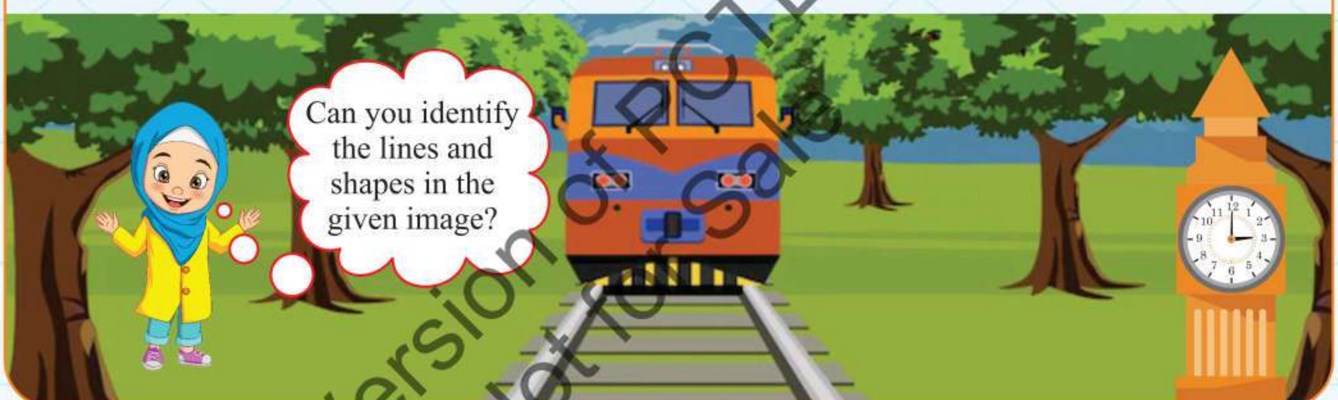
Domain 4 GEOMETRY

Sub-domain (i) Practical Geometry

Students' Learning Outcomes

After studying this sub-domain, students will be able to:

- Know that the perpendicular distance from a point to a line is the shortest distance to the line.
- Construct different types of triangles. (equilateral, isosceles, scalene, acute-angled, right-angled and obtuse-angled)

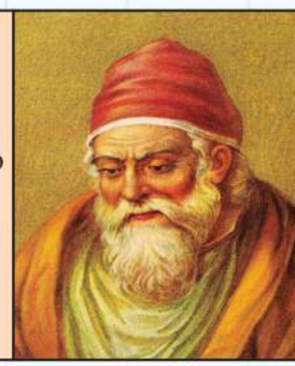


INTRODUCTION OF GEOMETRY

Geometry is the branch of mathematics which deals with the study of points, lines, surfaces and solids.

Geometry helps us what design to make which plays an important role in the construction of houses, buildings, dams etc. Art and architecture are based on geometry.

Euclid (300 BC) was the first mathematician who made remarkable achievements in geometry.



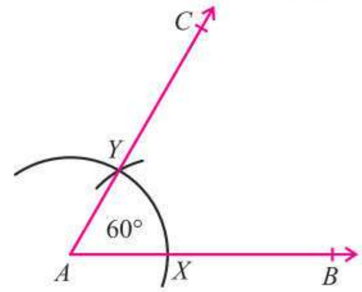
RECALL

Construction of Angles Using Compass and Ruler

We have learnt in previous class about the construction of angles 30°, 45°, 60°, 75°, 90°, 105° and 120°. We will revise here the construction of 60° and 90°.

Example 1 **Construction of an angle of 60°**

Solution **Steps of construction**



i. Draw a ray AB of any suitable length.

ii. Taking point A as centre, draw an arc of any suitable radius with compass intersecting $\overrightarrow{AB}$ at point X.

iii. Taking point X as centre, draw an arc of same radius cutting the previous arc at point Y.

iv. Draw $\overrightarrow{AC}$ passing through point Y. We get an angle of 60° i.e., $m\angle BAC = 60°$

Try yourself

In previous class we have also learnt how to bisect the given angle. Bisect the angle of 60°.

Important Information

Bisection of angle 60° is 30°. Bisection of an angle means to find an angle, half of the given angle

Example 2 **Construction of an angle of 90°**

Solution **Steps of construction**

Remember!

For the construction of angle 90°, we use the angles 60° and 120°.

i. Draw a ray CD of any length.

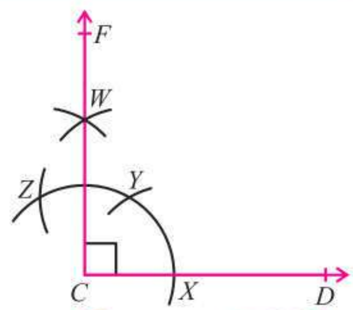
ii. With point C as centre, draw an arc of any suitable radius with compass which intersecting $\overrightarrow{CD}$ at point X.

iii. With point X as centre, draw an arc of same radius intersecting the previous arc at point Y.

iv. Taking point Y as centre, draw an arc of same radius intersecting the first arc at point Z.

v. Taking points Y and Z as centres draw two arcs of any suitable radius cutting each other at point W.

vi. Draw $\overrightarrow{CF}$ through point W. We get an angle of 90°. i.e., $m\angle DCF = 90°$.



Do you know?

If we draw a ray through Z we will get an angle of 120°.

Try yourself

What is bisection of angles 30°, 60° and 90°?

Brain Teaser!

If we bisect the angle 90° which angle do we get?

4.1.1 Triangles and their Construction

Triangles are closed figures with three line segments (called sides) and three angles inside it. Triangles are used in patterns for design and in construction.

Remember!

A triangle has 3 sides, 3 angles and 3 vertices.

Egyptians built pyramids that are triangular in shape.

i Types of Triangles

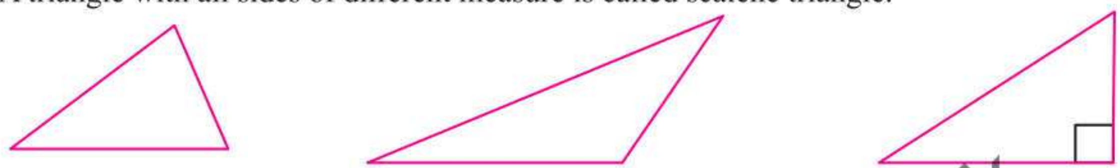
Triangles are mainly divided into two types:

(a) Triangles with respect to sides (b) Triangles with respect to angles

(a) Triangles with respect to sides

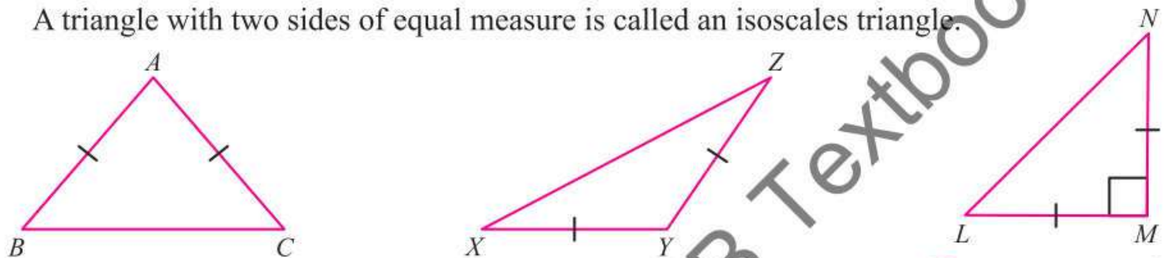
Scalene Triangle

A triangle with all sides of different measure is called scalene triangle.



Isosceles Triangle

A triangle with two sides of equal measure is called an isoscales triangle.



In ΔABC, $m\overline{AB} = m\overline{AC}$ and $\angle B$ and $\angle C$ are called base angles.

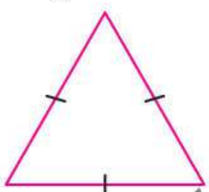
$\overline{BC}$ is base and $\angle A$ is called vertical angle.

Try yourself

In ΔXYZ and ΔLMN, name the base angles and vertical angle.

Equilateral Triangle

A triangle with all sides of equal measure is called an equilateral triangle.



Remember!

Each interior angle in an equilateral triangle is 60°.

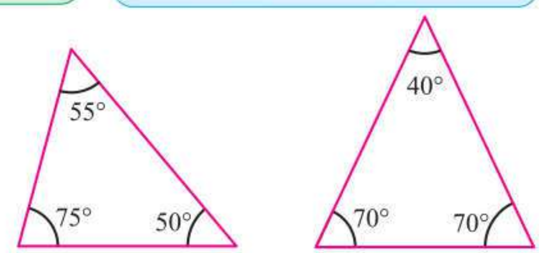
Need to Know!

The sum of all interior angles of a triangle is 180°.

(b) Triangles with respect to angles

Acute Angled Triangle

A triangle is called an acute angled triangle if all angles are less than 90°.



Need to Know!

An acute angled triangle can be equilateral or isosceles.

Remember!

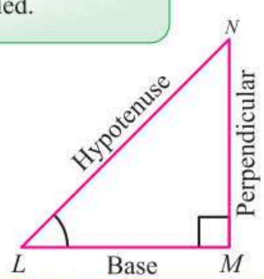
If sum of two angles in a triangle is greater than 90°, the triangle is acute angled.

Right Angled Triangle

A triangle is called a right angled triangle if exactly one angle is equal to 90°.

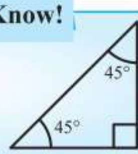
In right angled triangle LMN,

$\overline{LN}$ is the hypotenuse, $\overline{LM}$ is base and $\overline{MN}$ is perpendicular.



Need to Know!

A right angled triangle can be isosceles.

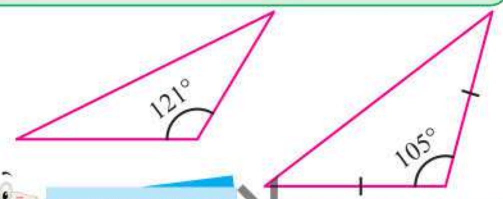


Remember!

In right angled triangle, one angle is 90° and the sum of the other two is 90°. i.e., other two angles are complementary.

Obtuse Angled Triangle

A triangle is called obtuse angled triangle if exactly one angle is of measure greater than 90°.



Remember!

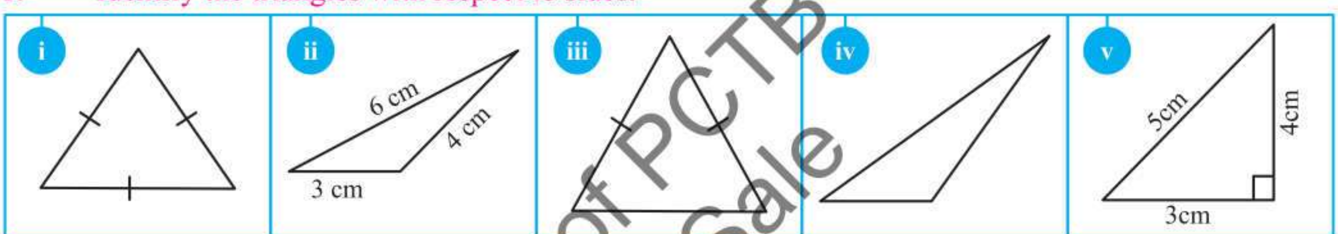
- An obtuse angled triangle can not be equilateral.
- An obtuse angled triangle may be isosceles or scalene.

Need to Know!

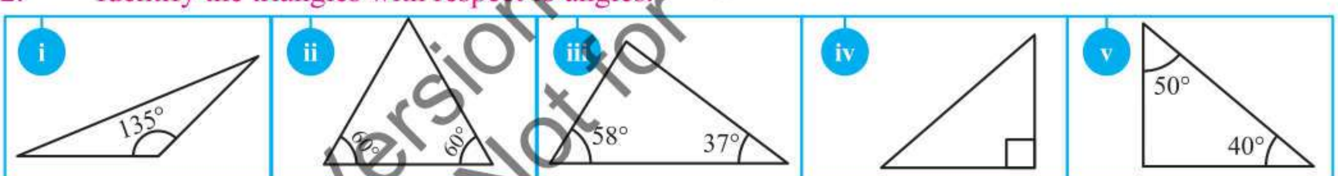
If the sum of two angles is less than 90°, then the triangle will be obtuse angled triangle.

EXERCISE 4.1

1. Identify the triangles with respect to sides.



2. Identify the triangles with respect to angles.



4.1.2 Construction of Triangles

It is very important to know how triangles are constructed. We construct triangles with respect to sides and angles.

Need to Know!

The sum of the length of any two sides in a triangle is always greater than the length of the third side.

Example 3 **Construction of Equilateral Triangle**

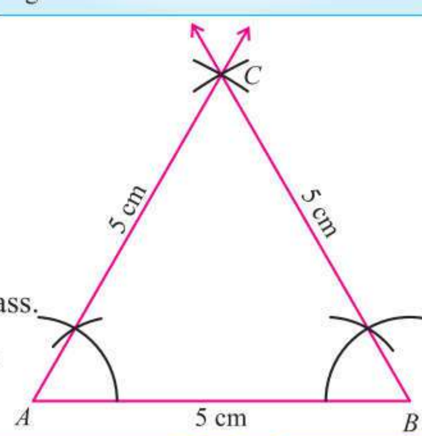
Construct an equilateral triangle ABC of a side length 5cm.

1st Method

Solution **Steps of construction**

i. Draw a line segment AB of length $5cm$.
ii. Construct an angle of 60° at points A and B with the help of compass.
iii. Draw two rays from points A and B meeting each other at point C.

Therefore, ΔABC is the required an equilateral triangle.



Remember!

We can construct the following angles with compass:
30°, 45°, 60°, 75°, 90°, 105°, 120°, 135°, 150°

2^{nd} Method

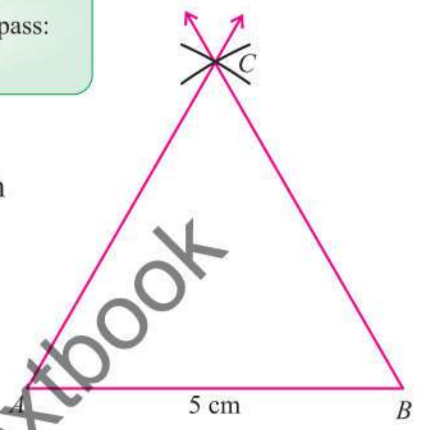
Steps of construction

i. Draw a line segment AB of length $5cm$.

ii. With centres at points A and B, draw arcs of radius $5cm$ with compass which intersecting each other at point C.

iii. Draw rays AC and BC.

Therefore, ΔABC is the required an equilateral triangle.



Example 4 **Construction of an isosceles triangle.**

Construct a triangle ABC with $m\overline{AB}$ = 6 cm as base and base angle 50°.

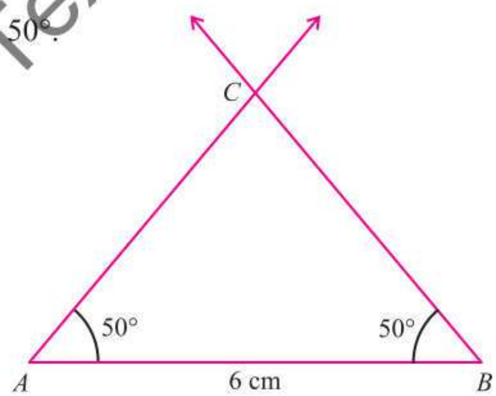
Solution **Steps of construction**

i. Draw $m\overline{AB} = 6cm$ with ruler.

ii. Using protractor, draw an angle of 50° at points A and B.

iii. Draw rays from points A and B meeting at point C.

Therefore, ΔABC is the required an isosceles triangle with $m\overline{AC} = m\overline{BC}$.



Example 5 **Construction of a right angled triangle.**

Construct a right angled triangle XYZ.

(a) $m\overline{XY}$ = 5.2 cm, $m\overline{YZ}$ = 4.5 cm and $m\angle Y = 90°$

(b) length of hypotenuse $\overline{YZ}$ = 7 cm and $m\overline{XY}$ = 6 cm

Brain Teaser!

Can you find $m\angle C$?

Solution **(a) Steps of construction**

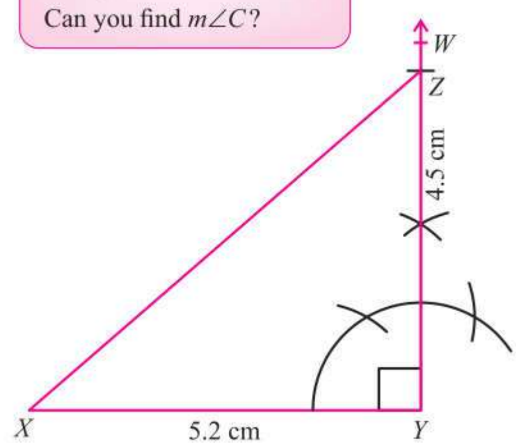
i. Draw $m\overline{XY}$ = 5.2 cm with ruler.

ii. Taking centre at point Y, construct an angle of 90° and draw a ray YW i.e., $m\angle XYW = 90°$.

iii. Draw an arc of radius 4.5 cm with centre at point Y cutting $\overrightarrow{YW}$ at Z.

iv. Join points X and Z.

Therefore, We get a right angled triangle XYZ with hypotenuse $\overline{XZ}$.



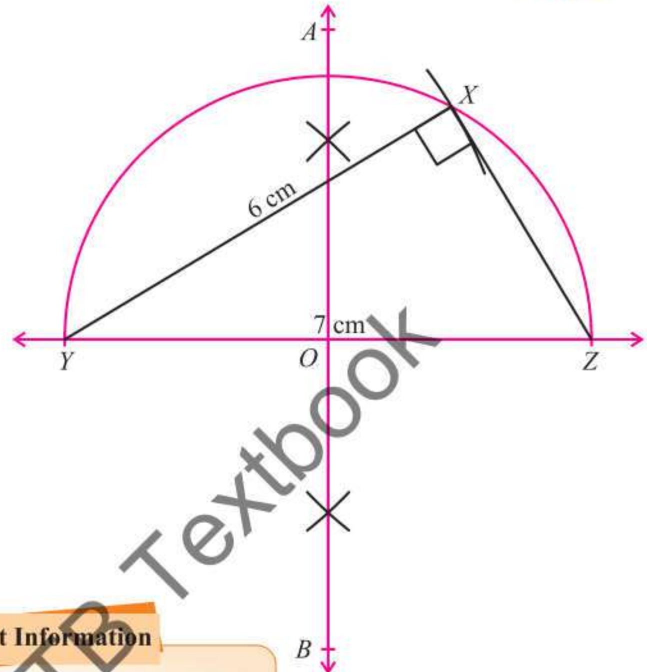
Brain Teaser!

Measure $\overline{XZ}$

Solution **(b) Steps of construction**

i. Draw $m\overline{YZ} = 7$ cm.
ii. Draw perpendicular bisector $\overleftrightarrow{AB}$ of $\overline{YZ}$ with the help of compass and ruler and mark the point O where $\overleftrightarrow{AB}$ intersects $\overline{YZ}$ (Point O is on $\overline{YZ}$).
iii. Taking the point O as centre, draw a semi-circle of radius $m\overline{OZ}$ using compass.
iv. Taking the point Y as centre draw an arc of radius 6 cm cutting the semi-circle at point X.
v. Join point X with points Y and Z.

We get ΔXYZ with $m\angle X = 90°$



Important Information

Since point O is the midpoint of $\overline{YZ}$, so $m\overline{OY} = m\overline{OZ}$

4.1.3 Construction of Scalene Triangles

We construct a scalene triangle when

(a) Length of two sides and their included angle are given.
(b) Length of two sides and any angle are given.
(c) Two angles and their included side are given.

Remember!

A scalene triangle can be acute angled, right angled or obtuse angled.

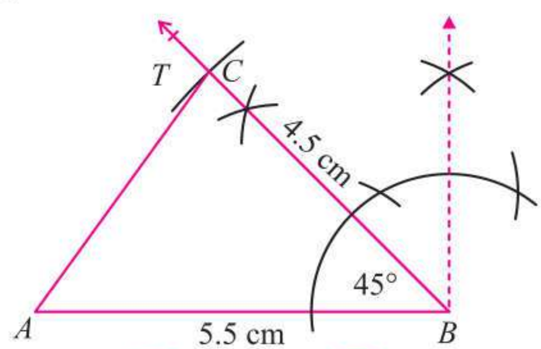
(a) Length of two sides and their included angle are given:

Example 6 Construct a triangle ABC when $m\overline{AB} = 5.5$ cm, $m\overline{BC} = 4.5$ cm and $m\angle B = 45°$

Solution **Steps of construction**

i. Draw a line segment AB of 5.5 cm long.
ii. Construct an angle of 45° at point B with compass and draw a ray BT.
iii. With centre at point B, draw an arc of radius 4.5 cm with compass which intersecting $\overrightarrow{BT}$ at point C.
iv. Join point A with point C.

We get a scalene triangle ABC.



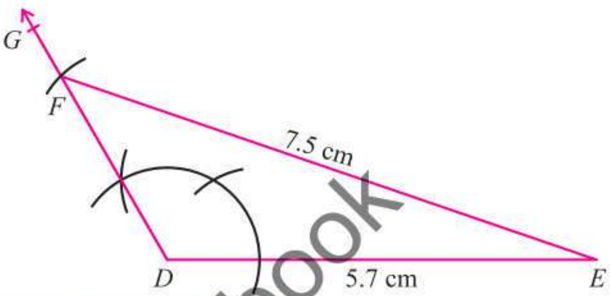
Try yourself

Construct a triangle with measure of sides 3.8 cm, 4.2 cm and 5 cm using compass and ruler.

(b) Length of two sides and an angle are given

Example 7 Draw a ΔDEF such that $m\overline{DE} = 5.7$ cm, $m\overline{EF} = 7.5$ cm and $m\angle D = 120°$

Solution **Steps of construction**



i. Draw $m\overline{DE} = 5.7$ cm.

ii. Construct an angle 120° at point D using a compass and draw $\overrightarrow{DG}$ such that $m\angle EDG = 120°$

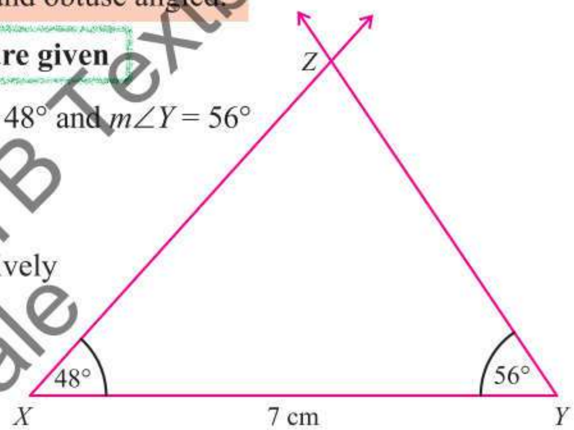
iii. With centre at point E, draw an arc of radius 5.7 cm with compass which intersecting $\overrightarrow{DG}$ at point F.

iv. Join points E and F. We get ΔDEF, which is scalene and obtuse angled.

(c) Two angles and the length of their included side are given

Example 8 Draw a ΔXYZ such that $m\overline{XY} = 7$ cm, $m\angle X = 48°$ and $m\angle Y = 56°$

Solution **Steps of construction**



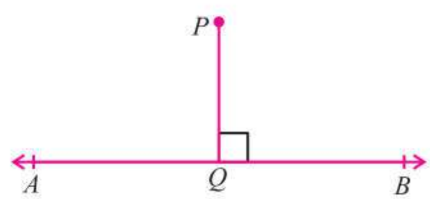
i. Draw a line segment XY of measure 7 cm.

ii. Construct angles 48° and 56° at points X and Y respectively using protractor and draw two rays from points X and Y.

iii. These rays intersect each other at point Z.

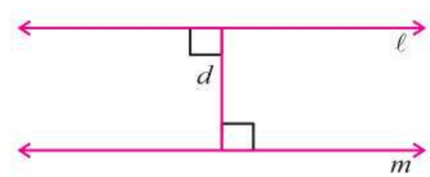
We get ΔXYZ which is scalene and acute angled.

Shortest Distance We have learnt in the previous class how to draw perpendicular from a point to a line i.e., in the figure. $\overline{PQ}$ is perpendicular to the line $\overline{AB}$ at point Q and $m\angle PQB = 90°$.



It is clear from the figure that $m\overline{PQ}$ is the shortest distance from point P to $\overleftrightarrow{AB}$. Hence we conclude that:

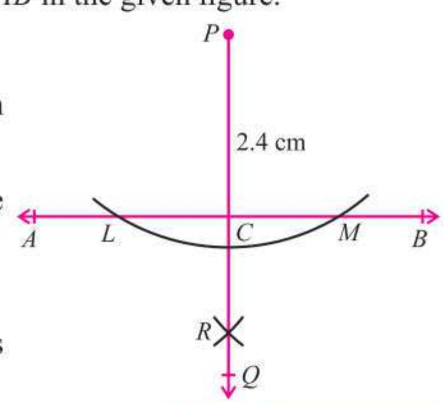
The shortest distance is always perpendicular.



In parallel lines ℓ and m, the shortest distance is d which is perpendicular to ℓ and m.

Example 9 Find the shortest distance of a point P from the line AB in the given figure.

Solution **Steps of construction**



i. With centre at point P draw an arc of any suitable radius which intersecting $\overleftrightarrow{AB}$ at points L and M.

ii. With centres at points L and M draw two arcs of any suitable radius which intersecting each other at point R.

iii. Draw a ray PQ passing through point R intersecting $\overleftrightarrow{AB}$ at point C.

iv. $\overline{PQ}$ is the perpendicular and $m\overline{PC} = 2.4$ cm (measured by ruler) is the shortest distance from point P to the line AB.

Important Information

A perpendicular bisector is a line which divides a line segment into two equal parts.

Activity

Draw the perpendicular bisector on side AB of a triangle ABC taking any suitable measures of its sides.

EXERCISE 4.2

1. Using compass and ruler, construct the following angles:
   (i) 30° (ii) 45° (iii) 75° (iv) 105° (v) 150°
2. Construct a right angled triangle XYZ in which $m\overline{XY} = 6.5$ cm, $m\angle Y = 40^\circ$ and right angle at point X.
3. Construct a right angled triangle ABC with hypotenuse AB of measure 7.1 cm and measure of angle A is 52°.
4. Construct an equilateral triangle of side length 4.7 cm.
5. Construct an isosceles triangle DEF given that $m\overline{DE} = 8.2$ cm is the base and equal sides are $\overline{EF}$ and $\overline{DF}$ each of length 6.5 cm.
6. Construct an isosceles triangle GHI with vertical angle at point G of measure 100° and $m\overline{GH} = 6$ cm.
7. Construct a triangle XYZ with $m\overline{YZ} = 5.3$ cm, $m\angle Y = 45^\circ$ and $m\angle Z = 30^\circ$.
8. Construct a triangle PQR with $m\overline{PQ} = 7$ cm, $m\overline{QR} = 5.5$ cm and $m\angle Q = 75^\circ$.
9. Construct a triangle FGH with $m\overline{GH} = 6.3$ cm, $m\angle H = 55^\circ$ and $m\overline{FG} = 6.9$ cm.
10. Draw a line segment PQ of measure 6 cm. Take a point A above $\overline{PQ}$ and draw a perpendicular from the point A to $\overline{PQ}$. What is the shortest distance of point A from $\overline{PQ}$?

SUMMARY

- Right angle is of measure 90°.
- A triangle is a 3-sided closed figure.
- Types of triangle:
  - with respect to angles are: acute angled , right angled and obtuse angled.
  - with respect to sides are: scalene, isosceles and equilateral.
- Triangle can be constructed if:
  - (a) length of all sides are given.
  - (b) when length of two sides and their included angle are given.
  - (c) when two angles and their included side are given.
  - (d) when length of two sides and any angle are given.
- The shortest distance from a point to a line is always perpendicular.

Sub-domain (ii)

Angle Properties of Polygons

Students' Learning Outcomes

After studying this sub-domain, students will be able to:

- Recognise quadrilaterals and their characteristics (parallel sides, equal sides, equal angles, right angles, lines of symmetry etc.) square, rectangle, parallelogram, rhombus, trapezium and kite.
- Differentiate between convex and concave polygons.
- Calculate unknown angles in quadrilaterals using the properties of quadrilaterals. (square, rectangle, parallelogram, rhombus, trapezium and kite).
- Understand the relationship between interior and exterior angles of polygons and between opposite interior and exterior angles in a triangle.
- Calculate the interior and exterior angles of a polygon and the sum of interior angles of a polygon.
- Calculate unknown angles in a triangle.

This is the honeybee hive.
Can you see geometrical patterns on it?
Who have taught the skill to little bee for making such patterns?

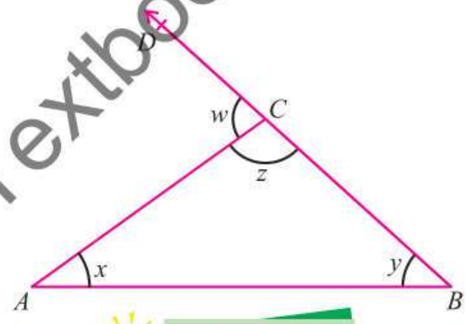
Can you see geometrical patterns on snake's skin?

4.2.1 Introduction

There are many 2D figures around us in triangular, square, rectangular, pentagonal or hexagonal in our daily life. These figures are used to make patterns in different 3D objects.

These figures are base of geometry which are useful to teach in elementary education.

4.2.2 Angle Properties of Triangle



The sum of all interior angles in a triangle is equal to 180°.

i.e., $x + y + z = 180°$ in ΔABC

If we extend the side $\overline{BC}$ to $\overrightarrow{BD}$ we get $m\angle ACD = w$ as exterior angle of ΔABC.

We see that interior and exterior angles at point C are supplementary

$$z + w = 180° \rightarrow \text{supplementary angles}$$

$$\Rightarrow \quad w = 180° - z$$

$$\text{But} \quad x + y + z = 180°$$

$$x + y = 180° - z$$

$$\because \quad x + y = w \qquad \because w = 180° - z$$

Exterior angle = Sum of two interior opposite angles

Remember!

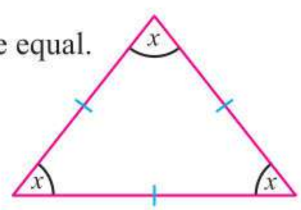
Angle formed between a side and extended side is called exterior angle.

Skill Practice

In ΔABC, find x, if $y = 36°$ and $w = 100°$

i Equilateral Triangle

In equilateral triangle all angles and sides are equal.



In the figure $\quad x + x + x = 180°$

$$3x = 180°$$

$$x = 60° \quad \text{(each angle is equal to 60°)}$$

Important Information

$m\angle A = 180° - 2m\angle B$ (vertical angle)

$m\angle B = \dfrac{180° - m\angle A}{2}$ (base angle)

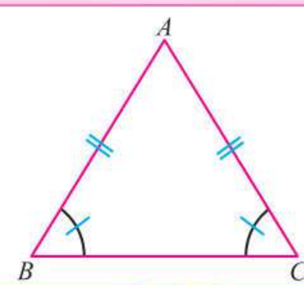
Skill Practice

Find vertical angle of an isosceles triangle if base angle is 75°.

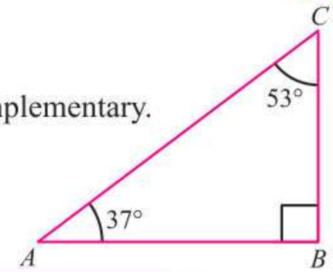
ii Isosceles Triangle

In ΔABC, $\quad m\overline{AB} = m\overline{AC}$ and $\quad m\angle B = m\angle C$

$\angle B$ and $\angle C$ are called base angles and $\angle A$ is called vertical angle of isosceles triangle.



iii Right Angled Triangle



In right angled triangle, exactly one angle is 90° and other two angles are complementary.

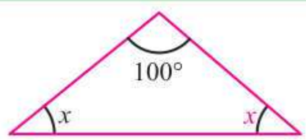
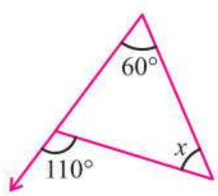
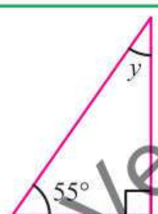
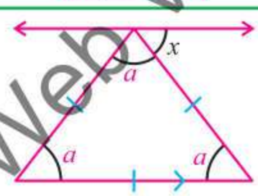
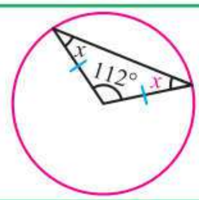
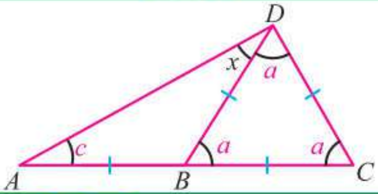
In ΔABC, $m\angle B = 90°$, $m\angle A = 37°$ and $m\angle C = 53°$

Where $\angle A$ and $\angle C$ are complementary angles.

Skill Practice

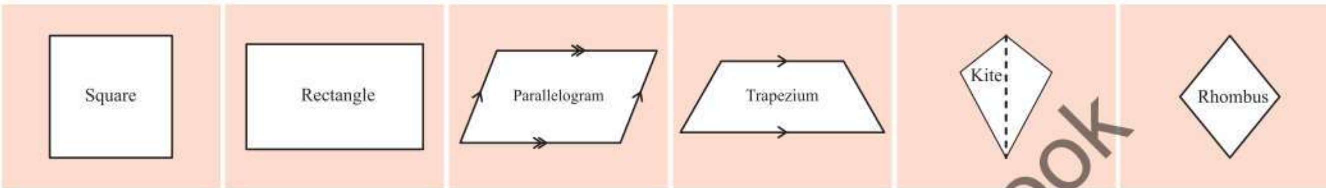
If complementary angles in a right angle triangle are equal to each other, then find these angles.

Example 1 Find unknown angles in the following figures:

	Figure	Solution
i		Sum of angles in a triangle = 180° $x + x + 100° = 180°$ $2x = 180° - 100° \Rightarrow x = \dfrac{80}{2} = 40°$
ii		Exterior angle = Sum of two interior opposite angles $110° = 60° + x$ $x = 110° - 60°$ $x = 50°$
iii		Triangle is right angled, So $y + 55° + 90° = 180°$ $y + 145° = 180°$ $y = 180° - 145° \Rightarrow y = 35°$
iv		The given triangle is equilateral $\because\ a + a + a = 180° \Rightarrow 3a = 180° \Rightarrow a = 60°$ a and x are alternate. i.e., $x = a = 60°$
v		The given triangle is isosceles $x + x + 112 = 180°$ $2x = 180° - 112° \Rightarrow 2x = 68° \Rightarrow x = 34°$
vi		ΔBCD is equilateral $3a = 180° \Rightarrow a = 60°$ also $c = x$ (ΔABD is isosceles) $c + x = a \Rightarrow x + x = 60°$ (The exterior angle is equal to the sum of two opposite interior angles) $2x = 60° \Rightarrow x = 30°$

4.2.3 Identification of Quadrilaterals

Any four sided closed figure is called quadrilateral. It has four angles and four vertices. There are following different types of quadrilaterals:

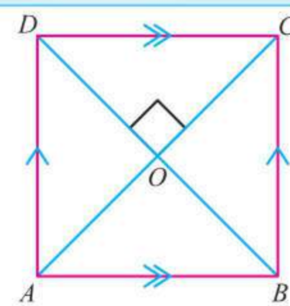


Now we discuss their properties in detail.

Need to Know!

The sum of interior angles of any quadrilaterals is equal to 360°.

Square



(i) All four sides are of equal length.

i.e., $m\overline{AB} = m\overline{BC} = m\overline{CD} = m\overline{AD}$

(ii) Four angles, each of measure equal to 90°.

i.e., $m\angle A = m\angle B = m\angle C = m\angle D = 90°$

(iii) Two pairs of parallel sides i.e., $\overline{AB} \parallel \overline{DC}$ and $\overline{BC} \parallel \overline{AD}$.

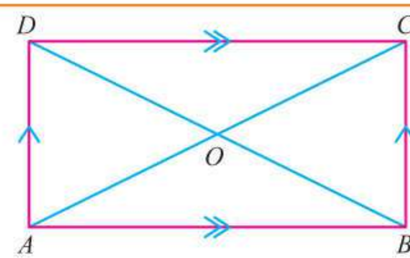
(iv) Diagonals are of equal length i.e., $m\overline{AC} = m\overline{BD}$.

(v) Diagonals bisect each other at point O i.e., midpoint of both diagonals is same.

(vi) Diagonals are perpendicular to each other i.e., $m\angle AOB = m\angle BOC = m\angle COD = m\angle AOD = 90°$

(vii) Since $\overline{AB} \parallel \overline{DC}$, so alternate angles formed are equal i.e., $m\angle CAB = m\angle ACD$ etc.

Rectangle



(i) It has four sides. Opposite sides are of equal length.

i.e., $m\overline{AB} = m\overline{DC}$ and $m\overline{AD} = m\overline{BC}$

(ii) Each interior angle is equal to 90°.

i.e., $m\angle A = m\angle B = m\angle C = m\angle D = 90°$

(iii) Two pairs of parallel sides i.e., $\overline{AB} \parallel \overline{DC}$ and $\overline{BC} \parallel \overline{AD}$.

(iv) The following alternate angles are formed with the diagonals

$m\angle BAC = m\angle DCA$ and $m\angle ABD = m\angle CDB$ etc.

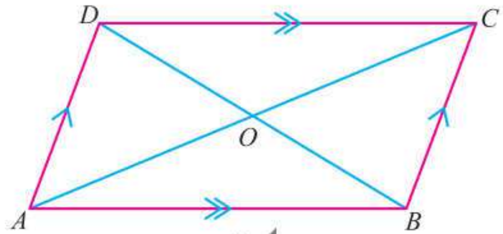
(v) Diagonals are of equal length i.e., $m\overline{AC} = m\overline{BD}$.

(vi) Diagonals bisect each other at point O and the midpont of diagonal AC = midpoint of diagonal BD.

Think

Is every square a rectangle?

Parallelogram



(i) It has four sides. Opposite sides are of equal length.

i.e., $m\overline{AB} = m\overline{DC}$ and $m\overline{AD} = m\overline{BC}$

(ii) Two pairs of parallel sides. (Opposite sides are parallel)

i.e., $\overline{AB} \parallel \overline{DC}$ and $\overline{BC} \parallel \overline{AD}$

(iii) The following alternate angles are formed with the diagonals

$m\angle BAC = m\angle DCA$ and $m\angle ABD = m\angle CDB$

Remember!

Every square is a parallelogram and every rectangle is a parallelogram.

(iv) Opposite angles are equal i.e., $m\angle A = m\angle C$ and $m\angle B = m\angle D$

(v) Adjacent angles are supplementary due to parallel lines i.e.,

$m\angle A + m\angle D = 180°$ and $m\angle B + m\angle C = 180°$

($m\angle A$, $m\angle D$) and ($m\angle B$, $m\angle C$) are two pairs of interior angles of parallel lines.

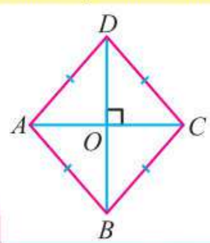
(vi) Diagonals bisect each other at point O and the mid point of diagonal AC = mid point of diagonal BD.

Rhombus

(i) All four sides are of equal length. i.e., $m\overline{AB} = m\overline{BC} = m\overline{CD} = m\overline{AD}$

(ii) Opposite sides are parallel. i.e., $\overline{AB} \parallel \overline{DC}$ and $\overline{BC} \parallel \overline{AD}$.

Alternate and interior angles are formed due to parallel lines.



(iii) Diagonals are not of equal length and they bisect the angles $m\angle A$, $m\angle B$, $m\angle C$ and $m\angle D$.

(iv) Diagonals bisect each other at right angle at point O and midpoint of diagonal AC = midpoint of diagonal BD.

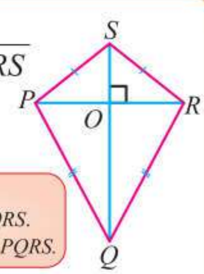
(v) Opposite angles are equal in measure

i.e., $m\angle A = m\angle C$ and $m\angle B = m\angle D$

Think

What is the difference between a rhombus and a square?

Kite



(i) Two pairs of adjacent sides are of equal length i.e., $m\overline{PQ} = m\overline{QR}$ and $m\overline{PS} = m\overline{RS}$

(ii) One pair of opposite angles are equal. i.e., $m\angle P = m\angle R$

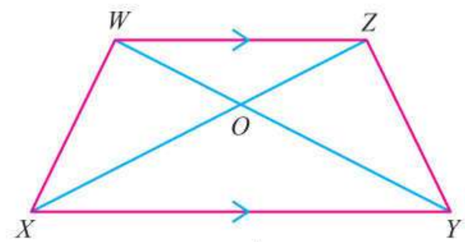
(iii) Diagonals are perpendicular to each other.

(iv) One diagonal QS bisect the other diagonal PR.

Try yourself!

- Name two isosceles triangles in kite $PQRS$.
- Name two right angled triangles in kite $PQRS$.

Trapezium



(i) Only one pair of parallel sides i.e., $\overline{XY} \parallel \overline{WZ}$

(ii) Two pairs of interior angles of parallel lines

i.e.,$m\angle WXY + m\angle XWZ = 180°$ and

$m\angle XYZ + m\angle WZY = 180°$

(iii) The pairs of alternate angles of parallel lines.

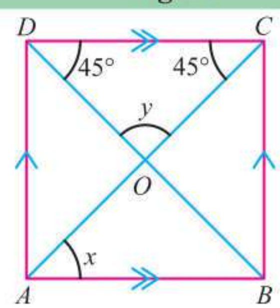
i.e.,$m\angle YXZ = m\angle WZX$ and $m\angle XYW = m\angle ZWY$

(iv) Vertically opposite angles at point O are: $m\angle XOY = m\angle ZOW$; $m\angle XOW = m\angle ZOY$

Example 2 Find the unknown angles *x, y* or *z* in the following:

i

Figure



ABCD is a square

Solution

Since the diagonals bisect the angles of square, so

$$x = \frac{90°}{2} = 45°$$

In ΔCOD, $m\angle C = m\angle D = 45°$

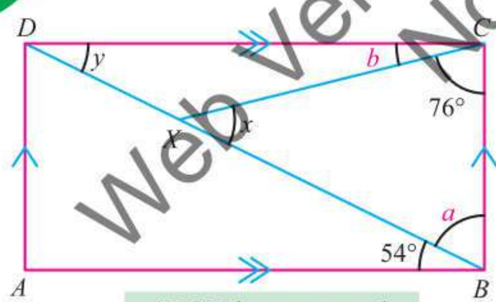
So, $y + 45° + 45° = 180°$

$y + 90° = 180°$

$y = 90°$

ii

Figure



ABCD is a rectangle

Solution

Since each angle is 90°,

So, $m\angle B = 90°$

$a = 90° - 54°$

$= 36°$

Mark angles *a* and *b* as shown

From ΔBCX: $x + 76° + a = 180°$

So, $x + 76° + 36° = 180°$

$x = 180° - 112°$

$= 68°$

Since $m\angle BCD = 90°$

So, $b + 76° = 90°$

$b = 90° - 76°$

$= 14°$

In ΔCDX,

Interior angle = Sum of two opposite interior angles

$\because$ $x = b + y$

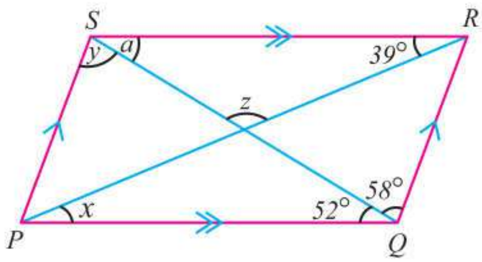
So, $68° = 14 + y$

$y = 68° - 14°$

$= 54°$

	Figure	Solution

iii



PQRS is a parallelogream

Mark angle a as shown

Since, $\overline{PQ} \parallel \overline{SR}$

So, $x = 39°$ Alternate angles of parallel lines

Since, opposite angles in parallelogram are equal, so

$$m\angle PSR = m\angle PQR$$

$$y = 52° + 58° = 110°$$

$$a = 52° \quad \text{Alternate angles of parallel lines}$$

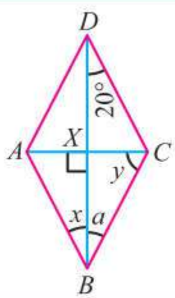
In ΔSRX,

$$a + z + 39° = 180°$$

$$52° + z + 39° = 180°$$

$$z = 180° - 91° = 89°$$

iv



ABCD is a rhombus

Mark angle a as shown

Since, diagonals bisect the angles

so, $x = 20°$ They are also alternate angles of parallel lines

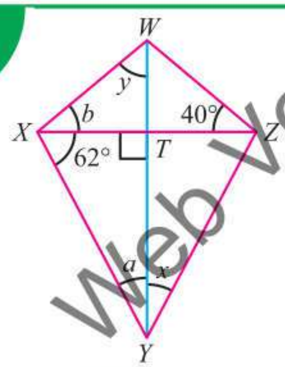
Also ΔBXC is right angled

$$y + a + 90° = 180°$$

$$y + 20° + 90° = 180°$$

$$y = 180° - 110° = 70°$$

v



XYZW is a kite

Mark angles a and b as shown. Diagonals are at right angle to each other. So, a and 62° are complementary angles in right angled $\Delta\, XTY$

$$a + 62 = 90°$$

$$a = 90° - 62° = 28°$$

Since $\Delta\, XZW$ is isosceles, so $b = 40°$

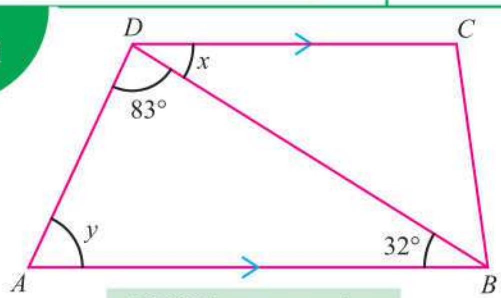
In right angled $\Delta\, XWT$, b and y are complementary

$$b + y = 90°$$

$$40° + y = 90°$$

$$y = 50°$$

vi



ABCD is a trapezium

Diagonals intersect parallel lines $\overline{AB}$ and $\overline{DC}$

$$x = 32° \quad \text{Alternate angles of parallel lines}$$

In $\Delta\, ABD$,

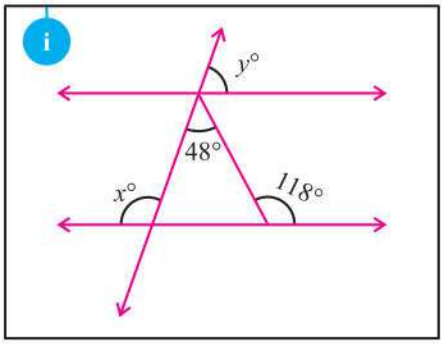
$$y + 32° + 83° = 180°$$

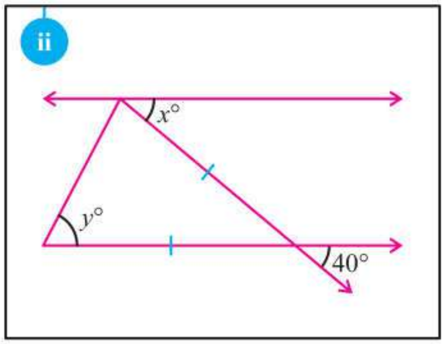
$$y = 180° - 115°$$

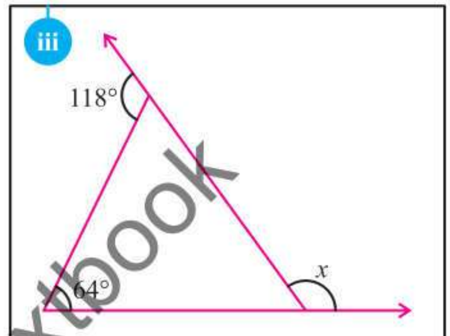
$$y = 65°$$

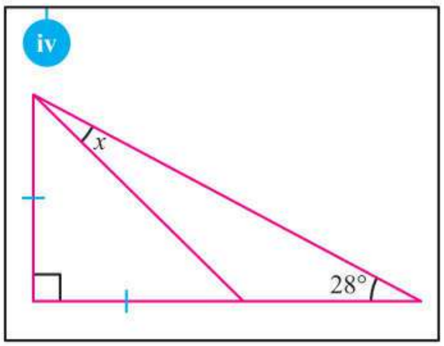
EXERCISE 4.3

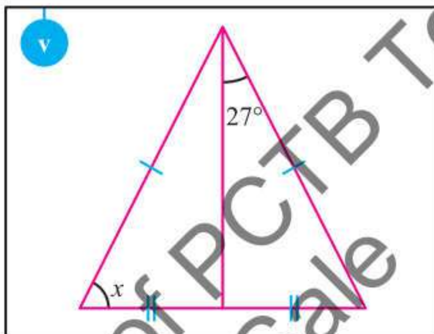
1. Find unknown angles in the following figures.

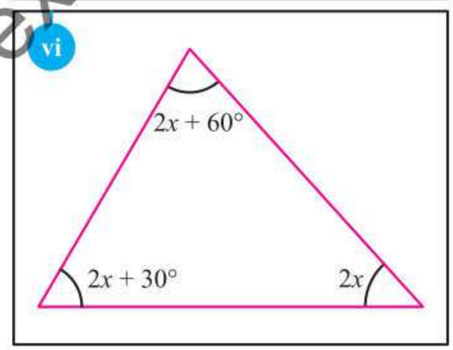




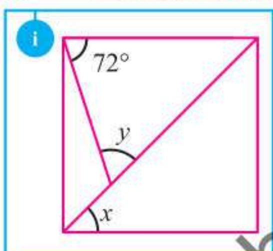


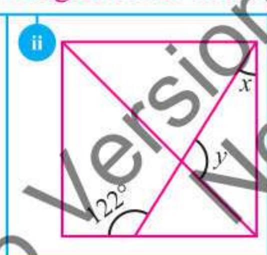


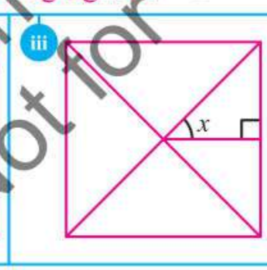


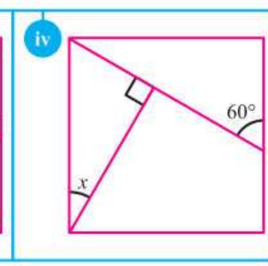


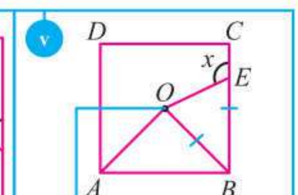
2. Find unknown angles in the following figures.





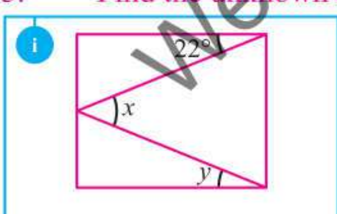




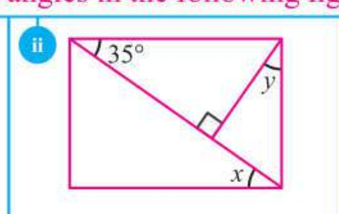


Point O is the midpoint of the diagonals and $m\overline{OB} = m\overline{BE}$

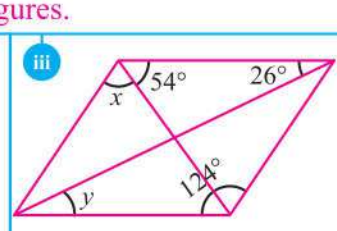
3. Find the unknown angles in the following figures.



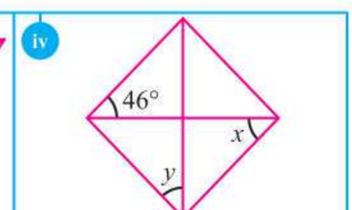
Rectangle



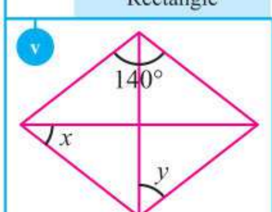
Rectangle



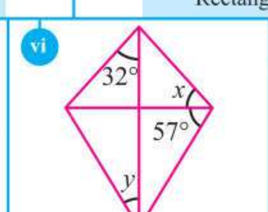
Parallelogram



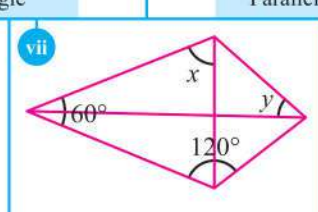
Rhombus



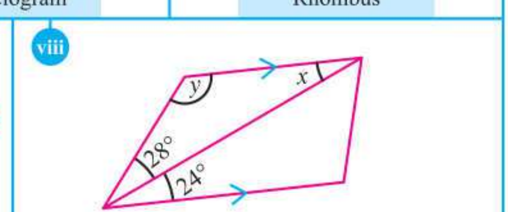
Rhombus



Parallelogram



Kite



Trapezium

4.2.4 Polygons

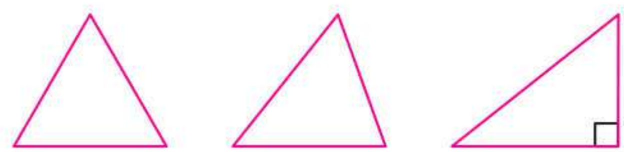
The design of floor or walls are usually square, triangular, pentagonal or hexagonal. Due to symmetry, their designs are fascinating.

Remember!

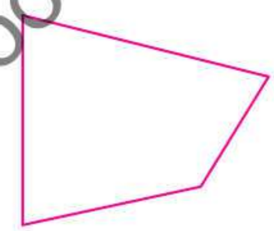
The sides of polygon are line segments.

Any closed 2D shape with three or more sides is called polygon. If a polygon has n-sides, then this polygon has n-angles and n-vertices.

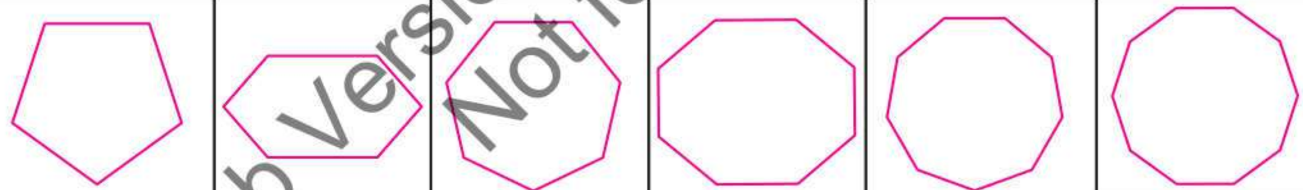
Polygon with three sides is called a triangle.



Polygon with four sides is called quadrilateral

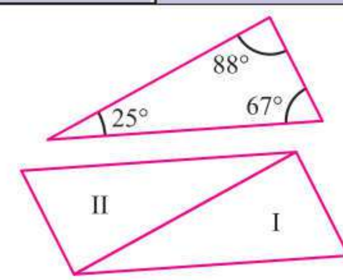


We have already studied different types of quadrilateral.



Pentagon (5-sides) | Hexagon (6-sides) | Heptagon (7-sides) | Octagon (8-sides) | Nonagon (9-sides) | Decagon (10-sides)

i The Sum of Interior Angles of a Polygon



The number of sides of a polygon is equal to the number of angles. We know that the sum of interior angles of a triangle is 180°.

We can find the sum of interior angles of a quadrilateral by dividing the figure into two triangles.

The sum of interior angles of the quadrilateral = Sum of interior angles of triangle-I and II

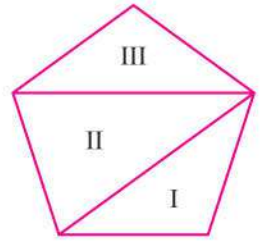
$$= 180° + 180°$$

or $= 2 \times 180°$

or $= (4 - 2) \times 180°$

$$= 360°$$

If we divide the pentagon into three triangles then the sum of interior angles of pentagon = Sum of interior angles of triangles I, II and III.



$= 180° + 180° + 180°$
$= 3 \times 180° = 540°$
$= (5 - 2) \times 180°$

We have seen that

Shape	Sum of interior angles	
Triangle	$180° = 1 \times 180°$	$(3 - 2) \times 180°$
Quadrilateral	$360° = 2 \times 180°$	$(4 - 2) \times 180°$
Pentagon	$540° = 3 \times 180°$	$(5 - 2) \times 180°$
n-sided polygon		$(n - 2) \times 180°$

Hence, the sum of interior angles of n-sided polygon = $(n - 2) \times 180°$

Example 3 Find the sum of interior angles of:

(i) Hexagon (ii) Octagon

Solution

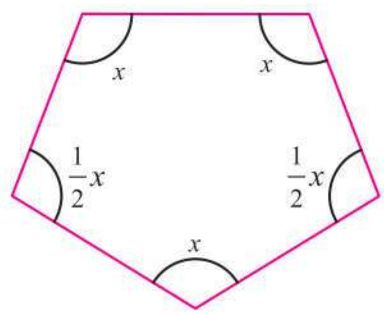
(i) Here $n = 6$ (Hexagon has 6 sides)

Sum of interior angles $= (n - 2) \times 180°$
$= (6 - 2) \times 180°$
$= 4 \times 180°$
$= 720°$

(ii) Here $n = 8$ (Octagon has 8 sides)

Sum of interior angles $= (n - 2) \times 180°$
$= (8 - 2) \times 180°$
$= 6 \times 180°$
$= 1080°$

Example 4 Find unknown value x.



Solution The given figure is a pentagon

Sum of interior angles of a pentagon $= (n - 2) \times 180°$
$= (5 - 2) \times 180°$
$= 3 \times 180° = 540°$

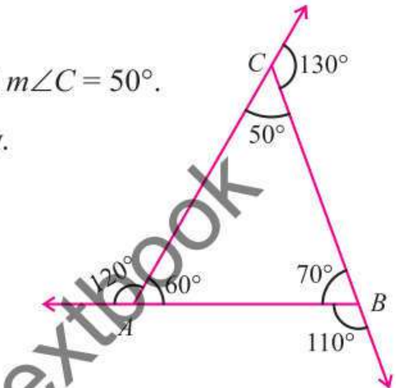
Sum of interior angles $= x + x + x + \frac{1}{2}x + \frac{1}{2}x$
$= 3x + x$
$= 4x$

$\because \quad 4x = 540°$

$x = \frac{540}{4}$
$= 135°$

ii The Sum of Exterior Angles of a Polygon

An exterior angle of polygon is formed when one of its side is extended. Then the angle formed with the adjacent side is called exterior angle.



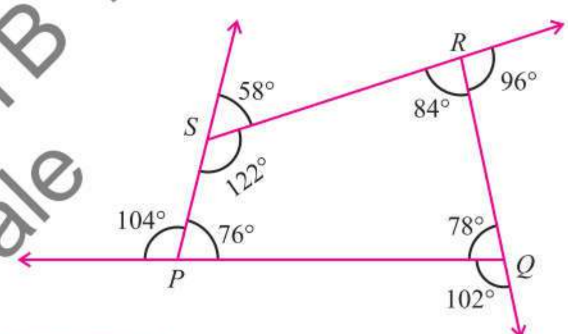
For example in ABC, interior angles are $m\angle A = 60°$, $m\angle B = 70°$ and $m\angle C = 50°$.

Exterior angles at point A, B, C are 120°, 110° and 130° respectively.

We notice that sum of exterior angles in a triangle

$$= 120° + 110° + 130°$$
$$= 360°$$

In case of quadrilateral $PQRS$ the exterior angles at P, Q, R and S are 104°, 102°, 96° and 58° respectively.



Sum of these exterior angles $= 104° + 102° + 96° + 58°$

$$= 360°$$

We have seen that the sum of exterior angle is 360° for both triangle and quadrilateral. So,

Sum of exterior angles of any polygon = 360°

(a) Rotation between Interior and Exterior Angles of a Polygon

From the above examples we noted that interior and exterior angles are adjacent and they are supplementary. So, the sum of interior and exterior angles at a point is equal to 180°.

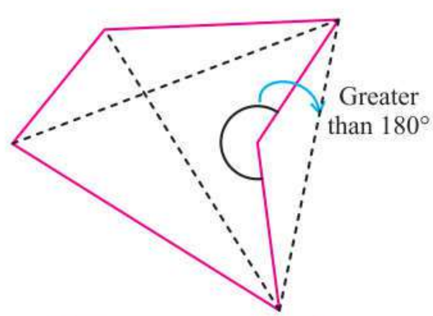
i.e., Interior angle + Exterior angle = 180°

4.2.5 Types of Polygons

There are two main types of polygons (a) Concave polygon (b) Convex polygon

(a) Concave Polygon

A polygon which has at least one reflex angel is said to be a concave polygon.

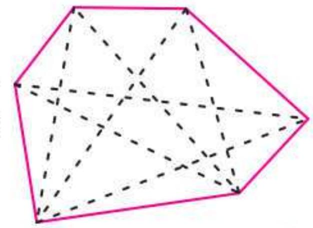


Concave polygon

For example: In the figure, one angle is of measure greater than 180° (i.e., one angle is reflex). So, it is a concave polygon. If we draw diagonals, we see that one of its diagonals lies outside the polygon.

(b) Convex Polygon

A polygon which has all of its angles less than 180° is called convex polygon. All of its diagonals lie inside the polygon as shown in the figure. Each angle is less than 180°.



Convex polygon

Activity

The number of diagonals in a polygon can be calculated by the formula $\frac{n(n-3)}{2}$ where n is the number of sides of the polygon. Find the number of diagonals of pentagon and octagon and also draw these figures with their diagonals.

Brain Teaser!

Draw a polygon which has two reflex angles.

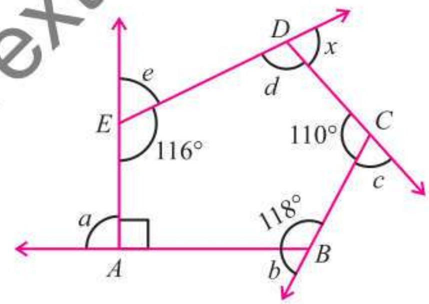
Example 5

(i) Identify the exterior angles of pentagon $ABCDE$.

(ii) Find the exterior angles at vertices A, B, C and E.

(iii) Find the value of x.

(iv) Identify the polygon (concave or convex)



Solution

(i) The angles marked at vertices A, B, C, D and E as a, b, c, x and e are exterior angles of pentagon $ABCDE$. Because they are formed with the extended side of the polygon.

(ii) The exterior angle at point A, $a = 180° - 90° = 90°$

The exterior angle at point B, $b = 180° - 118° = 62°$

The exterior angle at point C, $c = 180° - 110° = 70°$

The exterior angle at point E, $e = 180° - 116° = 64°$

Important Information

The exterior angle at point D can be written as

$$x = 180° - d$$

(iii) To find the value of x, first we find the value of d.

The sum of interior angles of pentagon $= (n-2) \times 180°$

$$= (5-2) \times 180°$$

$$= 3 \times 180° = 540°$$

Explore

Find whether the sum of exterior angles of pentagon $ABCDE$ is equal to 360°.

Adding all the interior angles of pentagon

$$90° + 118° + 110° + d + 116° = 540°$$

$$434° + d = 540° \quad \Rightarrow \quad d = 106°$$

Since x is exterior angle, so $x = 180° - 106°$

$$= 74°$$

(iv) Since each of the interior angle of the pentagon is less than 180°, so it is a convex polygon.

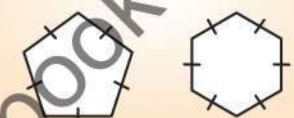
4.2.6 Regular Polygon

A polygon is said to be regular if all of its sides are equal in measure. A regular polygon also has all of its interior angles (or exterior angles) equal in measure.

An equilateral triangle and square are regular polygons because each side is of equal length and all of the interior angles are equal.

Important Information

A regular pentagon and a regular hexagon are:



(a) Interior Angle of Regular Polygon

Interior angle can be found by dividing sum of the interior angles by number of sides (n) i.e.,

$$\text{Interior angle} = \frac{\text{Sum of all interior angles}}{\text{Number of sides}}$$

$$= \frac{(n-2)\times 180^\circ}{n}$$

$\because$ Each interior angle is equal in measure

Skill Practice

Draw a pattern made of repeating equilateral triangle.

(b) Exterior Angle of Regular Polygon

We know that sum of exterior angles of every polygon with n-sides is equal to 360°. In regular polygon all exterior angles are equal in measure. So,

$$\text{Exterior angle} = \frac{360^\circ}{n}$$

n is the number of sides

Example 6 Find interior and exterior angles of a regular polygon with given number of sides (n).

(i) $n = 6$ (ii) $n = 9$

Solution **(i) $n = 6$ (Hexagon)**

Sum of interior angles of hexagon $= (n-2)\times 180^\circ$

$= 4\times 180^\circ$

$= 720^\circ$

$$\text{Interior angle} = \frac{720^\circ}{6} = 120^\circ$$

$$\text{Exterior angle} = \frac{360^\circ}{6} = 60^\circ$$

Working

Exterior angle = 180° – interior angle

= 180° – 120° = 60°

Working

The sum of all exterior angles of hexagon = 6 × 60° = 360°

(i) $n = 9$ (Nonagon)

$$\text{Interior angle} = \frac{(n-2)\times 180^\circ}{n}$$

$$= \frac{(9-2)\times 180^\circ}{9}$$

$$= \frac{7\times \cancel{180^\circ}^{20}}{\cancel{9}_1} = 140^\circ$$

Exterior angle = 180° – interior angle

= 180° – 140° = 40°

Working

Sum of all exterior angles of nonagon = 9 × 40° = 360°

Important Information

Bee honeycomb is a network of regular hexagon

Example 7

(i) The interior angle of a regular polygon with n-sides is 144°. Find the value of n.

(ii) The exterior angle of a regular polygon is 30°. Find the number of sides of the polygon.

Solution

(i) Interior angle = 144°

Number of sides = n

Interitor angle = $(n-2) \times 180°$

$$144° = \frac{(n-2) \times 180°}{n}$$

$$144n = 180n - 360$$

$$180n - 144n = 360$$

$$n = \frac{360}{36}$$

$$n = 10$$

(ii) Exterior angle = $\frac{360°}{n}$

$$30° = \frac{360°}{n}$$

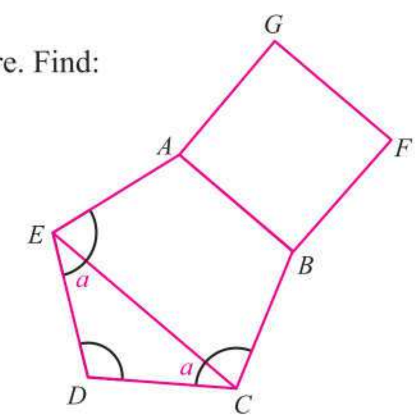
$$n = \frac{360°}{30°}$$

$$n = 12$$

Example 8 $ABCDE$ is a regular pentagon and $ABFG$ is a square. Find:

(i) Interior angle of pentagon

(ii) $m\angle CBF$ (iii) $m\angle DCE$



Solution

(i) A pentagon has $n = 5$ sides.

Interior angle = $\frac{(n-2) \times 180°}{n}$

Interior angle = $\frac{(5-2) \times 180°}{5}$

$$= \frac{3 \times \cancel{180°}^{36}}{\cancel{5}_1}$$

$$= 108°$$

(ii) $m\angle ABC = 108°$

$$m\angle ABF = 90°$$

Reflex angle CBF

$$m\angle ABC + m\angle ABF = 108° + 90°$$

$$= 198°$$

$$m\angle CBF = 360° - 198°$$

$$= 162° \text{ (which is obtuse)}$$

(iii) ΔDCE is isosceles because of regular pentagon

Let $m\angle CDE = 108°$ and $m\angle DCE = a$

So, $108° + a + a = 180°$

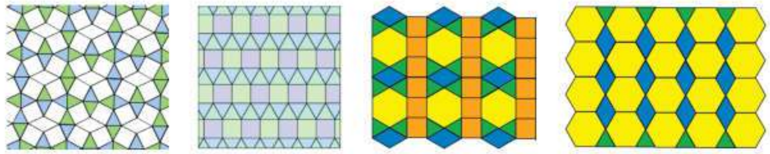
$$2a = 180° - 108°$$

$$a = \frac{72°}{2}$$

$$m\angle DCE = 36°$$

Important Information

A tessellation is a repetitive pattern of polygons that fit together without overlapping and without gaps between them. Find a tessellation anywhere in your house or city.



EXERCISE 4.4

1. Find the sum of interior angles of a polygon with given number of sides (n)

   (i) $n = 7$ (ii) $n = 9$ (iii) 13

2. Find an interior angle of a regular polygon with given number of sides (n):

   (i) $n = 6$ (ii) $n = 8$ (iii) $n = 15$

3. Find exterior angle of a regular polygon with given number of sides (n):

   (i) $n = 7$ (ii) $n = 8$ (iii) $n = 10$

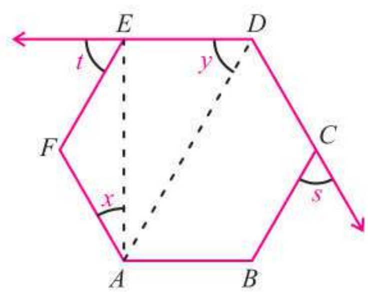
4. Find number of sides (n) of a regular polygon with given an interior angle:

   (i) 108° (ii) 140° (iii) 150°

5. Find number sides (n) of regular polygon with given an exterior angle:

   (i) $32\frac{8}{11}°$ (ii) $25\frac{5}{7}°$ (iii) 24°

6. The figure *ABCDEF* is a regular hexagon:



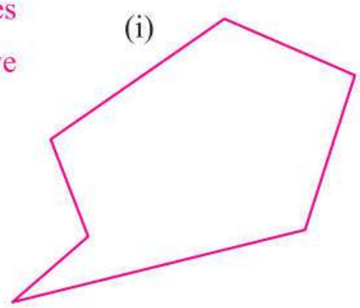
   (i) Find the value of x and y

   (ii) Identify two exterior angles in the polygon.

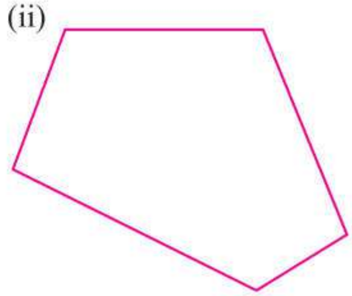
   (iii) Find the value of s and of t.

7. Draw diagonals in the given figures to identify which polygon is concave and which is convex?

(i)



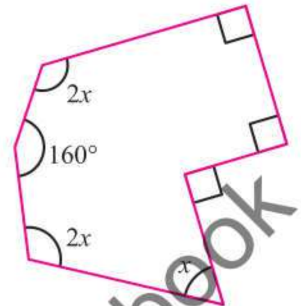
(ii)



8. The interior angles of a pentagon are in the ratio 1:2:3:4:2. Find these angles.

9. The size of each interior angle is 8 times the exterior angle in a regular polygon. Find the number of sides of the polygon.

10. The given polygon is a heptagon. Find the value of x.



SUMMARY

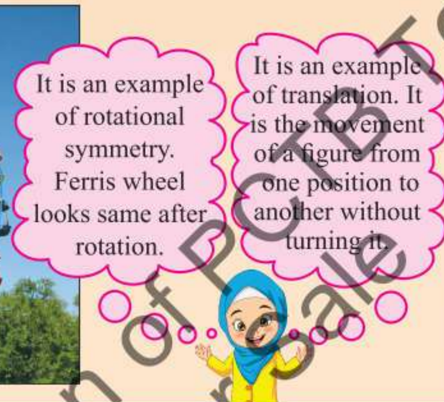
- In square, rectangle, parallellogram and rhombus, diagonals bisect each other and midpoint of one diagonal = midpoint of the other diagonal.
- In trapezium, there are two parallel lines.
- Any closed 2D figure with three or more sides is called a polygon.
- A polygon which has at least one reflex angel is said to be a concave polygon.
- A polygon is said to be a regular polygon if all the sides (or angles) are of equal measure. Equilateral triangle and square are regular polygon.
- The sum of interior angles of any polygon is $(n-2) \times 180$, with n-sides.
- The sum of exterior angles of any polygon is 360°.
- The size of interior angle of a regular polygon is $\frac{(n-2) \times 180°}{n}$.
- The size of exterior angle of a regular polygon is $\frac{360°}{n}$.
- The sum of each interior and corresponding exterior angles of any polygon is 180°.
- The exterior angle of triangle is equal to the sum of two opposite interior angles.

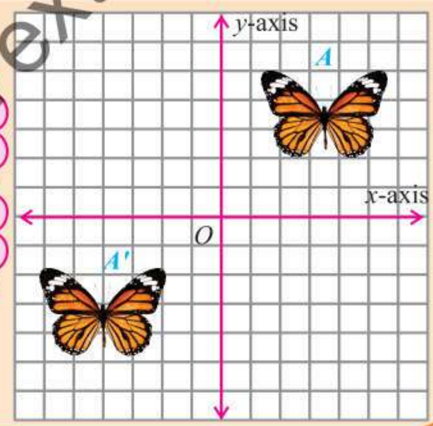
Sub-domain (iii) Transformation

Students' Learning Outcomes

After studying this sub-domain, students will be able to:

- Recognise identity and draw lines of symmetry in 2D shapes and rotate objects using rotational symmetry and find the order of rotational symmetry.
- Translate an object and give precise description of transformation





4.3.1 Symmetry

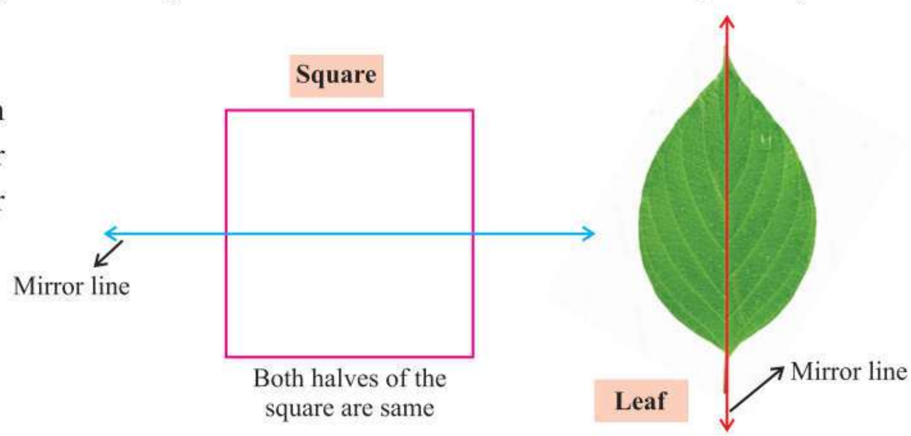
Most of the things in our life are symmetrical. Symmetry looks beautiful. Allah made symmetrical objects which creates harmony and order. We will discuss here line symmetry, rotational symmetry and translation symmetry.

i Types of Symmetry

Two types of symmetry are mostly used in daily life which are reflective and rotational symmetry.

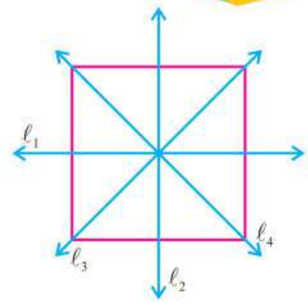
(a) Reflective Symmetry

It is a type of symmetry in which half of the shape reflects the other half. It is also known as mirror symmetry.



Lines of Symmetry in a Square

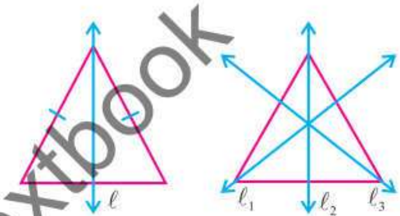
Mirror line is also called line of symmetry. There are following lines of symmetry that can be drawn in a square.



ℓ_1 is horizontal, ℓ_2 is vertical, ℓ_3 and ℓ_4 are diagonal lines of symmetry.

Lines of Symmetry in Isosceles and Equilateral Triangles

In an isosceles triangle there is only one line of symmetry. The equilateral triangle has three lines of symmetry.



Consider the line of symmetry ℓ_1 in equilateral triangle. If we fold on the line ℓ_1 we see that half of the figure matches the other half.

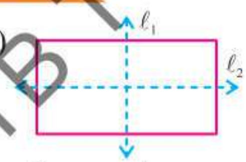
Example 1 Draw lines of symmetry in the following figures:

(i)

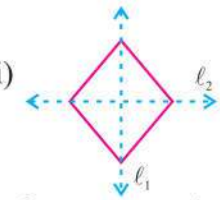
(ii)

Solution

(i)



(ii)



We can draw only two lines of symmetry in a rectangle and rhombus.

Identify Line of Symmetry

The figure shows a rectangle. If we fold the rectangle over the line ℓ_1, then upper half of figure does not match with lower half and also same in case of ℓ_3. So ℓ_1 and ℓ_3 are not lines of symmetry.

If the figure is folded over the line ℓ_2 both halves match each other. So ℓ_2 is the line of symmetry.

(b) Rotational Symmetry

We have seen that many figures have lines of symmetry but many have no line of symmetry. Some figures look the same after rotation.

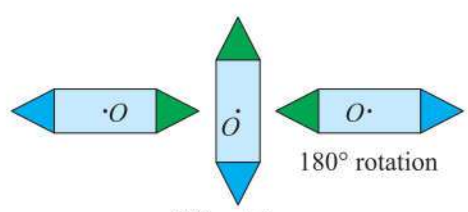

Ferris wheel looks same after rotation

If a figure is rotated around its centre and it looks same as it was before rotation, it is said to be a rotational symmetry.

The point about which the figure is rotated is called centre of rotation.

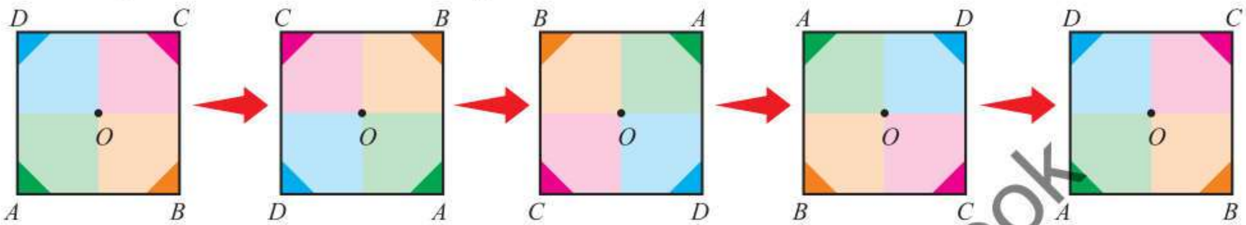
Identify Rotational Symmetry

State whether the figure has rotational symmetry or not? We see that 90° rotation clockwise or anticlockwise about point O we do not get the original figure. If we rotate 180° about point O, we will get the original figure again. So, the figure has rotational symmetry because it gets its shape two times when rotated in full rotation.



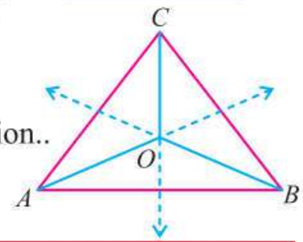
Order of Rotation

The order of rotation is the number of times a shape becomes same in one full rotation. We rotate the square about point O anticlockwise through 90°.



0° rotation | 90° rotation | 180° rotation | 270° rotation | 360° rotation

After 360° rotation the square is rotated four times and looks same after each of 90° anticlockwise rotation. So, the square has rotational symmetry of order 4.



Example 2 In the given equilateral triangle ABC, find centre and angle of rotation..

Solution $\overrightarrow{AO}$, $\overrightarrow{BO}$ and $\overrightarrow{CO}$ are lines of symmetry.

In the ΔOAB,

$m\angle OAB = m\angle OBA = 30°$

So, $m\angle AOB = 180° - 30° - 30° = 120°$

Similarly, $m\angle BOC = m\angle AOC = 120°$

If we rotate $\Delta\ ABC$ about point O through 120°, we get the original figure again. So,

Angle of rotation = 120°

Centre of rotation = point O.

Order of rotational symmetry $= \dfrac{360°}{120°} = 3$

The centre of rotation can be found by drawing lines of symmetry. The point where the lines of symmetry meet, is the point of rotation.

Need to Know!

In regular polygon with n-sides, the number of lines of symmetry are n and order of rotational symmetry is also n.

Angle of rotational symmetry $= \dfrac{360°}{n}$

4.3.2 Transformation

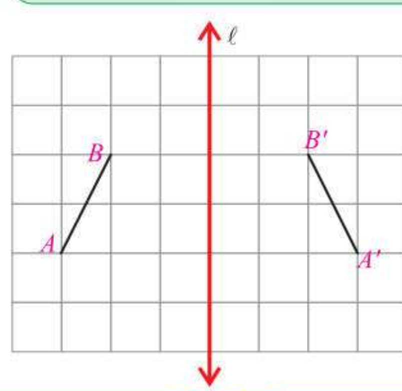
It is the process of moving a geometric figure from one place to another in a plane. The image is the figure which is obtained after transformations. Reflection, rotation and translation are all types of transformation. We first discuss reflection and then translation.

Do you know?

Transformation is the mapping of a geometric figure.

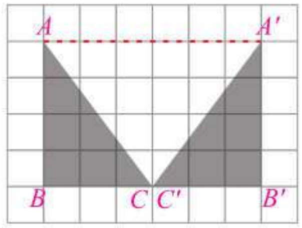
i Reflection

The mirror image produced by flipping a figure over a line is called reflection. The distance of any point on the object is equal to the distance of the corresponding point on the image.

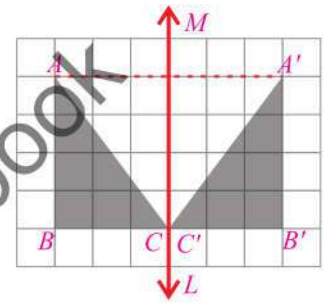


We see that A and A' are 3 units from line of reflection ℓ and B and B' are 2 units from the line of reflection ℓ. $\overline{A'B'}$ is the image of $\overline{AB}$ in the line of reflection ℓ.

Example 3 Draw line of reflection in the following figure.



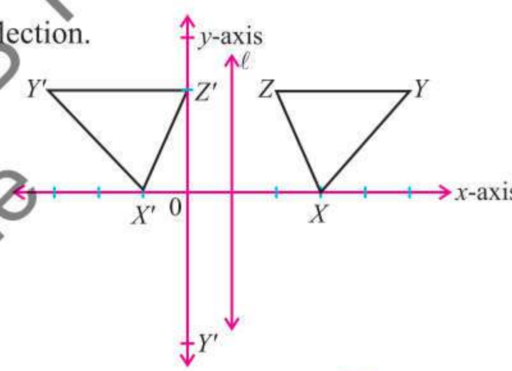
Solution



ΔABC has an image $\Delta A'B'C'$. We see that $m\overline{BC} = m\overline{B'C'}$. So, points C or C' is the midpoint of $\overline{BB'}$. Similarly midpoint of AA' is M. Draw $\overleftrightarrow{ML}$ which is the required line of reflection.

If a point lies on the line of reflection, the image of that point is same as the point. i.e., C and C' are same points.

Example 4 Complete the figure using the given line of reflection.



Solution

Image of X will be X' both 2 units away from line of reflection ℓ. Similarly Z , Z' are 1 unit away ℓ and Y , Y' are 4 units away from ℓ.

ii Translation

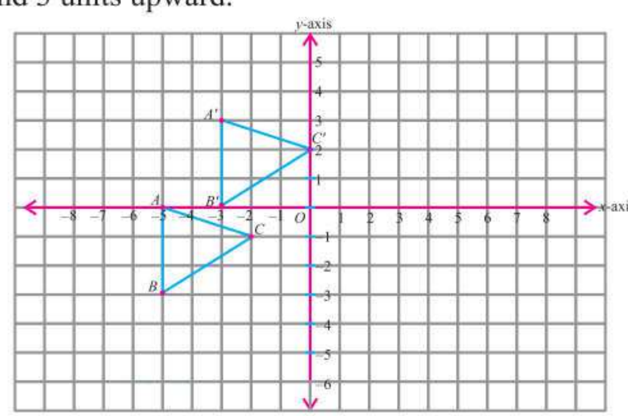
It is the movement of a figure from one position to another without turning it. In translation each point of the object is moved along straight line.

Need to Know!

- Translation is sometimes called slide.
- Translation is a type of transformation.

Example 5 Move the triangle ABC, 2 units right and 3 units upward.

Solution



The triangle ABC has coordinates $A(-5, 0)$, $B(-5, -3)$ and $C(-2, -1)$. Now move the point A, 2 units right and 3 units upward. We get $A'(-3, 3)$. Similarly the images of B and C are $B'(-3, 0)$ and $C'(0, 2)$.

So, $\Delta A'B'C'$ is the image of ΔABC under a translation 2 units right and 3 units upward.

We denote the translation by $T = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$, point by P and image by I then:

$P + T = I$ [i.e., Point + Translation = Image] i.e., for point A, $\begin{pmatrix}-5\\0\end{pmatrix}+\begin{pmatrix}2\\3\end{pmatrix}=\begin{pmatrix}-3\\3\end{pmatrix}$

For point B, $\begin{pmatrix}-5\\-3\end{pmatrix}+\begin{pmatrix}2\\3\end{pmatrix}=\begin{pmatrix}-3\\0\end{pmatrix}$

For point C, $\begin{pmatrix}-2\\-1\end{pmatrix}+\begin{pmatrix}2\\3\end{pmatrix}=\begin{pmatrix}0\\2\end{pmatrix}$

Remember!

- 2 units upward is written as +2
- 2 units down is written as –2
- 2 units right is written as +2
- 2 units left is written as –2

Example 6 Find translation when the point $P(-1, 1)$ is mapped on to $P'(3, -2)$

Solution We know that $P + T = P'$

$$T = P' - P$$
$$= \begin{pmatrix}3\\-2\end{pmatrix}-\begin{pmatrix}-1\\1\end{pmatrix}$$
$$= \begin{pmatrix}3+1\\-2-1\end{pmatrix}=\begin{pmatrix}4\\-3\end{pmatrix}$$

So, translation is 4 units right and 3 units downward.

Skill Practice

The image of a point P is $P'(-6, 8)$ under a translation $T=\begin{pmatrix}4\\-4\end{pmatrix}$. Find the point P.

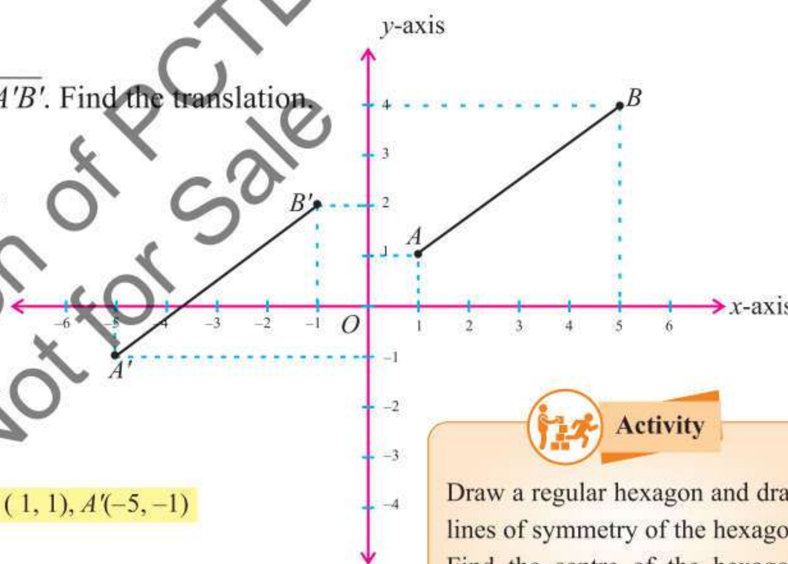
Example 8

Given a line segment AB and its image $\overline{A'B'}$. Find the translation.

Solution

By looking at points A and A' (also B and B'), we see that point A' is 6 units left and 2 units down from point A.

So, translation is $\begin{pmatrix}-6\\-2\end{pmatrix}$. We can also find translation T as:



T = Image – Point $A = (1, 1), A'(-5, -1)$

$$= \begin{pmatrix}-5\\-1\end{pmatrix} - \begin{pmatrix}1\\1\end{pmatrix}$$
$$= \begin{pmatrix}-5-1\\-1-1\end{pmatrix} = \begin{pmatrix}-6\\-2\end{pmatrix}$$

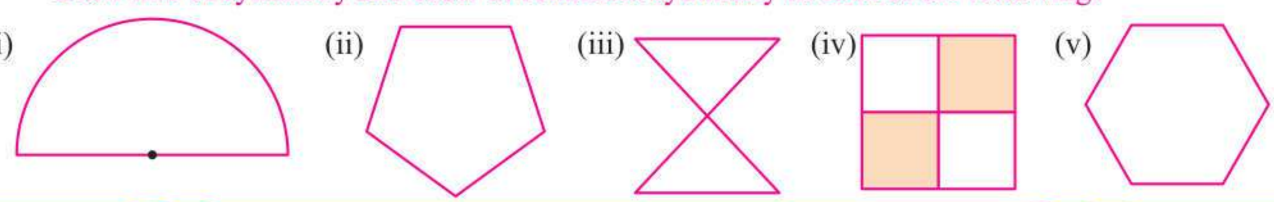
Activity

Draw a regular hexagon and draw lines of symmetry of the hexagon. Find the centre of the hexagon. What is the angle of rotation?

EXERCISE 4.5

1. Draw line of symmetry and order of rotational symmetry in each of the following:

(i) (ii) (iii) (iv) (v)

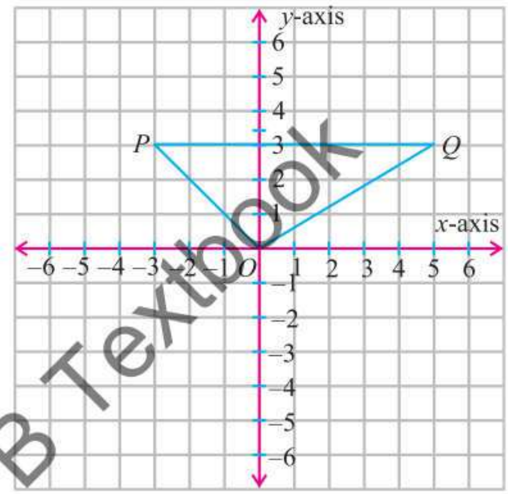


2. Find the point of rotation of regular hexagon and decagon. Also find the angle of rotation.

3. Find the translation if the point $B(-3, 2)$ is mapped on to the point $B'(2, 6)$.

4. A triangle ABC with vertices $A(-3, 0)$, $B(0, -3)$ and $C(3, 1)$ is mapped on to the triangle DEF with a translation of $\begin{pmatrix} 3 \\ 2 \end{pmatrix}$.

5. Translate the ΔOPQ with a translation of $\begin{pmatrix} 0 \\ -3 \end{pmatrix}$



SUMMARY

- In reflective symmetry, half of the shape is the mirror image of the other half.
- The line which divides the shape into two equal halves is called line of symmetry.
- The point and image are at equal distances from the line of symmetry.
- When a shape is rotated about its centre through an angle and the shape attains its original position after rotating, this type of symmetry is known as rotational symmetry.
- Order of rotational symmetry is the number of times, the shape remains same between 0° and 360°.
- The centre of rotation in regular polygons is the point of intersection of its lines of symmetry.
- Transformation is the process of moving a geometric figure from one position to another. Reflection, rotation and translation are types of transformation.

REVIEW EXERCISE 4

1. Choose the correct option.

   (i) A right angled triangle can not be __________ triangle.

   (a) isosceles (b) equilateral

   (c) scalene (d) both isosceles and scalene

(ii) In right angled triangle, one angle is 90° and the other two angles are __________ .

(a) complementary (b) supplementary (d) obtuse (d) reflex

(iii) In an isosceles triangle, if base angles are equal to 42° each, then vertical angle is________.

(a) 42° (b) 76° (c) 84° (d) 96°

(iv) Which of the following angles so formed with transversal and parallel lines are supplementary?

(a) alternate angles (b) interior angles

(c) vertically opposite angles (d) corresponding angles

(v) If two angles of an isosceles triangle are 40° and 100° the third angle is __________.

(a) 40° (b) 80° (c) 100° (d) 140°

(vi) The diagonals in the quadrilateral __________ do not bisect each other.

(a) square (b) rectangle (c) kite (d) rhombus

(vii) In which quadrilateral, there are no parallel lines?

(a) square (b) rectangle (c) kite (d) rhombus

(viii) In which quadrilateral, the opposite angles are not equal?

(a) square (b) rectangle (c) rhombus (d) trapezirm

(ix) A polygon is said to be __________ if atleast one angle is reflex.

(a) regular (b) concave (d) convex (d) closed

(x) The exterior angle of regular octagon is __________.

(a) 35° (b) 45° (c) 90° (d) 135°

(xi) If a figure is divided into the two equal parts, it is known as __________.

(a) reflection (b) rotation (c) translation (d) image

(xii) The order of rotational symmetry of hexagon is __________.

(a) 2 (b) 4 (c) 16 (d) 8

(xiii) Which of the following quadrilateral has no line of symmetry?

(a) rectangle (b) rhombus (c) kite (d) square

(xiv) Which of the following quadrilateral has no rotational symmetry?

(a) rectangle (b) rhombus (c) kite (d) square

(xv) The movement of an object from one position to another along straight line is called __________.

(a) translation (b) rotation (c) reflection (d) measurement

2. Using compass and ruler construct the following angles:

(i) 15° (ii) $22\frac{1}{2}^\circ$ (iii) $37\frac{1}{2}^\circ$ (iv) 135° (v) 165°

3. Construct a right angled triangle with side lengths 4.5 cm, 6 cm, 75 mm.

4. Construct an isosceles triangle with base length 5.4 cm and base angles each of measure 75°.

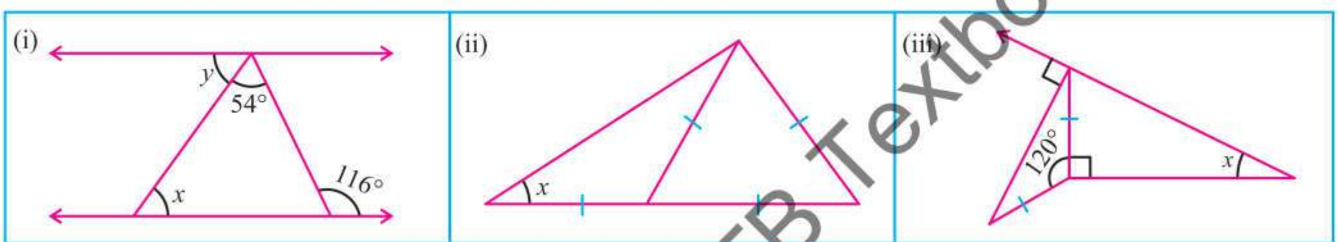
5. Construct triangle PQR if:

(i) $m\overline{PQ} = 7$ cm, $m\overline{PR} = 6.2$ cm and $m\angle QPR = 60°$

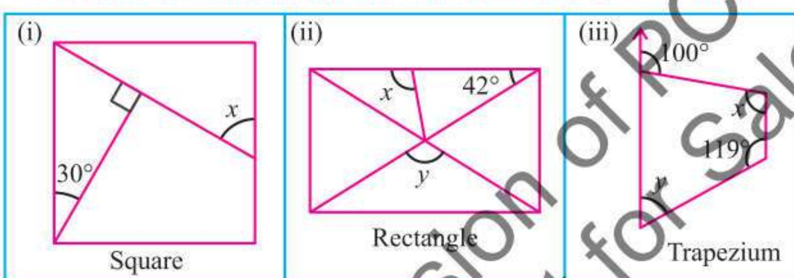
(ii) $m\overline{QR} = 6$ cm, $m\angle Q = 50°$ and $m\angle R = 120°$

(iii) $m\overline{PR} = 5.5$ cm, $m\angle R = 45°$ and $m\overline{PQ} = 6.7$ cm

6. Find the unknown angles in the following figures:



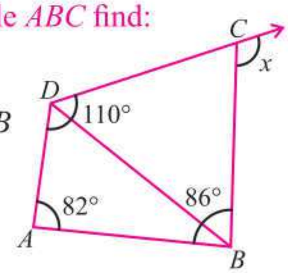
7. Find the unknown angles in the following quadrilaterals:



8. $ABCD$ is a quadrilateral and BD bisects the angle ABC find:

(a) x

(b) $m\angle ADB$

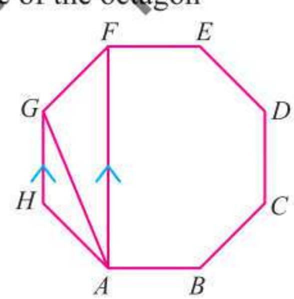


9. $ABCDEFGH$ is a regular octagon. Find

(i) Interior angle of the octagon

(ii) $m\angle HAG$

(iii) $m\angle AFG$

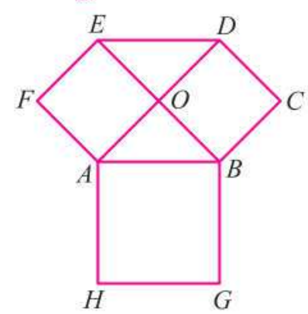


10. $ABCDEF$ is a regular hexagon and $ABGH$ is a square. Find:

(i) $m\angle ABC$

(ii) $m\angle CBG$

(iii) $m\angle AOB$



11. Find interior angle of regular

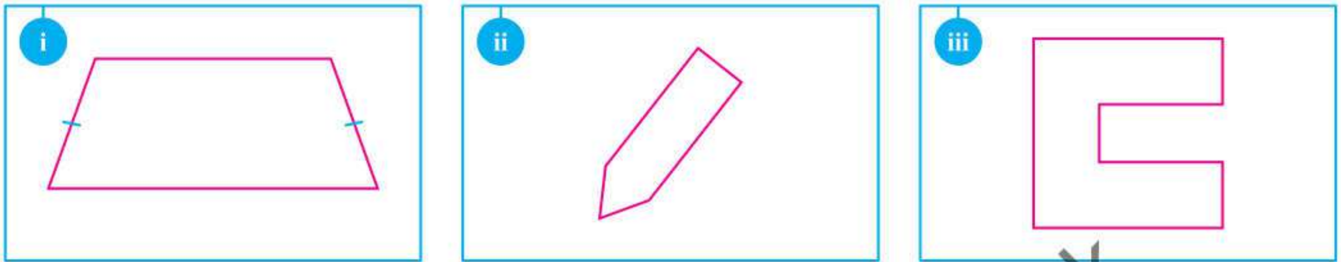
(i) 24-gon (ii) 27-gon

12. Find number of sides of a regular polygon with given interior angle:

(i) 162° (ii) 170°

13. The size of each interior angle is 14 times the exterior angle of a regular polygon with n-sides. Find the value of n.

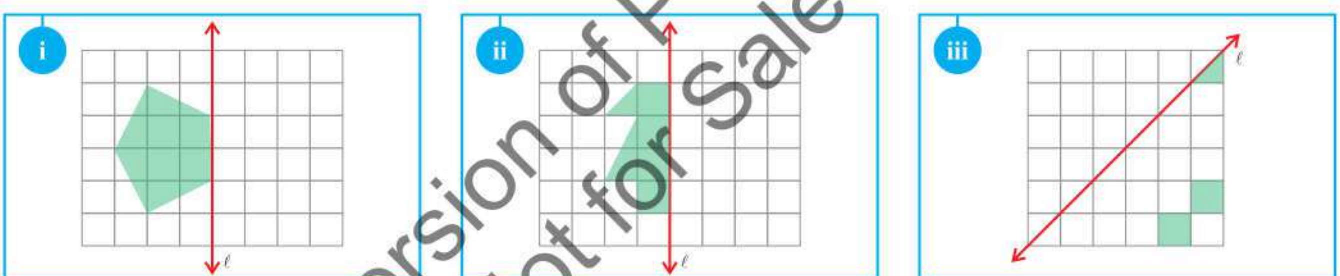
14. Draw one line of symmetry in each of the following:



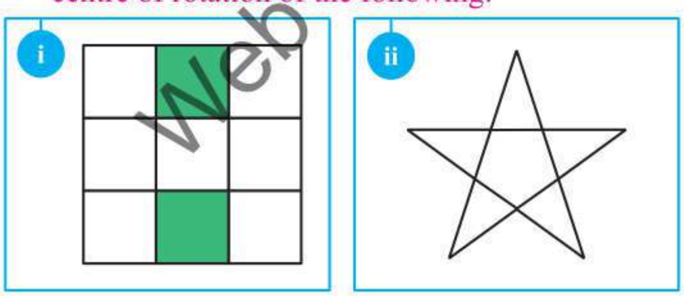
15. Which of the following letters have:

(a) line symmetry (b) rotational symmetry

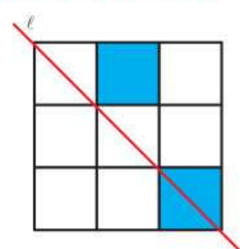
16. Complete the following diagrams using the given line of reflection:



17. Find the order of rotational symmetry and the centre of rotation of the following:

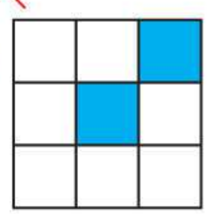


18. Shade one square so that the diagram has the given line of symmetry ℓ.



19. (i) How many squares should be shaded in the given diagram to get a rotational symmetry of order 4? Draw the figure.

(ii) Shade one square so that the diagram has two lines of symmetry



20. Find a translation when a point $A(1, -2)$ is moved to the point $A'(-3, 5)$.

21. Translate ΔABC with vertices $A(-2, 0)$, $B(-5, 2)$ and $C(-3, -6)$, 7 units right and 2 units upward.