reach the destination.

Inverse Proportion by Using Table

Example 4: 5 persons build a house in 600 days and 10 persons build the same house in 300 days. How many workers can build the same house in 100 days? Also complete the given table.

No. of workers	x	5	10				
No. of days	y	600	300	200	150	120	100

Solution: As, the number of workers is inversely proportional to the number of days. In other words, x is inversely proportional to y.

To complete the table, the pair of values x as 5 and y as 600.

- As, these variables are in inverse proportion, we will multiply 5 by 2 to get 10 which means we will divide 600 by 2 to get 300.

×2

No. of workers	x	5	10	15			
No. of days	y	600	300	200	150	120	100

÷2

Remember!

In inverse proportion, the variables move in opposite direction. So, If x value is multiplied then y will be divided.

Step 1: We divide 600 by 3 to get 200. As these variables are in inverse proportion, which means we will multiply 5 by 3 to get 15.

× 3

No. of workers	x	5	10	15			
No. of days	y	600	300	200	150	120	100

÷ 3

Step 2: We divide 600 by 4 to get 150, which means we will multiply 5 by 4 to get 20.

× 4

No. of workers	x	5	10	15	20		
No. of days	y	600	300	200	150	120	100

÷ 4

Step 3: We divide 600 by 5 to get 120. Which means we will multiply 5 by 5 to get 25.

× 5

No. of workers	x	5	10	15	20	25	
No. of days	y	600	300	200	150	120	100

÷ 5

Step 4: We divide 600 by 6 to get 100. Which means we will multiply 5 by 6 to get 30

× 6

No. of workers	x	5	10	15	20	25	30
No. of days	y	600	300	200	150	120	100

÷ 6

Thus, 30 workers can build the same house in 100 days.

Inverse Proportion By Using Equation

Example 5: 5 workers take 12 days to weed a field. How many days would 6 workers take to weed it?

Solution: Suppose 6 workers will take x days to weed a field.

We can see that when the number of workers is increased, the less number of days will be required. So, it is an inverse proportion.

Ratio in workers :: Ratio in days

5 : 6 :: 12 : x

$$\frac{5}{6} = \frac{x}{12}$$

$$(6)(x) = (5)(12)$$

$$x = \frac{5 \times \cancel{12}^{2}}{\cancel{6}_{1}}$$

$$x = 5 \times 2$$

$$x = 10 \text{ days}$$

In vertical, form we can write it as:

Days	:	Workers
12 ↑	:	5 ↓
x	:	6

$$\frac{x}{12} = \frac{5}{6} \quad \text{or} \quad \frac{12}{x} = \frac{6}{5}$$

$$x \times 6 = 12 \times 5$$

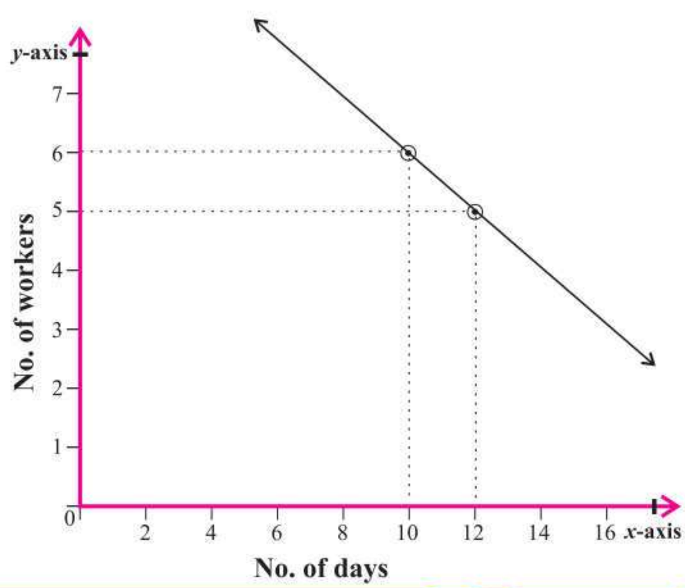
$$x = \frac{\cancel{12}^{2} \times 5}{\cancel{6}_{1}}$$

$$\boxed{x = 10 \text{ days}}$$

Thus, 6 workers will take 10 days.

Graphical Representation of Inverse Proportion

The given example can also be represented by using a graph.



Exercise 1.17

1. Write the ratios in the simplest form.

(i) 75 : 100 (ii) 12 is to 120 (iii) 2 kg is to 800 gm

(iv) $5:\frac{2}{3}$ (v) $\frac{1}{4}:\frac{1}{6}$ (vi) $\frac{1}{99}:\frac{1}{66}:\frac{2}{33}$

2. Find the unknown in the following proportion:

(i) $12 : p :: 3 : 6$ (ii) $8 : x :: x : 18$ (iii) $5 : 8 :: 15 : m$

(iv) $7 : m :: m : 28$ (v) $x : 2 :: 150 : 100$ (vi) $8 : 12 :: 6 : y$

(vii) $\frac{2}{5}=\frac{p}{20}$ (viii) $\frac{10}{3}=\frac{5}{p}$

3. If 150 shirts can be stitched on 6 sewing machines in a day, how many machines are required to stitch 225 shirts in a day? Also draw its graph.
4. If the price of 12 eggs is Rs. 180, how many eggs can be bought with Rs. 270? Also draw its graph.
5. If 30 labourers dig 800 cm^3 of earth in 3 hours, how many labourers will be required to dig the same earth in 2 hours? Also draw its graph.
6. 10 men have ration for 21 days in a camp. If 3 men leave the camp, for how many days will the ration be sufficient for the remaining men? Also draw its graph.
7. Shayan takes 200 steps for walking a distance of 160*m*. Find the distance covered by him in 350 steps.
8. In 25 minutes a train travels 20 *km*, how far will it travel in 5 minutes?
9. A car travelling at 50 *km/h* makes a journey in 70 minutes. How long will the journey take at 70 *km/h*? Also draw its graph.
10. A bag of 10 kg potatoes lasts for a week with a family of 7 people. Assuming all eat the same amount. How long will the potatoes last if there are only two in the family? Also draw its graph.
11. If *x* and *y* are in direct proportion, then complete the table and draw its graph.

(i)

x	2	6	8	10	12	14
y	8		32		48	

(ii)

x	5	10	15	20		30
y	100			400		

12. If *x* and y are in inverse proportion, then complete the table and draw its graph.

(i)

x	1	2	4	8	
y	80	40	20	10	

(ii)

x	1	2	4	5	8	
y	200	100			25	20

1.5.3 Compound Proportion

The relationship between two or more proportions is known as compound proportion.

Real Life Problems Involving Compound Proportion

The procedure of solving questions relating to the compound proportion is illustrated below with the help of examples.

Example 6: If 35 labourers dig 805 cm^3 of earth in 5 hours, how much of the earth will 30 labourers dig in 6 hours?

Solution: As the number of labourers decrease, the earth dug will also decrease. It is a direct proportion.

As the working time increase, the earth dug will also increase. It is also a direct proportion.

Let the earth dug be x cm^3

Labourers	:	**Hours**	:	**Earth (cm^3)**
35 ↑		5 ↑		805 ↑
30		6		x

$$\Rightarrow \quad \frac{x}{805} = \frac{6}{5} \times \frac{30}{35}$$

$$\Rightarrow \quad x = \frac{6 \times \overset{6}{\cancel{30}} \times \overset{\overset{23}{\cancel{161}}}{\cancel{805}}}{\underset{1}{\cancel{5}} \times \underset{\underset{1}{\cancel{7}}}{\cancel{35}}}$$

$$x = 6 \times 6 \times 23$$

$$x = 828\ cm^3$$

Thus, 828 cm^3 earth will be dug.

Example 7: Rs.8,000 are sufficient for a family of 4 members for 40 days. For how many days Rs. 15,000 will be sufficient for a family of 5 members?

Solution: We see that as amount increases the number of days also increases. So, it is direct proportion.

As the members of a family increase, the number of days decrease. So, it is an inverse proportion.

Let the number of days be x

Rupees	:	**Members**	:	**Days**
8,000 ↑		4 ↓		40 ↑
15,000		5		x

$$\Rightarrow \quad \frac{x}{40} = \frac{4}{5} \times \frac{15000}{8000}$$

$$\text{or} \quad = \frac{4 \times 15\,\cancel{000} \times \overset{\overset{1}{\cancel{5}}}{\cancel{40}}}{\underset{1}{\cancel{5}} \times \underset{1}{\cancel{8\,000}}}$$

$$x = 4 \times 15 = 60 \text{ days}$$

Thus, the amount shall be sufficient for 60 days.

Example 8: If 4200 men have sufficient food for 32 days at a rate of 12 gram per person, how many men may leave so that the same food may be sufficient for 42 days at a rate of 16 gram per person?

Solution: As the number of days increase, the number of men decreases. So, it is an inverse proportion.

As the quantity of food increases the number of men decreases. So, it is also inverse proportion.

Let the number of men be x.

Days	:	Food	:	Men
32 ↓		12 ↓		4200 ↑
42		16		x

$$\Rightarrow \quad \frac{x}{4200} = \frac{32}{42} \times \frac{12}{16}$$

$$\text{or} \qquad = \frac{\overset{2}{\cancel{32}} \times 12 \times \overset{100}{\cancel{4200}}}{\underset{1}{\cancel{42}} \times \underset{1}{\cancel{16}}}$$

$$x = 2 \times 12 \times 100 = 2400 \text{ men}$$

Thus, the food will be sufficient for 2400 men. So, 4200 − 2400 = 1800 men may leave.

Exercise 1.18

1. 30 men repair a road in 56 days by working 6 hours daily. In how many days 45 men will repair the same road by working 7 hours daily?
2. If 60 women spin 48 kg of cotton by working 8 hours daily, how much cotton will 30 women spin by working 12 hours daily?
3. If the price of a carpet 8 metres long and 3 metres wide is Rs. 6288, what will be the price of 12 metres long and 6 metres wide carpet?
4. If 15 labourers earn Rs. 67,500 in 9 days, how much money will 10 labourers earn in 12 days?
5. 70 men can complete a wall of 150 metres long in 12 days. How many men will complete the wall of length 600 metres in 30 days?
6. If the fare of 12 quintal luggage for a distance of 18 km is 12 rupees, how much fare will be charged for a luggage of 9 quintals for a distance of 20 km?
7. 14 masons can build a wall of 12 metres high in 12 days. How many masons will be needed to build a wall of 120 metres high in 7 days?
8. 15 machines prepare 360 sweaters in 6 days. 3 machines get out of order. How many sweaters can be prepared in 10 days by the remaining machines?

9. 1440 men had sufficient food for 32 days in a camp. How many men may leave the camp so that the same food is sufficient for 40 days when the ration is increased by $1\frac{1}{2}$ times?

[Hint: The 1st element (food) is 1 and the 2nd element (food) is $\frac{3}{2}$]

10. Ten men can assemble 400 cycles in 8 days. How many cycles will 5 men assemble if they work for 16 days?

SUMMARY

- The relation of equality of two ratios is called proportion, it is denoted by "::"
- Ratio is a comparison of two quantities of same kind. Ratio is denoted by ":" e.g., ratio a is to b is written as $a:b$, ratio c is to d is written as $c:d$.
- If four quantities a, b, c and d are written as:

  $$a : b :: c : d$$

  then a is called the first, b is the second, c is the third and d is the fourth proportional.
- If three quantities a, b and c are written as:

  $$a : b :: b : c$$

  then these quantities are in continued proportion and b is called the mean proportional.
- There are two types of proportion.

  (i) Direct Proportion (ii) Inverse Proportion.
- Direct proportion is a relation in which increases / decreases one quantity causes proportional increases / decreases in the other quantity, e.g., if we increase the number of eggs, then the cost of eggs will increase.
- Inverse proportion is a relation in which if one quantity increases / decreases then other quantity decreases / increases proportionally. For example, if we increase the speed of the car, then less time will be required to reach the destination.
- Direct proportion in table form, apply the same operation on both the quantities.
- Inverse proportion in table form, the variables move in opposite direction. If x value is multiplied, then y will be divided.

Sub-Domain (vi): Percentage and Financial Arithmetic

Students' Learning Outcomes

After completing this sub-domain, the students will be able to:

Currency conversion

- convert Pakistani currency to well-known international currencies and vice versa

Profit, Loss and Discount

- calculate profit percentage and loss percentage
- calculate percentage discount
- solve problems from real life situations involving successive transactions

Profit and Markup

- differentiate profit and markup
- calculate:
  - ❖ the profit/ markup
  - ❖ the principal amount
  - ❖ the profit/ markup rate, time period
- solve problems from real life situation involving profit/markup

Insurance

- solve real life problems involving
  - ❖ Insurance
  - ❖ Partnership
  - ❖ Inheritance (according to Islamic principles)

Application of percentage (%) in real life

Quantity per Serving	% Daily value
Total Fat	1%
Saturated Fat	0%
Cholesterol	0%
Sodium	11%
Total Carbohydrate	7%

1.6.1 Conversion of Currencies

A foreign currency exchange rate is a price that represents how much it costs to buy the currency of one country using the currency of another country.

Convert Pakistani Currency to Well-Known International Currencies

Currency conversion rates are not permanent but these change day by day. We use these currency rates to convert Pakistani currency to different international currencies. (rate of one US \$ is equal to Rs. 181.70)

Example 1: Saud wants to exchange Pakistani Rupees (PKR) 50,000 to US Dollars. How many US Dollars will he receive? (Rate of one US \$ = Rs. 181.70)

Solution:

Amount to be converted = Rs. 50,000

Rate of one US Dollar = Rs. 181.70

Number of US Dollars $= \dfrac{50{,}000}{181.70}$ = US \$ 275.2

Example 2: Convert Rs. 75,810 into UK £ (1 UK Pound = Rs. 237.5).

Solution:

Amount to be converted = Rs. 75,810

Rate of one UK Pound = Rs. 237.5

Number of UK Pounds $= \dfrac{75{,}810}{237.5}$ = UK £ 319.2

Table below shows current exchange rates of some currencies.

Country	Currency	Symbol	Buying (PKR)	Selling (PKR)
US	Dollar (\$)	\$	181.70	183
UK	Pound (£)	£	237.5	240
Saudi Arabia	Riyal (SR)	SAR	48	48.75
India	Rupee	₹	2.03	2.10

Exercise 1.19

1. Convert Rs. 70,000 into US \$ if the conversion rate is 1 US \$ = Rs.181.70.
2. Convert Rs. 75,000 into UK £. (Rate 1 UK £ = Rs. 237.5).
3. Converts. 50,000 into Saudi Riyal. (Rate 1 SAR = Rs. 48).
4. Convert Rs. 48,000 into Indian Rupee. (1 INR = Rs. 2.03).
5. Convert Rs. 35,000 into Australian Dollar. (1 Australian Dollar = Rs. 134).
6. Convert Rs. 80,000 into Chinese Yaun. (1 Chinese Yaun = Rs. 23.55).
7. Convert Rs. 50,000 into Canadian Dollar. (1 Canadian Dollar = Rs. 142.50).
8. Convert Rs. 70,000 into Turkish Lira. (1 Turkish Lira = Rs. 12.24).

1.6.2 Percentage

Percentage is widely used in our everyday life. Percentage is another way of expressing fraction or decimal.

The percentage means "per hundred".

The symbol used for percentage is %.

Try yourself!

Change 60% to a fraction.

Change $\frac{4}{5}$ to a percentage.

1.6.3 Profit

If the selling price (S.P.) is higher than the cost price (C.P.) then profit occurs. It can be written as:

Profit = Selling Price – Cost Price

Profit = S.P – C.P

Try yourself!

Change 10% to a decimal.

Profit Percentage

Profit percentage is always expressed in terms of cost price. To find out the profit percentage, we use the following formula.

$$\text{Profit percentage} = \frac{\text{profit}}{\text{cost price}} \times 100$$

Example 3: Abid bought a motor-cycle for Rs. 50,000 and sold it for Rs. 56,000. Find his percentage profit.

Solutions:

Cost Price (C.P.) = Rs. 50,000

Selling Price (S.P.) = Rs. 56,000

Profit = S.P – C.P

= 56,000 – 50,000

= Rs. 6,000

$$\text{Profit\%} = \frac{\text{profit}}{\text{C.P}} \times 100$$

$$= \frac{6000}{50000} \times 100$$

= 12%

1.6.4 Loss

If the cost price (C.P.) is higher than the selling price (S.P.), then loss occurs. It can be written as:

Loss = Cost Price – Selling Price

Loss = C.P. – S.P.

Loss Percentage

Loss percentage is also expressed in terms of cost price. To find out the loss percentage, we use the following formula:

$$\text{Loss percentage} = \frac{\text{loss}}{\text{cost price}} \times 100$$

Example 4: Hameed bought a piece of land worth Rs, 300000 and sold it for Rs. 240000. Find his profit / loss percentage?

Solution:

Cost Price (C.P) = Rs. 300000

Sale Price (S.P) = Rs. 240000

Loss = C.P. – S.P.

= 300000 − 240000

= Rs. 60,000

$$\text{Loss percentage} = \frac{\text{loss}}{\text{C.P.}} \times 100$$

$$= \frac{60,000}{300,000} \times 100$$

= Rs. 20%

1.6.5 Discount

Discount means to reduce the price of an article from its marked price which is also called list price or regular price. After reduction the amount is known as the selling price. The discount is the amount you saved in buying an article.

Discount = Marked price – Selling price

Percentage Discount

The discount is usually expressed as the percentage discount of the market price. To find the percentage discount. we use the following formula:

$$\text{Percentage discount} = \frac{\text{discount}}{\text{marked price}} \times 100$$

Example 5: Ali bought some articles of worth Rs. 2,500. He was allowed 15% discount on his purchase. Find price he paid of the said articles.

Solution:

Marked price = Rs. 2,500

Discount = 15%

$$\text{Discount on the articles} = \frac{25\cancel{00} \times 15}{1\cancel{00}}$$

= Rs. 375

So, the selling price = 2500 − 375

= Rs. 2, 125

Example 6: The marked price of an article is Rs.1,700. The selling price of the article is Rs. 1,360. Find the percentage discount.

Solution:

$$\begin{aligned}
\text{Market price} &= \text{Rs. } 1{,}700 \\
\text{Sale price} &= \text{Rs. } 1{,}360 \\
\text{Discount} &= \text{M.P} - \text{S.P} \\
&= 1700 - 1360 \\
&= \text{Rs. } 340 \\
\text{Percentage Discount} &= \frac{\text{Discount}}{\text{Market price}} \times 100 \\
&= \frac{\overset{20}{\cancel{340}}}{\cancel{1700}} \times \cancel{100} \\
&= 20\%
\end{aligned}$$

1.6.6 Solve Problems Involving Successive Transactions

Example 7: The cost price of an article is Rs. 6,000. The shopkeeper writes the market price of the article 15% above the cost price. The selling price of that article is Rs. 4600. Find percentage discount given to the customer.

Solution:

$$\begin{aligned}
\text{Cost Price} &= \text{Rs. } 6{,}000 \\
\text{Percentage increase} &= 15\% \\
\text{Total increase on cost price} &= \frac{6000 \times 15}{100} \\
&= \text{Rs. } 900 \\
\text{Market Price} &= 6000 + 900 \\
&= \text{Rs. } 6900 \\
\text{Selling Price} &= \text{Rs. } 4600 \\
\text{Discount} &= M.P - S.P \\
&= 6900 - 4600 \\
&= \text{Rs. } 2300 \\
\text{Percentage discount} &= \frac{\text{Discount}}{\text{Market price}} \times 100 \\
&= \frac{\overset{1}{\cancel{2300}}}{\underset{3}{\cancel{6900}}} \times 100 \\
&= \frac{100}{3} \\
&= 33\frac{1}{3}\%
\end{aligned}$$

Example 8: A wholesaler sold an article to a retailer at a profit of 10%. The retailer sold it for Rs. 1897.50 at a profit of 15%. What is the cost of wholesaler?

Solution:

Selling price of the retailer $=$ Rs. 1897.50 $=$ Rs. $\frac{3795}{2}$

Profit $= 15\%$

Cost price of retailer $=$?

Let the cost price of the retailer $=$ Rs. 100

Profit $= 15\%$

Sale price of retailer $= 100 + 15 =$ Rs. 115

If the selling price of retailer is Rs. 115, his cost price $=$ Rs. 100

If the selling price of retailer is Rs. 1, his cost price $= \frac{100}{115}$

If the selling price of retailer is Rs. $\frac{3795}{2}$ his cost price $= \frac{\overset{50}{\cancel{100}}}{\underset{1}{\cancel{\underset{23}{\cancel{115}}}}} \times \frac{\overset{33}{\cancel{\overset{759}{\cancel{3795}}}}}{\underset{1}{\cancel{2}}}$

$= 50 \times 33$

$=$ Rs. 1,650

The cost price of retailer $=$ The sale price of wholesaler

Sale price of wholesaler $=$ Rs. 1,650

Let the cost price of the wholesaler $=$ Rs. 100

Profit $= 10\%$

Sale price of wholesaler $= 100 + 10 =$ Rs. 110

If the selling price of wholesaler is Rs. 110, then his cost price $= 100$

If the selling price of the wholesaler is Rs. 1, then cost price $= \frac{100}{110}$

If the selling price of wholesaler is Rs. 1,650, the cost price is $= \frac{100}{\underset{1}{11\cancel{0}}} \times \overset{15}{\cancel{1650}}$

$=$ Rs. 1,500

The cost of wholesaler $=$ Rs. 1,500

Exercise 1.20

1. Haneef bought a car for Rs.550000. He sold it for Rs.605000 after some time. Find his profit percentage.
2. The marked price of an article is Rs.3000. Discount on this article is 20%. Find the selling price of the article.
3. A manufacturer sells an article which cost him Rs. 2500 at 20% profit. The purchaser sells the article at 30% gain. Find the final sale price of the article.
4. The marked price of every article was reduced by12% in sale at a store. A cash customer was given

a further 10% discount. What price would a cash customer pay for an article marked initially as Rs.2000?

5. Tahir purchased two toys for his children. He buys Spider Man and Barbie Doll for Rs.3000, and Rs.5000 respectively. If a discount of 20% is given on all toys, find the amount of discount and the selling price for each toy.

6. Tufail buys some items from a store. A special discount of 15% is offered on food items and 20% on other items. If he purchases food worth Rs. 1250 and other items worth Rs. 750, find the amount of discount and selling price of each separately.

7. A wholesaler sets his selling price by adding 15% to his cost price. The retailer adds 25% to the price he pays to the wholesaler to fix his selling price. At what price would a retailer sell an article which costs the wholesaler Rs. 400.

1.6.7 Profit / Markup

Profit

When we deposit money into a bank, the bank uses our money and in return pays an extra amount alongwith our actual deposit. The extra money which the bank gives for the use of our amount is called profit on the deposit.

Markup

When we borrow money from bank to run a business, the bank in return receives some extra amount alongwith the actual money given. This extra money which the bank receives is known as markup.

Principal Amount

The amount we borrow or deposit in the bank is called principal amount.

Profit / Markup Rate

The rate at which the bank gives share to its account holders is known as profit / markup rate. It is expressed in percentage.

Period

The time for which a particular amount is invested in a business is known as period.

Calculate the profit / markup, the principal amount, the profit / markup rate, the period

Calculate Profit / Markup

For calculation of profit / markup, we use the formula.

$$\text{Profit / markup} = \text{Principal} \times \text{Time} \times \text{Rate}$$

or

$$I = P \times R \times T = PRT$$

The use of this formula is illustrated with the help of examples.

Example 9: Younas borrowed Rs. 65,000 from a bank at the rate of 5% for 2 years. Find the amount of markup and the total amount to be paid.

Here principal (P) = Rs. 65,000

Rate (R) = 5%

Time (T) = 2 years

$$\text{Markup} = P \times R \times T$$

$$\text{Markup} = 65,000 \times \frac{5}{100} \times 2$$

$$= 650 \times 5 \times 2$$

$$= \text{Rs. } 6,500$$

So, Younas will have to pay Rs. 6,500 as markup.

Total amount to be paid = 65,000 + 6,500 = Rs. 71,500

Example 10: A student purchased a computer by taking loan from a bank on simple interest. He took loan of Rs. 25,000 at the rate of 10% for 2 years. Calculate the markup to be paid and the total amount to be paid back.

Solution: Here Principal (P) = Rs. 25,000

Rate (R) = 10%

Time (T) = 2 years

$$\text{Markup} = P \times R \times T$$

$$= 25000 \times \frac{10}{100} \times 2$$

$$= 250 \times 20 = \text{Rs. } 5,000$$

He has to pay Rs. 5,000 as markup.

Total amount to be paid = 25,000 + 5,000 = Rs. 30,000

Calculate Principal Amount

We have used formula of markup in the previous examples, we will use the same formula to calculate principal amount.

$$I = P \times R \times T$$

$$P = \frac{I}{R \times T}$$

Example 11: What principal amount is taken to bring in Rs. 640 as profit at the rate of 4% in 2 years?

Solution: Profit = Rs. 640

Rate (R) = 4%

Time (T) = 2 years

$$\text{Principal amount} = \frac{\text{Profit}}{R \times T}$$

$$= \frac{\overset{\overset{80}{\cancel{160}}}{\cancel{640}} \times 100}{\cancel{4} \times \cancel{2}}$$

$$= 80 \times 100$$

$$= Rs. 8,000$$

Thus, the principal amount = Rs. 8,000

Example 12: A person got some loan on which he has to pay Rs. 3,500 as markup at the rate of 10% for 3.5 years. What is the amount of loan?

Solution:

$$\text{Markup} = \text{Rs. } 3{,}500$$
$$\text{Rate (R)} = 10\%$$
$$\text{Time (T)} = 3.5\text{years} = \frac{7}{2}\text{ years}$$
$$\text{Principal (P)} = \frac{\text{Markup}}{\text{Rate} \times \text{Time}}$$
$$= \frac{\overset{50}{\cancel{3500}} \times 100 \times 2}{\cancel{10} \times \underset{1}{\cancel{7}}}$$
$$= 50 \times 200$$
$$= \text{Rs. } 10{,}000$$
$$\text{Thus, the amount of loan} = \text{Rs. } 10{,}000$$

Calculate Profit / Markup Rate

The formula for calculation of profit rate is

$$\text{Rate} = \frac{\text{Markup} \times 100}{\text{Principal} \times \text{Time}}$$

Example 13: At what annual rate percent of markup would the principal amount Rs. 68,000 become Rs. 86,360 in 3 years?

Solution:

$$\text{Total amount to be paid} = \text{Rs. } 86{,}360$$
$$\text{Principal (P)} = \text{Rs. } 68{,}000$$
$$\text{Markup} = 86{,}360 - 66{,}000$$
$$= \text{Rs. } 18{,}360$$
$$\text{Period / Time (T)} = 3 \text{ years}$$
$$\text{Rate (R)} = \frac{\text{Markup} \times 100}{\text{Principal} \times \text{Time}}$$
$$= \frac{\overset{612}{\cancel{18360}} \times \cancel{100}}{680\cancel{00} \times \cancel{3}}$$
$$= \frac{612}{68} = 9\%$$
$$\text{Rate of markup} = 9\%$$

Calculate the Period

Example 14: A person got loan from a bank at a rate of 3% per year for some period. In how much period his loan of Rs. 65,000 will become Rs. 68,900.

Solution:

$$\text{Total amount} = \text{Rs. } 68{,}900$$
$$\text{Principal (P)} = \text{Rs. } 65{,}000$$
$$\text{Markup} = 68{,}900 - 65{,}000$$
$$= \text{Rs. } 3{,}900$$
$$\text{Rate} = 3\%$$
$$\text{Period / Time (T)} = ?$$
$$\text{Period / Time (T)} = \frac{\text{Markup} \times 100}{\text{Principal} \times \text{Rate}}$$
$$= \frac{\cancel{3900}^{\cancel{300}^{\cancel{6}^{2}}} \times \cancel{100}}{\cancel{65000}_{\cancel{50}} \times \cancel{3}_{1}}$$
$$= 2 \text{ years}$$

Exercise 1.21

1. Find the profit on Rs. 40,000 at the rate of 3% per year for 4 years.
2. Saud borrowed Rs. 25,000 from bank at the rate of 6% per year for 3 years. Find the markup of the bank.
3. Find the principal amount invested by Riaz in a business if he receives a profit of Rs. 4200 in 3 years at the rate of 10% per year.
4. Ajmal invested some amount in a business. He receives a profit of Rs. 27,000 at the rate of 12% per year for 3 years. Find his original investment.
5. At what annual rate of markup would Rs.6,800 amount become Rs.9,044 in 11 years?
6. At what annual rate of profit would a sum of Rs.5,800 will increase to Rs.7,105 in 3 years' time?
7. How long would Rs. 15,500 have to be invested at a markup rate of 6% per year to gain Rs. 2790.
8. How long would Rs. 25,000 have to be deposited in the bank at 12% per year to receive back Rs. 31,000?

1.6.8 Insurance

Definition of Insurance:

Insurance is a system of protecting or safeguarding against risk or injuries. It provides financial protection for property, life, health, etc. against specified contingencies such as death, loss or damage and involving payment of regular premium in return for a policy guaranteeing. The contract is called the insurance policy. The party bearing the risk is the insurer or assurer and the party whose risk is covered is known as insured or assured.

There are many different types of insurance including health, life, property, etc. We will learn about only two types in this grade namely (i) Life insurance and (ii) Vehicle insurance

1.6.9 Solve Real Life Problems Regarding Life and Vehicle Insurance

Life Insurance

Life insurance is an agreement between the policy owner and the insurance company for an agreed time period. Insurance company agrees to pay back a sum equal to original amount and the profit at the end of agreed period or on the death or critical illness of the policy owner. In return the policy owner agrees to pay regular installments of premium.

Example 15: Saud got a life insurance policy of Rs. 500000. Rate of annual premium is 4.5% of the total amount of the policy whereas the policy fee is at the rate of 0.25%. Find the annual premium of the policy.

Solution:

Policy amount = Rs. 500000

Policy fee @ 0.25% $= \frac{25}{\cancel{100}} \times 5\,00\cancel{000} \times \frac{1}{\cancel{100}}$

= Rs. 1250

First premium @ 4. 5% $= \frac{45}{\cancel{10}} \times \frac{1}{\cancel{100}} \times 5\,00\cancel{000}$

= Rs. 22,500

Period / Time (T) $= \frac{\text{Markup}}{\text{Principal} \times \text{Rate}}$

Annual premium = First premium + policy fee

= 22,500 + 1,250 = Rs. 23,750

Example 16: A man purchased a life insurance policy for Rs. 300000. The annual premium is 4.5% of the policy amount whereas policy fee is at the rate of 0.25%. Calculate the annual premium and quarterly premium at 27% of the annual premium.

Solution:

Policy amount = Rs. 300000

Policy fee @ 0.25% $= \frac{25}{\cancel{100}} \times \frac{1}{\cancel{100}} \times 30\cancel{0000}$

= Rs. 750

First premium @ 4. 5% $= \frac{45}{\cancel{10}} \times \frac{1}{\cancel{100}} \times 300\cancel{000}$

= Rs. 13,500

Annual premium = First premium + policy fee

= 13,500 + 750

= Rs. 14,250

Quarterly premium $= \overset{285}{\cancel{1425}}\cancel{0} \times \frac{27}{\underset{2}{\cancel{10}}\cancel{0}}$

$= \frac{285 \times 27}{2}$ = Rs. 3847.50

Vehicle Insurance

Vehicle insurance provides a protection against risks to the vehicle. The amount of policy in this case depends upon the actual value of the vehicle.

Example 17: Aslam got his motorcycle insured for one year. The price of his motorcycle is Rs.50,000 and the rate of insurance is 4.5%. Find the amount of premium.

Solution:

The price of motorcycle = Rs. 50,000

Rate of insurance = Rs. 4.5%

Amount of premium $= \frac{4.5}{100} \times 50000$

$$= \frac{4.5}{1\cancel{0}} \times \frac{1}{10\cancel{0}} \times 50\cancel{0}\cancel{00}$$

$$= \text{Rs. } 2{,}250$$

Example 18: Khalid purchased an insurance policy for his car. The worth of the car is Rs. 750000. The rate of annual premium is 3% for two years and depreciation rate is 10%. Find the total amount he paid as premium.

Solution:

$$\begin{aligned}
\text{Worth of car} &= \text{Rs. } 750000\\
\text{Rate of annual premium} &= 3\%\\
\text{Depreciation rate} &= 10\%\\
\text{Time period} &= 2 \text{ years}\\
\text{First premium} &= \frac{3}{1\cancel{00}} \times 7500\cancel{00} = \text{Rs. } 22{,}500\\
\text{Depreciation after one year} &= 10\% \text{ of } 750000\\
&= \frac{10}{1\cancel{00}} \times 7500\cancel{00} = \text{Rs. } 75{,}000\\
\text{Depreciation after one year} &= 750000 - 75000\\
&= \text{Rs. } 67{,}5000\\
2^{\text{nd}} \text{ premium} &= 3\% \text{ of } 675000\\
&= \frac{3}{1\cancel{00}} \times 6750\cancel{00}\\
&= \text{Rs. } 20{,}250\\
\text{Depreciation after 2 years} &= 10\% \text{ of } 675000\\
&= \frac{10}{1\cancel{00}} \times 6750\cancel{00}\\
&= \text{Rs. } 67{,}500\\
\text{Depreciation after 2 years} &= 675000 - 67500\\
&= \text{Rs. } 60{,}7500\\
\text{Total amount paid as premium} &= 22{,}500 + 20{,}250\\
&= \text{Rs. } 42{,}750
\end{aligned}$$

Exercise 1.22

1. Usman purchased a car for Rs. 1250000 and insured it for one year at the rate of 4.5%. Find the annual premium.
2. Hameed got a life insurance policy of Rs.200000. Find the first premium he has to pay when the rate of annual premium is 5.2% and policy fee is 0.25%.
3. Zahid got a life insurance policy of Rs.500000 at the rate of 5.2% and the policy fee is 0.25%. Calculate half yearly premium at 52% of the annual premium.
4. Usama insured his life for Rs. 700000. Find annual premium at 4.5% of the policy amount with policy fee at the rate of 0.25%. Calculate monthly premium at 9% of the annual premium.
5. Saud bought a car for Rs.700000 and got it insured at 4.2% annual premium for 3 years. Calculate how much premium he paid in 3 years if depreciation rate is 12%.

6. A man has a car of worth Rs. 1400000. He got it insured for a period of 2 years at the rate of 4.5%. The depreciation rate is 10% per year. He has to pay the premium yearly. Find the total amount of premium he has to pay for a period of 2 years.

7. Asma insured her life for Rs. 1,000,000 at the rate of 5% per year. Find the amount of annual premium she has to pay.

1.6.10 Partnership

A business in which two or more persons run the business and they are responsible for the profit and loss is called the partnership.

If the partners start the business and close it together with same or different investment capital, this partnership is called a simple partnership.

If the partners contribute different capitals for different time periods or at least one partner contributes two or more capitals for different time periods, then this partnership is called a compound partnership. In this case, the profit or loss is divided in the ratio of monthly investments.

Example 19: Saud and Ammar started a business with capitals of Rs.56,000 and Rs. 64,000 respectively. After one year they earned a profit of Rs.22,500. Find the share of each one.

Solution: The simplified form of capital share ratio:

Saud's share	**:**	**Ammar's share**
56,000	:	64,000
56	:	64
7	:	8
Sum of ratios	=	7 + 8 = 15
Total profit	=	Rs. 22,500

Saud's profit $= \frac{7}{\cancel{15}_{1}} \times \cancel{22500}^{1500} = 7 \times 1500 = \text{Rs. } 10,500$

Ammar's profit $= \frac{8}{15} \times 22500$

$= 8 \times 1500 = \text{Rs. } 12,000$

Example 20: Tahir started a business with a capital of Rs. l5,000. After 5 months Umar also joined him with an investment of Rs. 30,000. At the start of 9 month's Usman joined them by investing Rs.45,000. At the end of the year, they earned a profit of Rs.406000. Find the share of each one.

Solution:

Tahir's investment for 12 months	=	Rs. 15,000
Tahir's effective investment for 1 month	=	15000 × 12
	=	Rs. 18,0000
Umar's investment for 7 months	=	Rs. 30,000
Umar's effective investment for 1 month	=	30,000 × 7
	=	Rs. 21,0000
Usman's investment for 3 months	=	Rs. 45,000
Usman's effective investment for 1 month	=	45,000 × 3
	=	Rs. 13,5000

Tahir	**:**	**Umar**	**:**	**Usman**
1,80,000	:	2,10,000	:	1,35,000
180	:	210	:	135
12	:	14	:	9

Sum of ratios = 12 + 14 +9 = 35

$$\text{Tahir's share} = \frac{12}{\underset{\underset{1}{\cancel{7}}}{\cancel{35}}} \times \overset{\overset{11600}{\cancel{81200}}}{\cancel{406000}}$$

$$= 12 \times 11600 = \text{Rs. } 139200$$

$$\text{Umar's share} = \frac{14}{\underset{1}{\cancel{35}}} \times \overset{11600}{\cancel{406000}}$$

$$= 14 \times 11600 = \text{Rs. } 162400$$

$$\text{Usman's share} = \frac{9}{\underset{1}{\cancel{35}}} \times \overset{11600}{\cancel{406000}}$$

$$= 9 \times 11600 = \text{Rs. } 104400$$

Example 21: Saud, Ali and Saad started a business with Rs.15,000, Rs.19,000 and Rs. 12,000 respectively. Saud manages the business and receives allowance of Rs. 16,000 for this assignment. After 5 months Ali withdraws Rs.9,000 and business is closed after 9 months. What did each receive in the profit of Rs.58,000.

Solution:

Saud's capital for 9 months	=	Rs. 15,000
Saud's effective capital for 1 months	=	15000×9
	=	Rs. 1,35,000
Ali's capital for 5 months	=	Rs. 19,000
Ali's effective capital for 1 months	=	19000×5
	=	Rs. 95,000
Ali's capital for 4 months	=	Rs. 10,000
Ali's effective capital for 1 months	=	10000×4
	=	Rs. 40,000
Ali's total capital	=	95,000 + 40,000
Saad's capital for 9 months	=	Rs. 12,000
Saad's effective capital for 1 months	=	12000×9
	=	Rs. 10,8,000
Total profit	=	Rs. 58,000
Saud's Allowance	=	Rs. 16,000
Net profit = 58,000 – 16,000	=	Rs. 42,000

Ratios of Capitals	Saud's	:	Ali	:	Saad's
	1,35,000	:	1,35,000	:	1,08,000
	135	:	135	:	108
	15	:	15	:	12
	5	:	5	:	4

Sum of ratios $= 5 + 5 + 4 = 14$

Tahir's share $= \frac{5}{\cancel{14}_{1}} \times \cancel{42000}^{3000}$

$= 5 \times 3000$

$=$ Rs. 15,000

Saud's allowance $=$ Rs. 16,000

Saud received $=$ Total of Saud's Profit + Allowance

$= 15,000 + 16,000$

$=$ Rs. 31,000

Exercise 1.23

1. Aslam and Akram invested Rs. 27,000 and Rs. 30,000 to start a business. If they earned a profit of Rs. 66,500 at the end of the year, find the profit of each one.
2. Amina and Maryam started a business with investment of Rs. 30,000 and Rs. 40,000 respectively in one year. At the end of the year they earned a profit of Rs. 8400. Find the share of each one.
3. Two partners contributed Rs.4000 and Rs. 3000. 1st contributed for 9 months and the 2nd contributed the amount for 7 months. Divide a profit of Rs. 11,590 between the partners.
4. Saad, Saud and Saeed started a business with capital of Rs. 12,000, Rs. 18,000 and Rs. 24,000 respectively. At the end of the year, they suffered with a loss of Rs.13,500. Find the share of each in this loss.
5. Akram and Asghar started a business with Rs. 9,000 and Rs. 11,000 respectively. Akram withdraws Rs. 1000 after 6 months. After 2 months of his withdrawal Asghar invested Rs.1000 more. After a year they earned a profit of Rs. 14,000. Find the share of each in the profit.
6. Three friends A, B and C started a firm with Rs. 20,000, Rs. 16,000 and Rs. 18,000 respectively. A kept his money for 4 months, B for 6 months and C for 8 months. Divide a profit of Rs. 12,000 among these friends.

1.6.11 Inheritance

When a person dies, then the assets left by him are called inheritance and it is distributed among his legal inheritors according to Islamic Shariah Law. In Islam, the principles of distribution of inheritance are given below.

- First of all his/her funeral expenses and all his/her all debt be paid.
- Then execute the will upto 1/3 of his/her property if asked for.
- Then distribute the remaining inheritance accordingly.

The procedure is illustrated with the help of following examples.

Example 22: A man left his property of Rs.640000. A debt of Rs.40,000 was due to him and Rs.5,000 was spent on his burial. Distribute the amount between his widow, 1 daughter and 2 sons according to the Islamic Law.

Solution:

Total amount of property = Rs. 640,000

His debt = Rs. 40,000

Burial Expenses = Rs. 5,000

Total amount paid = 40,000 + 5,000 – 45,000

Remaining amount = 640,000 – 45,000 = Rs. 595,000

Widow's share = $\frac{1}{8} \times 595{,}000$ = Rs. 74,375

Remaining Inheritance = 595,000 – 74,375 = Rs. 520,625

Now, Ratios of shares

Sons	**:**	**Daughter**
2	:	1
$2 \times 2 = 4$	:	$1 \times 1 = 1$

Sum of ratios = $4 + 1 = 5$

Share of 2 sons = $\frac{4}{5} \times 520{,}625$

= $4 \times 104{,}125$

= Rs. 416,500

Share of each son = $\frac{416{,}500}{2} = 208{,}250$

Share of one daughter = $\frac{1}{5} \times 520{,}625$

= Rs. 104,125

Example 23: Mst. Zainab Begum died leaving behind her a property of Rs.802500 which was to be distributed among her husband, her mother and two daughters. The husband got $\frac{1}{4}$, mother got $\frac{1}{6}$ and remaining for 2 daughters. Rs.7,500 was spent on her burial. Find the share of each one.

Solution:

Total amount left = Rs. 802,500

Expenditure on her burial = Rs. 7,500

Remaining amount = 802,500 – 7,500

= Rs. 795,000

Share of her husband = $\frac{1}{4} \times 795{,}000$

= Rs. 198,750

Share of her mother = $\frac{1}{6} \times 795{,}000$

= Rs. 132,500

Total share of her husband and her mother = 198,750 + 132,500

= Rs. 331,250

Remaining Inheritance = 795,000 – 331,250

= Rs. 493,750

Share of 2 daughters = Rs. 46,3750

Share of each daughter = $\frac{46,3750}{2}$

= Rs. 231,875

Exercise 1.24

1. A man left Rs. 240000 as inheritance. His heirs are 6 daughters and 2 sons. Find the share of each inheritor so that a son gets twice of his sister's share.
2. Allah Ditta died leaving a property of Rs. 850000. He left a widow, two sons and one daughter. Find the share of each in the inheritance if the burial expenditure was Rs. 50,000.
3. Akram left a wealth of Rs. 780000. His heir are a widow, 3 sons and 4 daughters. Calculate the share of each one if the funeral expenses is Rs. 30,000 and a loan of Rs. 50,000 is due to him.
4. A man died leaving a saving of Rs. 72,000 in the bank. Find the share of each: widow, one son and one daughter.
5. Aslam left a property worth Rs. 650000. He had to pay Rs. 50,000 as debt. The remaining amount was divided among his 2 sons and 2 daughters. Find the share of each.
6. A person died leaving behind inheritance of Rs. 300,000. Distribute the amount among 4 sons and 3 daughters so that each son gets double of what a daughter gets. Find the share of each when a debt of Rs. 80,000 was also to be paid.
7. Wife of Ahmad died leaving behind 2 daughters and a son. If Ahmad gets $\frac{1}{4}$ of the inheritance of Rs. 180,000. The remaining amount is to be distributed among her children so that a son receives twice of a daughter's share. Find the amount of share of each daughter, son and Ahmad.

SUMMARY

- A foreign currency exchange rate is a price that represents how much it costs to buy the currency of one country using the currency of another country.
- If the Selling Price (S.P.) is higher than the Cost Price (C.P.), then profit occurs.
- Profit percentage = $\frac{\text{Profit}}{\text{Cost Price}} \times 100\%$
- If the cost price (C.P.) is higher than the selling price (S.P.), then loss occurs.
- Loss percentage = $\frac{\text{Loss}}{\text{Cost price}} \times 100\%$
- Percentage Discount = $\frac{\text{Discount}}{\text{Market Price}} \times 100\%$

- When we borrow money from bank to run a business, the bank in return receives some extra amount along with the actual money given. This extra money which the bank receives is known as markup.
- The amount we borrow or deposit in the bank is called principal amount.
- The rate at which the bank gives share to its account holders is known as profit / markup rate. It is expressed in percentage.
- The time for which a particular amount is invested in a business is known as period.
- Profit / Markup = Principal × Time × Rate

  Or $I = P \times R \times T = PRT$
- $P = \dfrac{I}{R \times T}$
- $\text{Rate} = \dfrac{\text{Markup}}{\text{Principal} \times \text{Time}}$
- Life insurance is an agreement between the policy owner and the insurance company for an agreed time period.
- Vehicle insurance provides a protection against risks to the vehicle. The amount of policy in this case depends upon the actual value of the vehicle.
- A business in which two or more persons run the business and they are responsible for the profit and loss is called the partnership.
- When a person dies, then the assets left by him are called inheritance and it is distributed among his legal inheritors according to Islamic Shariah Law.
- The relation of equality of two ratios is called proportion, It is denoted by ::
- Ratio is a comparison of two quantities of same kind. Ratio is denoted by : e.g., ratio a is to b is written as $a:b$, ratio c is to d is written as $c:d$.
- If three quantities a, b and c are written as: $a : b :: b : c$, then these quantities are in continued proportion and b is called the mean proportional.

Review Exercise (1b)

1. Four options are given against each statement. Encircle the correct one.

(i) If a is not a member of the set A, then symbolically it is denoted by:

(a) $a \in A$ (b) $a \backslash A$ (c) $a \notin A$ (d) $a \cap A$

(ii) Which of the following is not a set?

(a) $\{1,2,3\}$ (b) $\{a, b, c\}$ (c) $\{2,3,4\}$ (d) $\{1,2,2,3\}$

(iii) A set consisting of all subsets of the set A is called:

(a) subset (b) universal set (c) power set (d) superset

(iv) If $U = \{1,2,3,...10\}$ and $A = \{2,4,6,....,10\}$ and $B=\{1,3,5,....,9\}$, then $(A-B)^c$ is equal to:

(a) U (b) B (c) A (d) ϕ

(v) If $P(A) = \{\phi, \{a\}, \{b\}, \{a, b\}\}$, then set A is equal to:
(a) ϕ (b) $\{a\}$ (c) $\{b\}$ (d) $\{a, b\}$

(vi) Proportion means:
(a) equality of two ratios (b) equality of quantities
(c) inequality of two ratios (d) inequality of quantities

(vii) Product of extremes is equals to:
(a) direct proportion (b) inverse proportion
(c) product of means (d) division of means

(viii) A relation in which increases one quantity causes proportional increases in the other quantity is called:
(a) compound proportion (b) ratio
(c) inverse proportion (d) direct proportion

(ix) The comparison of the quantities of same kind is called:
(a) proportion (b) ratio
(c) compound proportion (d) inverse proportion

(x) The relation between two or more proportions is called:
(a) compound proportion (b) direct proportion
(c) proportion (d) inverse proportion

(xi) A person who buys life insurance from an insurance company is called;
(a) insured (b) insurer (c) lesser (d) beneficiary

(xii) To reduce the price of an article from its market price is called:
(a) discount (b) profit (c) period (d) insurance

(xiii) The time for which a particular amount is invested in a business is known as:
(a) profit (b) markup rate (c) period (d) principal amount

(xiv) A business in which two or more persons run the business and they are responsible for the profit and loss is called the:
(a) profit (b) partnership (c) markup (d) principal amount

(xv) The cost price of a toy car is Rs. 100. If it is sold at 10% discount then the selling price of the toy car:
(a) Rs. 90 (b) Rs. 110 (c) Rs. 80 (d) Rs. 120

(xvi) When a person dies, then the assets left by him are called:
(a) period (b) principal amount
(c) inheritance (d) discount

2. Define the following:
(i) Direct proportion (ii) Inverse proportion (iii) Compound proportion
(iv) Loss percentage (v) Discount (vi) Markup
(vii) Inheritance (viii) Vehicle insurance (ix) Life insurance

3. Write all subsets of the following sets:
(i) $A = \{1, 2, 3, 4\}$ (ii) $B = \{a, b, c\}$

4. Write the power set of the following sets:
(i) $C = \{2, 4, 6\}$ (ii) $D = \{+, -, \times, \div\}$

5. If $U = \{1, 2, 3, \ldots\ldots\ldots\ldots, 10\}$, $A = \{1, 3, 5, 7\}$ and $B = \{2, 4, 6, 8, 10\}$, then verify $(A \cup B)^c = A^c \cap B^c$

6. In a survey of a primary school students, it was found that 20 students have cars, 15 students have motorbikes, 5 students have both and 3 don't have any of them. How many students are there in the survey?

7. Mrs. Razia paid her driver Rs. 2500 for 5 days. What amount will she pay him for 10 days? Also draw its graph.

8. If a motorbike needs 20 litres of petrol for a travel of 160 km. Find how much petrol will be required to travel 360 km. Also draw its graph.

9. If 15 men take 85 days to build a wall, how long would it take for 10 men? Also draw its graph.

10. 10 lions can eat 500 kg meat in 10 days. How many lions will eat the same meat in 5 days? Also draw its graph.

11. A factory can manufacture 200 items in a day using 10 machines. How many items can be manufactured by using 14 machines in 4 days?

12. 25 labourers can construct 5 rooms in 18 days. In how many days can 10 labourers complete 10 rooms of the same size?

13. A factory marked price of the articles 25% above the cost price. The cost price of an article is Rs.5000 and its selling price is Rs. 4500. Find the discount % given to the customer.

14. Saeed invests Rs.12,000 at $8\frac{1}{2}\%$ per year profit. How much would the amount be after 2 years and 6 months?

15. Faheem got his car insured at a rate of 3% for 3 years. The worth of his car is Rs. 850,000. Find the total amount paid as premium if rate of depreciation is 10% per year.

16. Aslam started a business with Rs. 35,000. After 3 months Akram joined the business with Rs.4000 and after 6 months Asghar invested Rs.5000. At the end of the year, they earned a profit of Rs.1620. Find the share of each in the profit.

17. Asghar Ali died leaving assets worth Rs. 655275. Funeral expenses were Rs. 5275. He had to pay Rs. 50,000 as debt. After making these payments, his widow shall get $\frac{1}{8}$ of the remaining property. Find the share of his son and one daughter when share of son is double the share of his daughter.