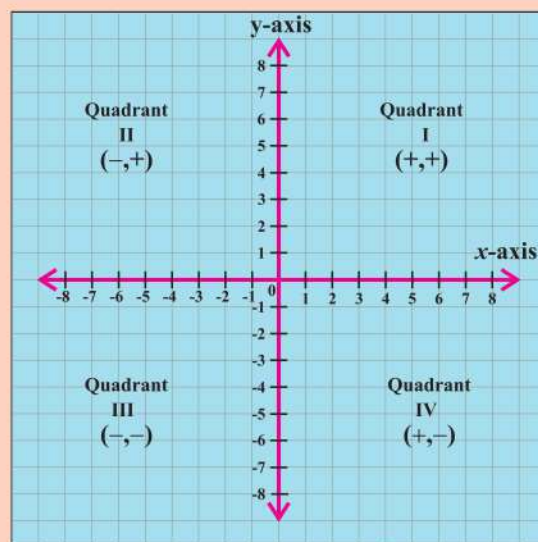
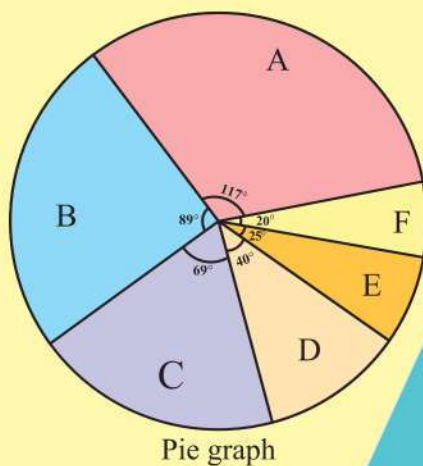
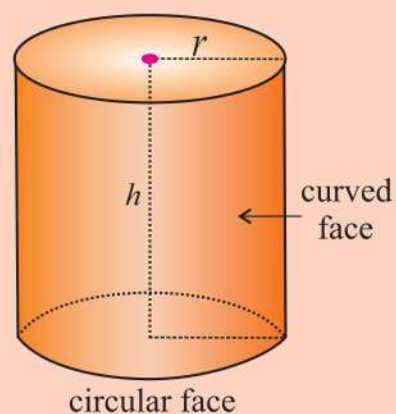
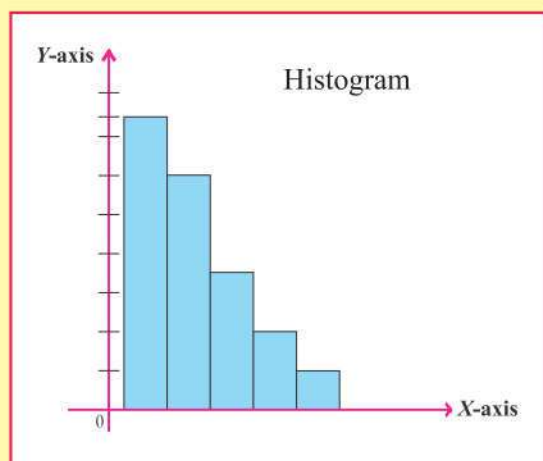


MATHEMATICS

Based on Single National Curriculum 2022

7



Punjab Curriculum and Textbook Board, Lahore

بِسْمِ اللَّهِ الرَّحْمَنِ الرَّحِيمِ

(In the Name of Allah, the Most Compassionate, the Most Merciful)

MATHEMATICS

7

Based on Single National Curriculum 2022

ONE NATION, ONE CURRICULUM



**PUNJAB CURRICULUM AND
TEXTBOOK BOARD, LAHORE**

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Experimental Edition

Domain

1

NUMBERS AND OPERATIONS

Sub-domain

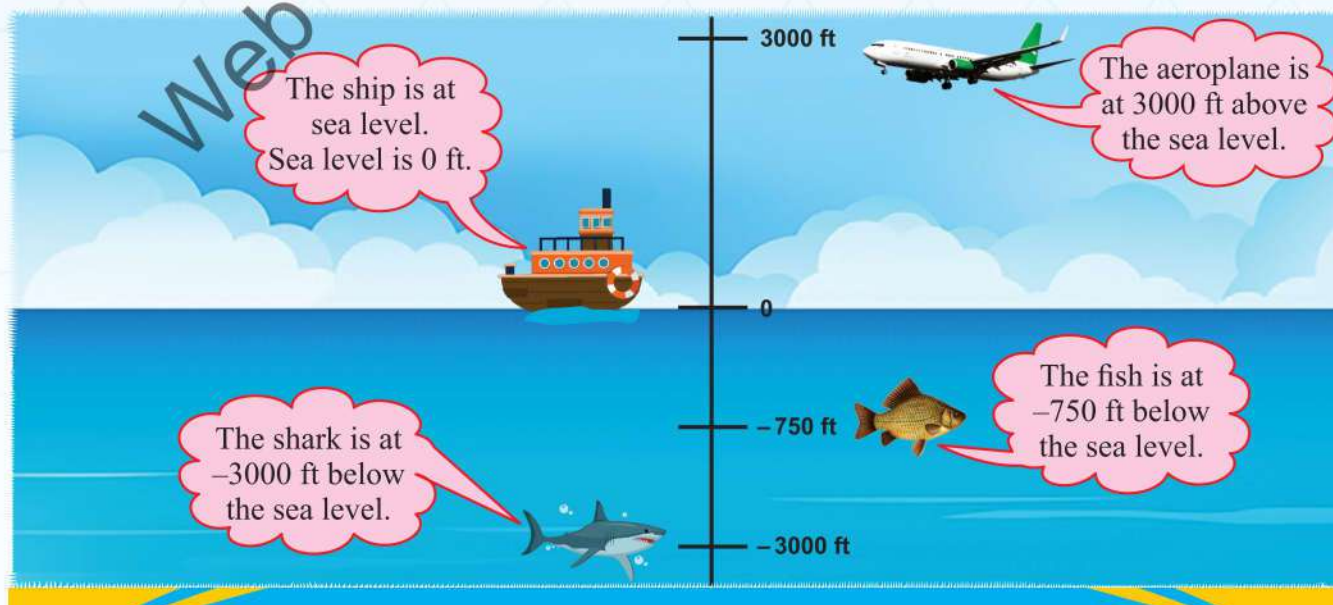
(i)

Rational Numbers and Decimal Numbers

Students' Learning Outcomes

After studying this sub-domain, students will be able to:

- Recall - Recognise, identify and represent integers (positive, negative and neutral integers) and their absolute or numerical value.
- Represent whole numbers, integers and decimal numbers on a number line.
- With increasing degree of challenge, use the concept of place value for whole numbers, integers, rational numbers and decimal numbers.
- Compare (using symbols $<$, $>$, $=$, $\leq$ and $\geq$) and arrange (in ascending or descending order) whole numbers, integers, rational numbers and decimal numbers.
- Identify and convert between various types of fractions.
- Recall H.C.F and L.C.M.
- Identify and represent (on a number line) rational numbers.
- Solve real-world word problems involving operations on rational numbers.
- Verify commutative, associative and distributive properties of rational numbers
- Round whole numbers, integers, rational numbers and decimal numbers to a required degree of accuracy, significance or decimal places (up to 3 decimal places).
- Use knowledge of rounding to give an estimate to a calculation; to check the reasonableness of the solution.

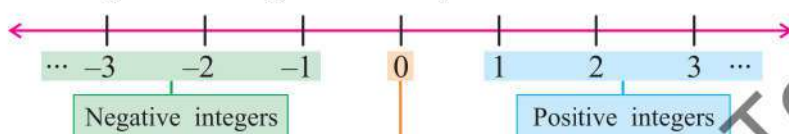


RECALL

You have already learnt about:

- Natural numbers (1, 2, 3, 4, ...) are also called positive integers and $-1, -2, -3, \dots$ are called negative integers. The positive integers and negative integers along with zero (0) are called integers. i.e., The set of integers $Z = \{0, \pm 1, \pm 2, \pm 3, \dots\}$.
- Natural numbers along with zero (0) are called whole numbers i.e., The set of whole numbers $W = \{0, 1, 2, 3, 4, \dots\}$.

When we arrange these integers on a line, then the line is called number line.



- 0 is neither positive nor negative.
- 0 is also known as neutral number/integer.



Keep in mind!

The distance from 0 to -3 and 0 to 3 is same. So, -3 and 3 have the same numerical or absolute value. The absolute value of -3 is denoted by $|-3|$. So, $|-5| = 5$; $-|-6| = -6$



History

In 1563, Arbermouth Holst introduced integers. He was a German Mathematician.



Think!

Give any five real life examples in which you use negative integers.



Skill Practice

Solve the following:

$$|-10|, -|-17|, |19| \text{ and } |-6|$$

Also arrange them in ascending and descending order.



Search out

https://www.math-play.com/Millionaire-Game-Absolute-Value/Millionaire-Game-Absolute-Value_html5.html



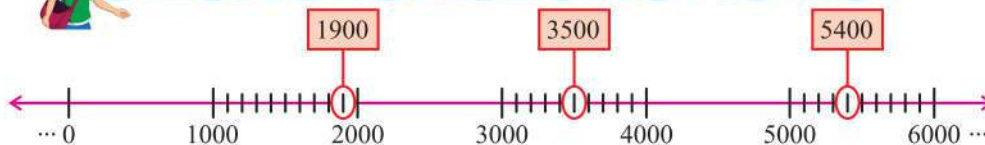
Activity

Display different temperature cards in front of the students, they will be asked to arrange them in order from the smallest to the greatest after taking their absolute values.

1.1.1 Representation of Whole Numbers on Number Line



Let us represent 1900, 3500 and 5400 on the number line.



Teachers' Guide

Recall and clear the difference among the natural numbers, whole numbers, integers and the concept of absolute value by using real life situations, in classroom.



Skill Practice

Represent the following whole numbers on the number line.

- (i) 880, 730, 540, 990, 670

1.1.2 Place Value of Whole Numbers

Look at the following place value chart:

Third period			Second period			First period		
Millions			Thousands			Ones		
Hundred million	Ten million	Million	Hundred thousands	Ten thousands	Thousands	Hundreds	Tens	Ones
			9	8	7	4	3	6
			9	2	3	5	3	0
			9	8	7	2	1	9

Let us consider 987436.

- ▶ The digit 9 is at “hundred thousands place”. Its value is 900000.
- ▶ The digit 8 is at “ten thousands place”. Its value is 80000.
- ▶ The digit 7 is at “thousands place”. Its value is 7000.
- ▶ The digit 4 is at “hundreds place”. Its value is 400.
- ▶ The digit 3 is at “tens place”. Its value is 30.
- ▶ The digit 6 is at “ones place”. Its value is 6.



Challenge

Find the 7-digit number with the help of the given clues: 2 is at ones place. The place value of 1 is 100. 7 is not at the tens place. 4 is at thousand place. The digit at the hundred thousands place is sum of digits at thousands and ones place. The digit at ten thousands is one more than the digit at thousand place. 7 digit are 1, 2, 3, 4, 5, 6 and 7 to make a 7-digit number.

1.1.3 Comparing and Ordering Whole Numbers

Let us consider the numbers 987219, 987436 and 923530 for comparing.

- In 923530, the digit at ten thousands place is smaller than the remaining two numbers. Therefore, 923530 is the smallest number.
- In 987436 and 987219, the digit at the hundreds place “4” is greater than “2”. Therefore, 987436 is greater than 987219.

$$987436 > 987219$$

$$\text{So, } 987436 > 987219 > 923530$$

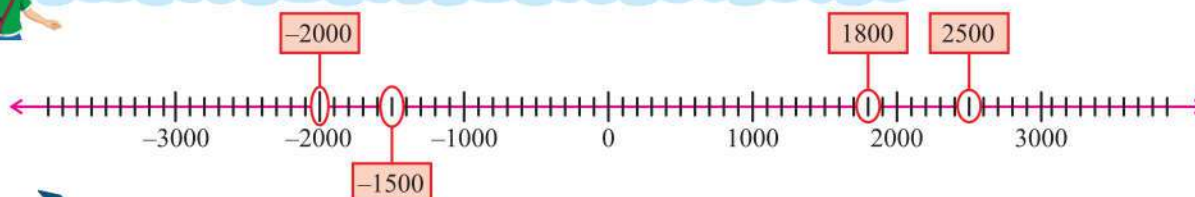
Ascending order: 923530, 987219, 987436

Descending order: 987436, 987219, 923530

1.1.4 Representation of Integers on the Number Line



Let us represent 2500, -1500, 1800, -2000 on a number line.



Teachers' Guide

Draw a number line on the writing board and ask the students to represent some whole numbers on the number line.

1.1.5 Comparing and ordering of Integers

Let us compare the given integers and write in ascending and descending order.

$-436, -538, 483, 836$

Integer that has negative sign, it will be smaller.

So, $836 > 483 > -436 > -538$

Ascending order: $-538, -436, 483, 836$

Descending order: $836, 483, -436, -538$



Skill Practice

Represent the following on the number line:

(i) $-700, 100, 900, -800, -500$

(ii) $2200, -1800, -900, 1500, 2500$



Remember!

When we write the negative sign ($-$) with a whole number, then it becomes negative integer except zero (0)



Search out

https://www.mathplayground.com/mobile/numberballs_fullscreen.htm



Skill Practice

Represent the following integers on number line:

$-50, 60, -30, -20, 10, 40$



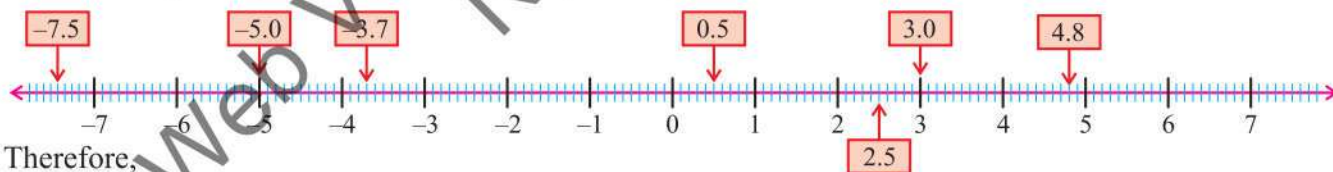
Activity

- Teacher will provide a handout with some blank spaces for 2-digit or 3-digit numbers with positive and negative signs and an empty box in between number blanks for comparison of integers.
- Player 1 will roll dice and fill one blank of question and then player 2 will roll dice and fill the other blank then according to positive and negative signs, players will determine which sign goes between numbers ($<$, $>$ or $=$).
- The player with greater number gets 1 point.

1.1.6 Representation of Decimal Numbers on the Number Line

Let us represent the given decimal numbers on the number line.

$-7.5, 2.5, 0.5, -3.7, 3.0, -5.0, 4.8$



Therefore,

Ascending order: $-7.5, -5.0, -3.7, 0.5, 2.5, 3.0, 4.7$, Descending order: $4.7, 3.0, 2.5, 0.5, -3.7, -5.0, -7.5$

1.1.7 Place Value of Decimal Numbers

Look at the following place value chart for decimal numbers:

Hundreds	Tens	Ones	Decimal point	Tenths	Hundredths	Thousandths
2	3	5	.	8	9	
2	3	5	.	7	0	
2	3	5	.	7	4	

1.1.8 Comparing and ordering of Decimal Numbers

Let us compare the given decimal numbers.

- (i) Compare the digits at the greatest place.
As, the digits at hundreds, tens and ones are same.
Now we move at tenths place.
- (ii) At tenths place digit 8 is greater than 7.
- (iii) So, 235.89 is greater than 235.70 and 235.74
- (iv) At hundredths place, the digit 4 is greater than 0.
So, 235.74 is greater than 235.70
By using symbols, $235.89 > 235.74 > 235.70$



Skill Practice

Write the following integers on your notebook and arrange them in ascending and descending order.

–890.550, –230.430, –410.890



Think!

On which day the price of one litre petrol was maximum and minimum?

Let us arrange these decimal numbers in ascending and descending order.

Ascending order: 235.70, 235.74, 235.89, Descending order: 235.89, 235.74, 235.70

EXERCISE 1.1

1. Represent the following on the number line:
 - (i) 230, 130, 150, 100, 170, 120
 - (ii) 70, 80, 95, 35, 40, 45, 60
 - (iii) 1600, 1000, 1700, 1500, 900, 2000
 - (iv) –50, 60, –25, –70, –10, –35, 45
 - (v) –225, 440, –400, 100, –150, 120, 110
 - (vi) –1100, 2100, –1500, –2500, 1600
 - (vii) –15.5, 20.7, 11.2, –10.5, –12.5
 - (viii) 0.5, –0.7, –0.1, 0.3, 0.9, –0.5
2. Compare by using symbol $>$, $<$. Also arrange them in ascending and descending order.
 - (i) 235691, 235090, 245091, 245192
 - (ii) 578901, 579803, 679807, 679817
 - (iii) 10028, 100026, 110028, 110027
 - (iv) 562389, 562399, 572390, 572381
3. Compare by using symbol $>$, $<$. Also arrange them in ascending and descending order.
 - (i) –8395, –8496, –8491, –8394
 - (ii) 503, 530, 556, –563
 - (iii) –137, –138, –1886, 1308
 - (iv) –87650, –78432, –78402, 78401
4. Compare by using symbol $>$, $<$. Also arrange them in ascending and descending order.
 - (i) 28.356, 28.343, 28.357, 28.532
 - (ii) 120.08, 130.08, 120.80, 120.01
 - (iii) 131.01, 131.08, 103.78, 113.08
 - (iv) 236.089, 236.219, 236.217, 236.207

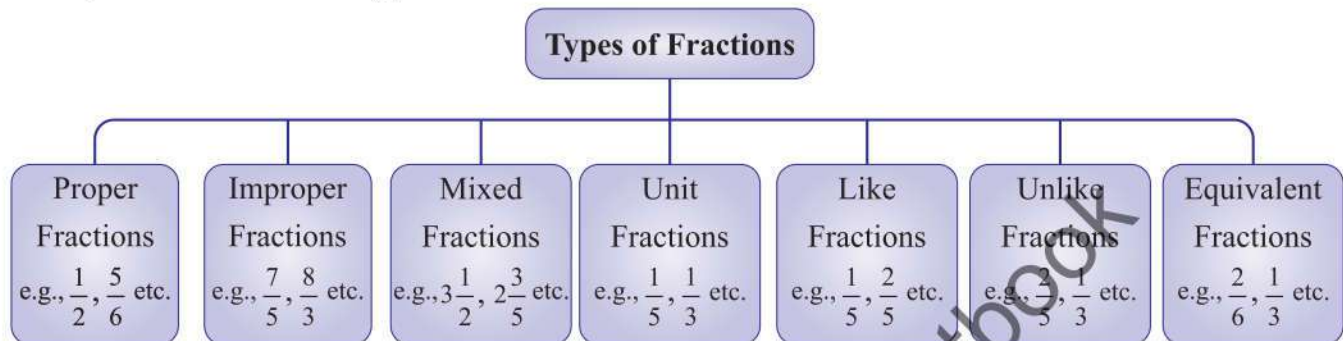


Teachers' Guide

Ask the students to come in front of writing board, write four to five decimal numbers on the writing board then ask them to compare these numbers and arrange them in ascending and descending order.

1.1.9 Identification and Conversion of Various Types of Fractions

Hence, there are total seven types of fractions such as:



The concept of fraction is illustrated as:

A whole pizza is divided into six equal parts. Here one piece of pizza is represented as $\frac{1}{6}$, where 1 is the numerator and 6 is the denominator.



Remember!

The first four fractions are defined for a single fraction but other three fractions are used to compare two or more fractions.



Important Information

Improper fractions are sometimes called the top heavy numbers.



Remember!

An improper fraction is always ≥ 1 .

To solve the unlike fractions and equivalent fractions, we will have to use LCM and HCF of the denominators.



Important Information

Mixed fractions can be converted into improper fractions and vice versa.



Important Information

Mixed number is basically an improper fraction > 1 .

RECALL

Least Common Multiples (LCM) and Highest Common Factors (HCF)

We have already learnt in previous grades to find out the LCM of two or more than two numbers.

LCM

2	72	48	24
2	36	24	12
2	18	12	6
2	9	6	3
3	9	3	3
3	3	1	1
	1	1	1

$$\text{LCM} = 2 \times 2 \times 2 \times 2 \times 3 \times 3 = 144$$

HCF

24	36	12	12
	24		12
	12	24	0
		24	
		0	

$$\text{HCF} = 12$$



Keep in mind!

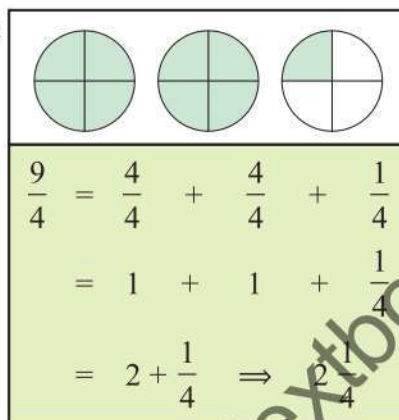
- Highest common factor is also known as greatest common divisor (GCD).
- The smallest common multiple of two or more than two numbers is called least common multiple (LCM).

i Conversion of Improper Fractions into Mixed Numbers

Example 1 Convert $\frac{9}{4}$ into mixed number.

Solution To convert $\frac{9}{4}$ into mixed number, we will divide 9 by 4.

$$\begin{array}{r} 2 \\ 4 \overline{) 9} \\ \underline{8} \\ 1 \end{array}$$

$$\frac{9}{4} = 2 + \frac{1}{4} = 2\frac{1}{4}$$


Keep in mind!

In $2\frac{1}{4}$

Whole part $\left(2\right)$ Fractional part $\left(\frac{1}{4}\right)$

ii Conversion of Mixed Number into Improper Fraction

We follow the following steps to convert a mixed number into an improper fraction

- Rule 1** Write the mixed number e.g., $2\frac{3}{5}$
- Rule 2** Multiply the whole number by the denominator i.e., $2 \times 5 = 10$
- Rule 3** Add the product to the numerator i.e., add $10 + 3 = 13$

So, the improper fraction is $\frac{13}{5}$.

In this case, we multiply 2 by 5



Activity

Students will work in groups of 4. Teacher provide cards of improper fractions and cards of mixed fraction equivalent fraction cards. Prepare at least 8 extra cards of improper fraction and mixed fraction also that do not match. Distribute the sets of cards to each group. Students will match the mixed fraction and improper fraction that are equal. The group who will match all the cards correctly will win.

1.1.10 Rational Numbers

We have learnt about different types of numbers such as natural numbers, whole numbers, integers and common fractions. All these numbers are called rational numbers.



A number that can be expressed in the form of $\frac{p}{q}$ where p, q are integers and $q \neq 0$ is called a rational number.

The relationship among different types of numbers is illustrated by the figure given below:

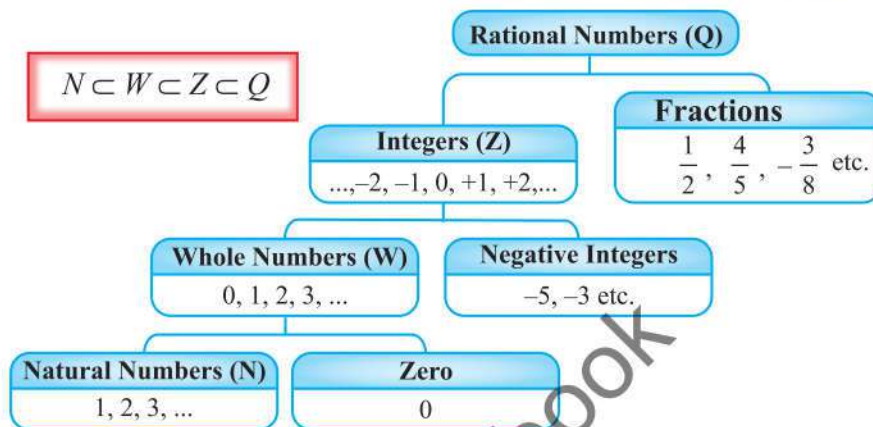


Important Information

There are many situations involve in fractional numbers. To include such numbers we need to extend our number system by introducing rational numbers.

**Important Information**

The set of rational number is denoted by Q .

**Important Information**

In $\frac{p}{q}$, the integer p is the numerator and the integer $q \neq 0$ is the denominator.

1.1.11 Representation of Rational Numbers on the Number Line**Example 2**

Represent $\frac{8}{3}$ and $-\frac{8}{3}$ on number line.

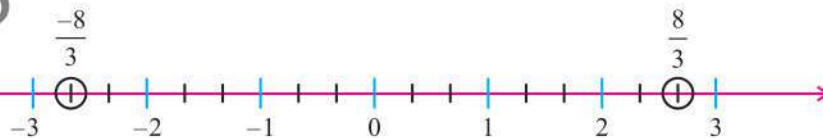
Solution

Since $\frac{8}{3}$ and $-\frac{8}{3}$ are improper fractions and $\frac{8}{3}$ is positive, so it will lie on right side of 0 on the number line and $-\frac{8}{3}$ is negative so, it will lie on left side of 0 on the number line. Let us first convert $\frac{8}{3}$ and $-\frac{8}{3}$ into mixed number.

$$-\frac{8}{3} = -2\frac{2}{3}$$

$$\frac{8}{3} = 2\frac{2}{3}$$

So, $\frac{8}{3}$ lies between 2 and 3. We will divide the line between 2 to 3 and -2 to -3 into 3 equal parts.



The rational numbers $-\frac{8}{3}$ and $\frac{8}{3}$ are represented on the number line as shown in the figure.

1.1.12 Comparison of Two Rational Numbers

Sadia used $\frac{5}{8}$ metre ribbon to decorate the gift box.

Tahir used $\frac{13}{16}$ metre ribbon to decorate the gift box.

Who did use more ribbon to decorate the gift box?

**Remember!**

- Every positive rational number is greater than zero.
- Every positive rational number is greater than every negative rational number.



Keep in mind!

- If the denominators of two rational numbers are same, then compare the numerators, the rational number with greater numerator is greater than other.
- If the denominators of two rational numbers are different, then convert them into like fractions by taking LCM of the denominators.

To solve this problem, we will compare both the rational numbers.

As the denominators of both the rational numbers are different, so we convert them into like fractions.

Multiply all the fractions by a number so that their denominators

become equal to their LCM

$$\frac{5}{8} = \frac{5 \times 2}{8 \times 2} = \frac{10}{16}$$

$$\frac{13}{16} = \frac{13 \times 1}{16 \times 1} = \frac{13}{16}$$

Now, compare numerators

As, $13 > 10$ So, $\frac{13}{16} > \frac{10}{16}$ i.e., $\frac{13}{16} > \frac{5}{8}$

Hence, Tahir used more ribbon than Sadia.



Working

Find LCM of 8 and 16

2	8 , 16
2	4 , 8
2	2 , 4
2	1 , 2
	1 , 1

$$\text{LCM} = 2 \times 2 \times 2 \times 2 = 16$$

1.1.13 Arrange the Rational Numbers in Ascending or Descending Order

To arrange the rational numbers in order, we will have the following steps:

Step 1

Write the given rational numbers with positive denominator.

Step 2

Take LCM of positive denominators.

Step 3

Write each rational number with LCM as the common denominators.

Example 3

Arrange the following rational numbers in ascending and descending order.

$$\frac{4}{5}, \frac{7}{30}, \frac{7}{15}, \frac{3}{10}$$

Solution

$$\frac{4}{5}, \frac{7}{30}, \frac{7}{15}, \frac{3}{10}$$

Multiply all the rational numbers by a number so that their denominators become equal to their LCM

$$\text{Now, } \frac{4}{5} = \frac{4 \times 6}{5 \times 6} = \frac{24}{30}$$

$$\frac{7}{30} = \frac{7}{30}$$

$$\frac{7}{15} = \frac{7 \times 2}{15 \times 2} = \frac{14}{30}$$

2	5	30	15	10
3	5	15	15	5
5	5	5	5	5
	1	1	1	1

$$\text{LCM} = 2 \times 3 \times 5 = 30$$

$$\frac{3}{10} = \frac{3 \times 3}{10 \times 3} = \frac{9}{30}$$

Now, since $24 > 14 > 9 > 7$

$$\text{So, } \frac{24}{30} > \frac{14}{30} > \frac{9}{30} > \frac{7}{30} \Rightarrow \frac{4}{5} > \frac{7}{15} > \frac{3}{10} > \frac{7}{30}$$

Ascending order: $\frac{7}{30}$, $\frac{3}{10}$, $\frac{7}{15}$, $\frac{4}{5}$

Descending order: $\frac{4}{5}$, $\frac{7}{15}$, $\frac{3}{10}$, $\frac{7}{30}$



Activity

Students will work in pairs. Provide each pair with blank paper chits (4 or 5). All pairs of students will write some rational numbers with different denominators. Each pair will now make all the denominators equal to their LCM. Now, each pair will arrange these rational numbers in the ascending or descending order.

EXERCISE 1.2

1. Represent the following rational numbers on number line:

(i) $\frac{1}{3}$

(ii) $\frac{3}{7}$

(iii) $\frac{-3}{5}$

(iv) $\frac{5}{3}$

(v) $\frac{5}{8}$

(vi) $\frac{3}{10}$

(vii) $-2\frac{1}{2}$

(viii) $-2\frac{3}{4}$

2. Convert:

(i) $1\frac{2}{3}$, $3\frac{4}{5}$, $2\frac{5}{11}$, $8\frac{3}{7}$ into improper fractions

(ii) $\frac{18}{7}$, $\frac{63}{14}$, $\frac{43}{13}$, $\frac{115}{9}$ into mixed fractions

3. Put the correct sign ($>$, $<$ or $=$) between the following rational numbers:

(i) $\frac{3}{5}$ $\frac{6}{25}$

(ii) $\frac{7}{10}$ $\frac{-7}{10}$

(iii) $\frac{-3}{8}$ $\frac{-5}{8}$

(iv) $1\frac{1}{7}$ $2\frac{3}{14}$

(v) $1\frac{5}{12}$ $\frac{7}{12}$

(vi) $\frac{-19}{25}$ $\frac{-7}{25}$

4. Compare the following rational numbers:

(i) $\frac{2}{7}$, $\frac{2}{5}$

(ii) $\frac{5}{9}$, $\frac{-3}{5}$

(iii) $\frac{1}{12}$, $\frac{-1}{12}$

(iv) $\frac{-2}{7}$, $\frac{-3}{11}$

(v) $\frac{-5}{11}$, $\frac{-7}{9}$

(vi) $\frac{3}{22}$, $\frac{-5}{22}$

(vii) $3\frac{5}{9}$, $4\frac{5}{18}$

(viii) $-1\frac{7}{25}$, $\frac{-3}{5}$

(ix) $3\frac{1}{2}$, $2\frac{1}{14}$

5. Arrange the following rational numbers in ascending order:

(i) $\frac{1}{4}$, $\frac{3}{5}$, $\frac{7}{8}$, $\frac{5}{4}$

(ii) $\frac{6}{7}$, $\frac{3}{14}$, $\frac{-9}{7}$, $\frac{13}{-14}$

(iii) $\frac{-7}{12}$, $\frac{5}{-4}$, $\frac{-2}{3}$, $\frac{5}{8}$

(iv) $\frac{7}{8}$, $\frac{-5}{4}$, $\frac{-7}{5}$, $\frac{13}{8}$

6. Arrange the following rational numbers in descending order:

(i) $\frac{2}{3}$, $\frac{4}{7}$, $\frac{-4}{-7}$, $\frac{-3}{4}$

(ii) $\frac{4}{5}$, $\frac{-13}{-30}$, $\frac{7}{-10}$, $\frac{7}{-15}$

(iii) $\frac{5}{8}$, $\frac{13}{24}$, $\frac{17}{-12}$, $\frac{9}{48}$

(iv) $\frac{-5}{11}$, $\frac{-9}{22}$, $\frac{-4}{33}$, $\frac{-41}{44}$

1.1.14 Operations on Rational Numbers

i Addition of Rational Numbers

Addition of rational numbers is similar to the addition of fractions.

Case I Rational numbers having the same denominators

To add rational numbers with the same denominators, we add their numerators and the denominator remains the same.

In other words, if $\frac{p}{q}$ and $\frac{r}{q}$ are two rational numbers, then

$$\frac{p}{q} + \frac{r}{q} = \frac{p+r}{q}$$

For example

$$\frac{5}{7} + \frac{1}{7} = \frac{5+1}{7} = \frac{6}{7}$$

$$\frac{2}{13} + \frac{4}{13} + \frac{3}{13} = \frac{2+4+3}{13} = \frac{9}{13}$$

Case II Rational numbers having different denominators

To add rational numbers with different denominators, first make the denominators same by taking the LCM as their common denominators.

Danish purchased a cake. He eats $\frac{3}{10}$ parts of the cake.

Danish's friend eats $\frac{1}{5}$ parts of the cake and his sister

eats $\frac{4}{15}$ parts of the cake. How much cake is eaten by them altogether?



Skill Practice

Solve the following:

$$\frac{1}{2} + \frac{7}{9} + \frac{1}{9} = \boxed{}$$



Challenge

If $\frac{p}{q}$ and $\frac{r}{s}$ are two rational numbers then:

$$\frac{p}{q} + \frac{r}{s} = \boxed{}$$



To solve this problem, we will have to add these rational numbers i.e., $\frac{3}{10} + \frac{1}{5} + \frac{4}{15}$

As, the rational numbers are unlike fractions. First of all, we will convert them into like fractions.

Step 1 Find LCM of all the denominators. LCM of 10, 5 and 15 is 30.

Step 2 Now, multiply all the rational numbers by a number to make their denominators equal to their LCM.

$$\frac{3}{10} = \frac{3 \times 3}{10 \times 3} = \frac{9}{30} ; \frac{1}{5} = \frac{1 \times 6}{5 \times 6} = \frac{6}{30} ; \frac{4}{15} = \frac{4 \times 2}{15 \times 2} = \frac{8}{30} \quad \text{These become like fractions.}$$

Step 3 Now, add these like fractions

$$\frac{9}{30} + \frac{6}{30} + \frac{8}{30} = \frac{9+6+8}{30} = \frac{23}{30}$$

Hence, $\frac{23}{30}$ parts of the cake is eaten by them.



Skill Practice

Can you find out how much cake is left with Danish?

ii Subtraction of Rational Numbers

Case I Rational numbers having the same denominators

To subtract rational numbers with the same denominators, we subtract the numerators, the denominators remain the same.

For example

Look at the following examples:

$$(i) \frac{5}{9} - \frac{1}{9} = \frac{5-1}{9} = \frac{4}{9}$$

$$(ii) \frac{5}{8} - \frac{3}{8} = \frac{5-3}{8} = \frac{2}{8} = \frac{2 \div 2}{8 \div 2} = \frac{1}{4}$$



If $\frac{p}{q}$ and $\frac{r}{q}$ are rational numbers then,

$$\frac{p}{q} - \frac{r}{q} = \frac{p-r}{q}$$

Case II Rational numbers having different denominators

In this case, make denominators same by taking the LCM as their common denominator.

Consider the following example:



Skill Practice

Solve the following:

$$\frac{11}{15} - \frac{9}{15} - \frac{2}{15} = \boxed{}$$



If $\frac{p}{q}$ and $\frac{r}{s}$ are two rational numbers, then

$$\frac{p}{q} - \frac{r}{s} = \frac{p \times s - r \times q}{q \times s}$$

Example 4 The sum of two rational numbers is $\frac{-3}{5}$. If one of them is $\frac{9}{10}$, find the other.

Solution Sum of the numbers = $\frac{-3}{5}$; One of the number = $\frac{9}{10}$

$$\begin{aligned}\text{The other number} &= \text{Sum of numbers} - \text{one of the number} \\ &= \frac{-3}{5} - \frac{9}{10} = \frac{-3 \times 2 - 9 \times 1}{10} = \frac{-6 - 9}{10} = \frac{-15}{10}\end{aligned}$$

Hence, the other number is $\frac{-15}{10}$.

iii Additive Inverse

An additive inverse of a rational number is the same original number with the opposite sign. For example, the additive inverse of $\frac{5}{6}$ is $\frac{-5}{6}$.



Skill Practice

Find the additive inverse: (i) $\frac{2}{3}$ (ii) $\frac{-4}{7}$

For each rational number $\frac{p}{q}$ there exist a number $\frac{-p}{q}$ such that

$$\frac{p}{q} + \left(\frac{-p}{q}\right) = \left(\frac{-p}{q}\right) + \frac{p}{q} = 0$$

Here, $\frac{-p}{q}$ is called additive inverse of $\frac{p}{q}$.

iv Multiplication of Rational Numbers

To find the product of two rational numbers $\frac{p}{q}$ and $\frac{r}{s}$.

$$\text{If } \frac{p}{q} \text{ and } \frac{r}{s} \text{ are two rational numbers, then } \frac{p}{q} \times \frac{r}{s} = \frac{p \times r}{q \times s} = \frac{pr}{qs}$$

i.e., Product of two rational numbers = $\frac{\text{Product of their numerators}}{\text{Product of their denominators}}$

Example 5 Solve: $\frac{2}{5} \times \left(\frac{-3}{4}\right) \times \left(\frac{-9}{5}\right)$

$$\begin{aligned}\text{Solution } \frac{2}{5} \times \left(\frac{-3}{4}\right) \times \left(\frac{-9}{5}\right) &= \frac{2 \times (-3) \times (-9)}{5 \times 4 \times 5} \\ &= \frac{2 \times 27}{100} = \frac{54}{100} = \frac{27}{50}\end{aligned}$$



Remember!

To multiply two rational numbers, multiply the numerator by numerator and denominator by the denominator.



Noor and Nazia planted their plants. The height of Noor's plant was $\frac{15}{4}$ cm while height of Nazia's plant was $\frac{7}{5}$ times of Noor's plant. Find the height of Nazia's plant.



Teachers' Guide

Write some real life examples on the writing board to clear how to find out the difference of two or more rational numbers having different denominators.



To find the height of Nazia's plant, we will have to multiply $\frac{7}{5}$ by $\frac{15}{4}$.

$$\text{Height of Nazia's plant} = \frac{7}{5} \times \frac{15}{4} = \frac{21}{4}$$

So, the height of Nazia's plant was $\frac{21}{4}$ cm.



Skill Practice

Ahmad's friend left $\frac{3}{8}$ of a whole pizza for him, Ahmad ate $\frac{1}{3}$ of what was left. How much of the whole pizza did Ahmad left?

v

Division of Rational Numbers

Division of rational numbers is the same as division of a fraction by another fraction.



If $\frac{p}{q}$ and $\frac{r}{s}$ are any two rational numbers such that $\frac{r}{s} \neq 0$, then:

$$\frac{p}{q} \div \frac{r}{s} = \frac{p}{q} \times \frac{s}{r} = \frac{p \times s}{q \times r} = \frac{ps}{qr}$$



Important Information

Division is the inverse process of multiplication.



Remember!

Division by 0 is undefined.

Example 6

Solve: $-4\frac{2}{5} \div \left(-\frac{6}{5}\right)$

Solution

$$\begin{aligned} -4\frac{2}{5} \div \left(-\frac{6}{5}\right) &= \frac{-22}{5} \div \left(-\frac{6}{5}\right) = \frac{-22}{5} \times \left(-\frac{5}{6}\right) = \frac{(-22) \times (-5)}{5 \times 6} \\ &= \frac{110}{30} = \frac{11}{3} = 3\frac{2}{3} \end{aligned}$$



Important Information

It is important to note that when divide rational numbers the order matters i.e.,

$$\frac{p}{q} \div \frac{r}{s} \neq \frac{r}{s} \div \frac{p}{q}$$



How many pieces of $\frac{15}{7}$ metres of ribbon can be cut from the ribbon which is $\frac{150}{7}$ metres long?



To find out the number of pieces of ribbon, we will have to divide $\frac{150}{7}$ by $\frac{15}{7}$.

$$\begin{aligned} \text{Number of pieces of ribbon} &= \frac{150}{7} \div \frac{15}{7} \\ &= \frac{150}{7} \times \frac{7}{15} = 10 \end{aligned}$$

So, the total length of ribbon can be cut into 10 equal parts of length $\frac{15}{7}$ metres.



Skill Practice

Solve the following:

$$\left(\frac{2}{3} \times \frac{9}{10}\right) \div \frac{3}{10} = \boxed{}$$

EXERCISE 1.3

1. Add the following rational numbers:

(i) $\frac{5}{9}$ and $\frac{4}{9}$

(ii) $\frac{-13}{4}$ and $\frac{9}{14}$

(iii) $\frac{-7}{11}$ and $\frac{-3}{22}$

(iv) $\frac{-6}{13}$ and $\frac{-8}{13}$

(v) $\frac{3}{4}$ and $\frac{-6}{5}$

(vi) 4 and $\frac{-3}{5}$

2. Solve the following:

(i) $\frac{3}{-40} + \left(\frac{-5}{14}\right)$

(ii) $\frac{-5}{9} + \left(\frac{-1}{3}\right)$

(iii) $\frac{3}{4} + \left(\frac{-2}{5}\right) + \left(\frac{-7}{20}\right)$

(iv) $-2 + \frac{7}{11} + \left(\frac{-4}{15}\right)$

(v) $\frac{-1}{8} + \frac{5}{12} + \left(\frac{-7}{24}\right)$

3. Subtract:

(i) $\frac{3}{4}$ from $\frac{1}{2}$

(ii) $\frac{-7}{6}$ from $\frac{5}{18}$

(iii) $\frac{-4}{9}$ from $\frac{-3}{5}$

(iv) $\frac{-7}{8}$ from -2

4. Solve the following:

(i) $\frac{4}{7} - \frac{5}{14}$

(ii) $1\frac{1}{5} - \frac{7}{14}$

(iii) $\frac{-4}{15} - \frac{8}{25}$

(iv) $\frac{-2}{5} - \frac{3}{14}$

(v) $\frac{4}{9} - \left(\frac{-5}{-9}\right)$

(vi) $1\frac{1}{12} - \left(\frac{-4}{9}\right)$

(vii) $\frac{-5}{-8} - \left(\frac{-3}{7}\right)$

(viii) $\frac{4}{11} - \left(\frac{-5}{22}\right)$

5. Solve the following:

(i) $\frac{7}{11} \times \frac{5}{6}$

(ii) $\frac{3}{7} \times \left(\frac{-4}{5}\right)$

(iii) $\frac{-2}{9} \times \frac{5}{12}$

(iv) $\frac{3}{14} \times \frac{-5}{7}$

(v) $\frac{-7}{6} \times (-18)$

(vi) $\frac{-13}{5} \times \frac{-25}{26}$

(vii) $\frac{-5}{11} \times \left(-1\frac{7}{22}\right)$

(viii) $-4\frac{1}{2} \times \left(-1\frac{3}{4}\right)$

(ix) $\frac{-5}{8} \times 48$

6. Solve the following:

(i) $\frac{5}{6} \div \frac{7}{12}$

(ii) $\frac{-7}{16} \div \frac{1}{8}$

(iii) $\frac{-6}{5} \div 8$

(iv) $\frac{16}{15} \div (-18)$

(v) $\frac{-3}{10} \div \left(\frac{-21}{40}\right)$

(vi) $4\frac{1}{16} \div -1\frac{5}{8}$

(vii) $-12 \div \left(\frac{-5}{6}\right)$

(viii) $-1\frac{10}{19} \div \left(\frac{-5}{58}\right)$

7. The product of two rational numbers is $\frac{-1}{10}$. If one number is $\frac{-3}{5}$, find the other.
8. By what rational number should we multiply $\frac{-15}{14}$ to get $\frac{-3}{7}$.
9. Find the additive inverse of:
- (i) $\frac{3}{5}$ (ii) $\frac{-7}{12}$ (iii) $-\left(\frac{-8}{15}\right)$ (iv) $-7\frac{5}{12}$ (v) $-2\frac{1}{13}$
10. Amna purchased $\frac{25}{3}$ m cloth to stitch a shirt. If she used $\frac{13}{9}$ m cloth to stitch the shirt. How much cloth was left with her?
11. If $\frac{75}{4}$ kilogram rice is to be packed in $\frac{25}{8}$ kilogram packets, then find:
- (a) How many packets of rice will be packed?
- (b) How much rice will be there in 20 packets of mass $\frac{24}{15}$ kilograms each?
12. Zara bought $\frac{235}{7}$ litres cooking oil. She used $\frac{2}{14}$ litres oil for cooking. How much oil was left with her?

1.1.15 Commutative, Associative and Distributive Properties of Rational Numbers

Rational numbers have several distinct properties, including commutative, associative and distributive properties.

i

Commutative Property

In commutative property, the order of operations doesn't matter. To understand this concept better, let's look at an example: If Rakib has three apples in his bag and adds six more, then it is the same as if Rakib has six apples and adds three more.

(a) Commutative Property of Rational Numbers w.r.t Addition

The addition of two rational numbers is commutative in general. If $\frac{a}{b}$ and $\frac{c}{d}$ are any two rational numbers, then:

$$\frac{a}{b} + \frac{c}{d} = \frac{c}{d} + \frac{a}{b}$$

Example 7

Prove that: $\frac{1}{2} + \frac{4}{5} = \frac{4}{5} + \frac{1}{2}$

Solution

Left Hand Side	Right Hand Side
$L.H.S = \frac{1}{2} + \frac{4}{5}$	$R.H.S = \frac{4}{5} + \frac{1}{2}$
$L.H.S = \frac{5+8}{10} = \frac{13}{10}$	$R.H.S = \frac{8+5}{10} = \frac{13}{10}$
So, L.H.S = R.H.S	

(b) Commutative Property of Rational Numbers w.r.t Multiplication

The commutative property of multiplication means changing the order of the numbers while multiplying does not change the answer. If $\frac{a}{b}$ and $\frac{c}{d}$ are any two rational numbers, then:

$$\frac{a}{b} \times \frac{c}{d} = \frac{c}{d} \times \frac{a}{b}$$

Keep in mind!

Commutative property does not hold in subtraction and division in nature.

Example 8

Prove that: $\frac{3}{4} \times \frac{5}{7} = \frac{5}{7} \times \frac{3}{4}$

Solution

Left Hand Side	Right Hand Side
$L.H.S = \frac{3}{4} \times \frac{5}{7}$	$R.H.S = \frac{5}{7} \times \frac{3}{4}$
$L.H.S = \frac{3 \times 5}{4 \times 7} = \frac{15}{28}$	$R.H.S = \frac{5 \times 3}{7 \times 4} = \frac{15}{28}$
So, L.H.S = R.H.S	

ii**Associative Property**

The associative property means that you can join or combine things in any order and still get the same answer. For example, Rakib went to the supermarket and bought ice cream for 12 rupees, bread for 8 rupees, and milk for 15 rupees. He gave the cashier 35 rupees for everything. He always got the same answer not matter in which order he added them.

(a) Associative Property of Rational Numbers w.r.t Addition

If $\frac{p}{q}$, $\frac{r}{s}$ and $\frac{t}{u}$ are three rational numbers, then according to associative properties of addition, we have:

$$\left(\frac{p}{q} + \frac{r}{s}\right) + \frac{t}{u} = \frac{p}{q} + \left(\frac{r}{s} + \frac{t}{u}\right)$$

Example 9

Prove that: $\left(\frac{3}{4} + \frac{5}{2}\right) + \frac{1}{2} = \frac{3}{4} + \left(\frac{5}{2} + \frac{1}{2}\right)$

Solution

Left Hand Side	Right Hand Side
$\begin{aligned} \text{L.H.S} &= \left(\frac{3}{4} + \frac{5}{2}\right) + \frac{1}{2} \\ &= \left(\frac{3+10}{4}\right) + \frac{1}{2} \\ &= \frac{13}{4} + \frac{1}{2} \\ \text{L.H.S} &= \frac{13+2}{4} = \frac{15}{4} \end{aligned}$	$\begin{aligned} \text{L.H.S} &= \frac{3}{4} + \left(\frac{5}{2} + \frac{1}{2}\right) \\ &= \frac{3}{4} + \left(\frac{5+1}{2}\right) \\ &= \frac{3}{4} + \frac{6}{2} \\ \text{L.H.S} &= \frac{3+12}{4} = \frac{15}{4} \end{aligned}$
So, L.H.S = R.H.S	

(b) Associative Property of Rational Numbers w.r.t Multiplication

When we multiply three numbers, it does not matter how we group them because the answer will be the same. If $\frac{p}{q}$, $\frac{r}{s}$ and $\frac{t}{u}$ are three rational numbers, then according to associative property of multiplication, we have

$$\left(\frac{p}{q} \times \frac{r}{s}\right) \times \frac{t}{u} = \frac{p}{q} \times \left(\frac{r}{s} \times \frac{t}{u}\right)$$

Example 10

Prove that: $\left(\frac{5}{7} \times \frac{3}{2}\right) \times \frac{1}{3} = \frac{5}{7} \times \left(\frac{3}{2} \times \frac{1}{3}\right)$

Solution

Left Hand Side	Right Hand Side
$\begin{aligned} \text{L.H.S} &= \left(\frac{5}{7} \times \frac{3}{2}\right) \times \frac{1}{3} \\ &= \left(\frac{5 \times 3}{7 \times 2}\right) \times \frac{1}{3} \\ &= \frac{15}{14} \times \frac{1}{3} \\ \text{L.H.S} &= \frac{15 \times 1}{14 \times 3} = \frac{15}{42} = \frac{5}{14} \end{aligned}$	$\begin{aligned} \text{L.H.S} &= \frac{5}{7} \times \left(\frac{3}{2} \times \frac{1}{3}\right) \\ &= \frac{5}{7} \times \left(\frac{3 \times 1}{2 \times 3}\right) \\ &= \frac{5}{7} \times \frac{3}{6} \\ \text{L.H.S} &= \frac{5 \times 3}{7 \times 6} = \frac{15}{42} = \frac{5}{14} \end{aligned}$
So, L.H.S = R.H.S	

iii

Distributive property of Rational Numbers w.r.t Multiplication over Addition and Subtraction

If $\frac{p}{q}$, $\frac{r}{s}$ and $\frac{t}{u}$ are three rational numbers, then according to distributive property of multiplication over addition and subtraction, we have:

$$(i) \quad \frac{p}{q} \times \left(\frac{r}{s} + \frac{t}{u} \right) = \left(\frac{p}{q} \times \frac{r}{s} \right) + \left(\frac{p}{q} \times \frac{t}{u} \right) \quad (ii) \quad \frac{p}{q} \times \left(\frac{r}{s} - \frac{t}{u} \right) = \left(\frac{p}{q} \times \frac{r}{s} \right) - \left(\frac{p}{q} \times \frac{t}{u} \right)$$

Example 11

Prove that: $\frac{2}{3} \times \left(\frac{1}{3} + \frac{4}{5} \right) = \left(\frac{2}{3} \times \frac{1}{3} \right) + \left(\frac{2}{3} \times \frac{4}{5} \right)$

Solution

Left Hand Side	Right Hand Side
$\begin{aligned} \text{L.H.S} &= \frac{2}{3} \times \left(\frac{1}{3} + \frac{4}{5} \right) \\ &= \frac{2}{3} \times \left(\frac{5+12}{15} \right) \\ &= \frac{2}{3} \times \frac{17}{15} \\ \text{L.H.S} &= \frac{2 \times 17}{3 \times 15} = \frac{34}{45} \end{aligned}$	$\begin{aligned} \text{L.H.S} &= \left(\frac{2}{3} \times \frac{1}{3} \right) + \left(\frac{2}{3} \times \frac{4}{5} \right) \\ &= \left(\frac{2 \times 1}{3 \times 3} \right) + \left(\frac{2 \times 4}{3 \times 5} \right) \\ &= \frac{2}{9} + \frac{8}{15} \\ \text{L.H.S} &= \frac{10+24}{45} = \frac{34}{45} \end{aligned}$
So, L.H.S = R.H.S	

Example 12

Prove that: $\frac{1}{4} \times \left(\frac{2}{6} - \frac{1}{8} \right) = \left(\frac{1}{4} \times \frac{2}{6} \right) - \left(\frac{1}{4} \times \frac{1}{8} \right)$

Solution

Left Hand Side	Right Hand Side
$\begin{aligned} \text{L.H.S} &= \frac{1}{4} \times \left(\frac{2}{6} - \frac{1}{8} \right) \\ &= \frac{1}{4} \times \left(\frac{8-3}{24} \right) \\ &= \frac{1}{4} \times \frac{5}{24} \\ \text{L.H.S} &= \frac{1 \times 5}{4 \times 24} = \frac{5}{96} \end{aligned}$	$\begin{aligned} \text{L.H.S} &= \left(\frac{1}{4} \times \frac{2}{6} \right) - \left(\frac{1}{4} \times \frac{1}{8} \right) \\ &= \left(\frac{1 \times 2}{4 \times 6} \right) - \left(\frac{1 \times 1}{4 \times 8} \right) \\ &= \frac{2}{24} - \frac{1}{32} \\ \text{L.H.S} &= \frac{8-3}{96} = \frac{5}{96} \end{aligned}$
So, L.H.S = R.H.S	

EXERCISE 1.4

1. Verify the following commutative properties:

$$(i) \quad \frac{-3}{5} + \frac{6}{7} = \frac{6}{7} + \frac{-3}{5}$$

$$(ii) \quad \frac{7}{10} + \frac{-2}{5} = \frac{-2}{5} + \frac{7}{10}$$

$$(iii) \quad \frac{3}{-2} \times \frac{-5}{7} = \frac{-5}{7} \times \frac{3}{-2}$$

$$(iv) \quad \frac{15}{26} \times \frac{15}{45} = \frac{15}{45} \times \frac{15}{26}$$

2. Verify the following associative properties:

$$(i) \quad \left(\frac{4}{3} + \frac{2}{5}\right) + \frac{3}{7} = \frac{4}{3} + \left(\frac{2}{5} + \frac{3}{7}\right) \quad (ii) \quad \left(\frac{10}{12} + \frac{11}{-2}\right) + \frac{-3}{4} = \frac{10}{12} + \left(\frac{11}{-2} + \frac{-3}{4}\right)$$

$$(iii) \quad \left(\frac{15}{20} + \frac{-5}{10}\right) + \frac{12}{40} = \frac{15}{20} + \left(\frac{-5}{10} + \frac{12}{40}\right) \quad (iv) \quad \left(\frac{-5}{10} + \frac{10}{-20}\right) + \frac{15}{30} = \frac{-5}{10} + \left(\frac{10}{-20} + \frac{15}{30}\right)$$

3. Verify the following distributive properties:

$$(i) \quad \frac{-2}{5} \times \left(\frac{3}{9} + \frac{8}{6}\right) = \left(\frac{-2}{5} \times \frac{3}{9}\right) + \left(\frac{-2}{5} \times \frac{8}{6}\right) \quad (ii) \quad \frac{10}{11} \times \left(\frac{6}{7} + \frac{9}{4}\right) = \left(\frac{10}{11} \times \frac{6}{7}\right) + \left(\frac{10}{11} \times \frac{9}{4}\right)$$

$$(iii) \quad \frac{15}{16} \times \left(\frac{8}{12} - \frac{2}{6}\right) = \left(\frac{15}{16} \times \frac{8}{12}\right) - \left(\frac{15}{16} \times \frac{2}{6}\right)$$

$$(iv) \quad \frac{-4}{6} \times \left(\frac{8}{10} - \frac{4}{5}\right) = \left(\frac{-4}{6} \times \frac{8}{10}\right) - \left(\frac{-4}{6} \times \frac{4}{5}\right)$$



Activity

Provide a handout of associative property with blanks. Students will roll 3 dice at once and will write the numbers in the same sequence on both sides and then they will solve both sides to find out both sides give the same answer.

1.1.16 Round Whole Numbers, Integers, Rational Numbers and Decimal Numbers



Nimra wants to buy the pack of tea. The price of the tea pack is Rs. 239.75. In such situation, how much amount does she pay?



To solve this situation, she will have to pay Rs. 240. This procedure is known as rounding.



Skill Practice

If the price of a packet of chips is Rs. 15.15, tell the price of packet of chips after rounding.

i

Round Whole Numbers



Keep in mind!

- First of all look at the digit next to the required degree of accuracy while rounding.
- If the digit is 5 or greater than 5, then we will add 1 to the digit of the required degree of accuracy.
- If the digit is less than 5, then we will leave the digit as it is given on the required degree of accuracy.



Let us round 3925 to the 1 significant figure, 2 significant figures, and 3 significant figures.

3925 = 4000 1 significant figure

3925 = 3900 2 significant figures

3925 = 3930 3 significant figures



Key fact!

The number which is rounded to a greater number of significant figures that number is more accurate.

**Skill Practice**

Round 48163 to the given degree of accuracy: (i) 2 significant figures (ii) 3-significant figures

ii Round Decimal Numbers

Let us round 24.3528 to the given degree of accuracy.

- (i) 2 decimal places (ii) 3 significant figures

(i) $24.3528 = 24.35$ (2 decimal places) (ii) $24.3528 = 24.4$ (3 significant figures)

Example 13 Round 0.028635 to the given degree of accuracy.

- (i) 3 significant figures (ii) 3 decimal places

Solution (i) $0.028635 = 0.0286$ (ii) $0.028635 = 0.029$

**Skill Practice**

Hamza purchased two litres of petrol for Rs. 438.75. Can you round the number to the given degree of accuracy?

- (i) 3 significant figures
(ii) 1 decimal place

iii Round Negative Integers

While rounding the negative integer, the number which is close to the zero (0) is greater number and which is far from the zero (0) is smaller number.



Let us round -43566 to the given degree of accuracy.

- (i) 2 significant figures
(ii) 3 significant figures

(i) $-43566 = -43000$ (2 significant figures)
 $= -44000$ (2 significant figures)

When round towards zero (0)

When round away from zero (0)

(ii) $-43566 = -43500$ (3 significant figures)
 $= -43600$ (3 significant figures)

When round towards zero (0)

When round away from zero (0)

**Keep in mind!**

In decimal part of a decimal number, all zeros before a non-zero digit are not significant but all zeros after a non-zero digit are significant.

**Remember!**

In negative integers, the procedure to round integers is different from the procedure to round whole numbers.

**Skill Practice**

How many significant does 0.0789 have?

**Skill Practice**

Round -53278 to the 2 significant figures and 3 significant figures.

**Teachers' Guide**

Write some negative integers and decimal numbers on writing board. Call out students in front of writing board. Ask them to round the numbers to the required degree of accuracy.

iv Round Rational Numbers



Aslam bought $\frac{25}{18}$ kilogram of rice. How can we round the number to the given degree of accuracy?

- (i) 3 decimal places (ii) 2 significant figures (iii) 3 significant figures

(i) $\frac{25}{18} = 1.388889 = 1.389$ (3 decimal places)

(ii) $\frac{25}{18} = 1.388889 = 1.4$ (2 significant figures)

(iii) $\frac{25}{18} = 1.38889 = 1.39$ (3 significant figures)



Skill Practice

Round $\frac{72}{33}$ to the:

- (i) 3 decimal places
(ii) 2 significant figures
(iii) 3 significant figures

1.1.17

Use Knowledge of Rounding to Give an Estimate to a Calculation; to check the Reasonableness of the Solution

Sometimes, in our real life, we are unable to find the actual value of any object or thing, then to solve this situation we have to round or estimate the value to the required degree of accuracy. When round a number, we obtain an estimated or approximated value that may or may be differ from the actual value.

If the difference between the approximated / estimated value and actual value is small, then we can say that the result or solution is reasonable.

$$\text{Error} = \text{Actual value} - \text{Approximated value}$$



Keep in mind!

The difference between the actual value and approximated / estimated value is called an error.

Example 14

Find the sum of 24.8356 and 15.7156 after rounding 3 decimal places. Also check the solution is reasonable or not?

Solution

First of all round 24.8356 and 15.7156 to the 3 decimal places.

$$24.8356 = 24.836 \text{ (3 decimal places)} ; 15.7156 = 15.716 \text{ (3 decimal places)}$$

$$\text{Approximated sum} = 24.836 + 15.716$$

$$= 40.552$$

$$\text{Actual sum} = 24.8356 + 15.7156$$

$$= 40.5512$$

$$\text{Error} = |\text{Actual sum} - \text{Approximated sum}|$$

$$= |40.5512 - 40.552|$$

$$= |-0.008|$$

$$\text{Error} = 0.008$$



Skill Practice

$$28.3 \times 15.2$$

Find the approximated value and error. Also explain regarding the reasonableness of the solution.

The value of error is small. Hence, the approximated sum is reasonable.



Activity

Students will work in pairs. One student will roll the dice to make a 4-digit number. Then his/her partner will roll the dice to make another 4-digit number. Estimate the sum of the first and the second number. Add the numbers. Make a table of 6 by 2 (6 rows and 2 columns) on the sheet with headings estimated sum and actual sum. Students will record their estimated sum and actual sum. Repeat steps 2 to 7 five times. Compare the answers with the estimated sums. Are the answers reasonable?

EXERCISE 1.5

1. Round the following to the given degree of accuracy:

- | | |
|--------------------------------------|------------------------------------|
| (i) 87932 (2 significant figures) | (ii) 3890 (3 significant figures) |
| (iii) 790253 (2 significant figures) | (iv) 10025 (3 significant figures) |
| (v) 18954 (3 significant figures) | (vi) 25980 (3 significant figures) |

2. Round the numbers away from zero (0) as given.

- | | |
|--------------------------------------|--|
| (i) -27.3289 (3 significant figures) | (ii) -59058 (2 significant figures) |
| (iii) -12569 (3 significant figures) | (iv) 17.238 (2 significant figures) |
| (v) 0.003857 (3 significant figures) | (vi) 0.0467453 (3 significant figures) |

3. Round the following to the stated number of decimal places or significant figures:

- | | |
|--|--|
| (i) $\frac{47}{8}$ (2 decimal places) | (ii) $\frac{5}{27}$ (3 significant figures) |
| (iii) $\frac{22}{27}$ (3 decimal places) | (iv) $\frac{27}{19}$ (3 significant figures) |
| (v) $\frac{37}{53}$ (3 decimal places) | (vi) $\frac{77}{83}$ (3 significant figures) |

4. Round the following to the required degree of accuracy:

- | | |
|---------------------------------------|---------------------------------------|
| (i) 25.359 (2 decimal places) | (ii) 0.002897 (3 significant figures) |
| (iii) 0.0878 (3 decimal places) | (iv) 17.00597 (3 significant figures) |
| (v) 0.0005246 (3 significant figures) | (vi) 13.5876 (3 decimal places) |

5. Compare your approximated value and accurate value. Also check is the solution reasonable or not?

- | | |
|---|---|
| (i) $4.281 \div 2.157$ (2 decimal places) | (ii) $3.348 - 2.104$ (2 decimal places) |
| (iii) 2.281×3.567 (2 decimal places) | |

SUMMARY

- The positive integers and negative integers along with zero (0) are called integers. i.e.,

$$Z = \{0, \pm 1, \pm 2, \pm 3, \dots\}$$
- The only whole number zero (0) is not a natural number.
- 0 is neither positive nor negative. 0 is also known as neutral number/integer.
- A number that can be expressed in the form of $\frac{p}{q}$ where p, q are integers and $q \neq 0$ is called a rational number. The set of rational number is denoted by Q .
- In $\frac{p}{q}$, the integer p is the numerator and the integer $q \neq 0$ is the denominator.
- If the denominators of two rational numbers are same, then compare the numerators, the rational number with greater numerator is greater than other.
- If the denominator of two rational numbers are different, then convert them into like fractions by taking LCM of the denominators.
- The smallest common multiple of two or more than two numbers is called least common multiple (LCM).
- Every positive rational number is greater than zero. Every positive rational number is greater than every negative rational number.
- The difference between the actual value and approximated/ estimated value is called an error.
- The following are the properties of rational numbers:
 - $\frac{a}{b} + \frac{c}{d} = \frac{c}{d} + \frac{a}{b}$ (commutative property of rational numbers w.r.t addition)
 - $\frac{a}{b} \times \frac{c}{d} = \frac{c}{d} \times \frac{a}{b}$ (commutative property of rational numbers w.r.t multiplication)
 - $\left(\frac{p}{q} + \frac{r}{s}\right) + \frac{t}{u} = \frac{p}{q} + \left(\frac{r}{s} + \frac{t}{u}\right)$ (Associative property of rational numbers w.r.t addition)
 - $\left(\frac{p}{q} \times \frac{r}{s}\right) \times \frac{t}{u} = \frac{p}{q} \times \left(\frac{r}{s} \times \frac{t}{u}\right)$ (Associative property of rational numbers w.r.t multiplication)
- Distributive property of rational numbers w.r.t multiplication over addition and subtraction.
 - $\frac{p}{q} \times \left(\frac{r}{s} + \frac{t}{u}\right) = \left(\frac{p}{q} \times \frac{r}{s}\right) + \left(\frac{p}{q} \times \frac{t}{u}\right)$ $\frac{p}{q} \times \left(\frac{r}{s} - \frac{t}{u}\right) = \left(\frac{p}{q} \times \frac{r}{s}\right) - \left(\frac{p}{q} \times \frac{t}{u}\right)$

Sub-domain

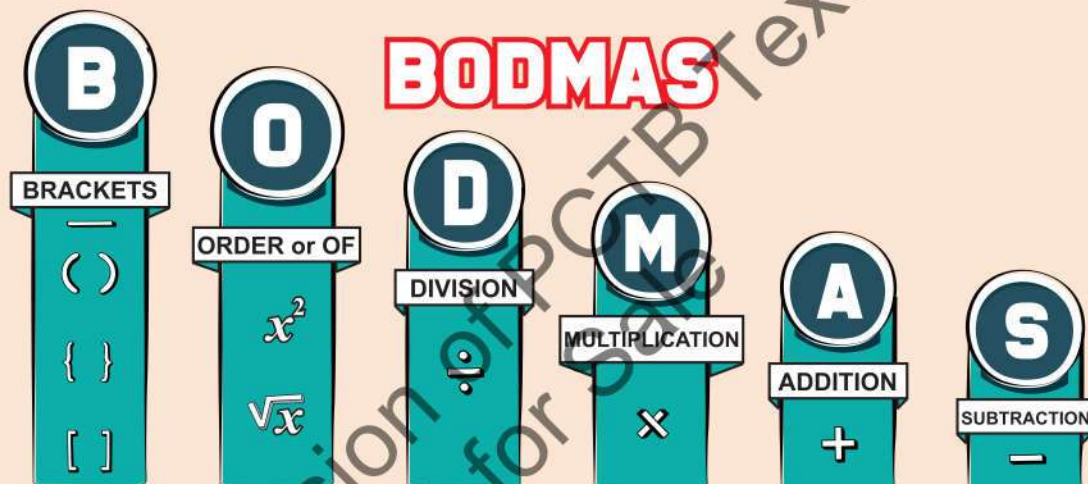
(ii)

Simplification

Students' Learning Outcomes

After studying this sub-domain, students will be able to:

- Recognise the order of operations and use it to solve mathematical expressions involving whole numbers, decimals, fractions and integers.



Mathematics involves directly to business, health and finance. A mathematical expression consist of fractions, integers, whole numbers and decimals along with mathematical operations. To solve the mathematical expression a special technique and conceptual procedure is required. technique. In a mathematical expression, the order to apply the mathematical operations plays a vital role. In order to solve the mathematical expression, BODMAS rule is used. It guides us which operation is used first.

Step 1	B	Brackets	$\{ \{ (\overline{\quad}) \} \}$
Step 2	O	order or of or power	x^2 or $\sqrt{x}$
Step 3	D	Division	$\div$
Step 4	M	Multiplication	$\times$
Step 5	A	Addition	$+$
Step 6	S	Subtraction	$-$



Remember!

The procedure to simplify the mathematical expression is known as simplification.



History

Achilles Reselfelt introduced the "BODMAS" rule.

1.2.1 Brackets

In a mathematical expression, brackets are used to group/gather along with mathematical operations.

i.e., +, −, × and ÷. To simplify a mathematical expression, the given order of brackets is used.

Step 1	Vinculum or bar	—
Step 2	Round, curved brackets	()
Step 3	Curly brackets or braces	{ }
Step 4	Box brackets or square brackets	[]



Search out

<https://www.liveworksheets.com/bp935523uz>



Let us solve the given mathematical expression using BODMAS rule and order of brackets.

Example 1

Simplify: $18 \div 2 - [3 \times 5 - \{12 - 6 - (-6 + 8 - 5)\}]$

Solution

$$\begin{aligned}
 &18 \div 2 - [3 \times 5 - \{12 - 6 - (-6 + 8 - 5)\}] \\
 &= 18 \div 2 - [3 \times 5 - \{12 - 6 - (-6 + 8 - 5)\}] \quad \text{solved bar or vinculum} \\
 &= 18 \div 2 - [3 \times 5 - \{12 - 6 - (-3)\}] \\
 &= 18 \div 2 - [3 \times 5 - \{12 - 6 + 3\}] \quad \text{solved round brackets} \\
 &= 18 \div 2 - [3 \times 5 - \{9\}] \\
 &= 18 \div 2 - [3 \times 5 - 9] \quad \text{solved curly brackets} \\
 &= 18 \div 2 - [15 - 9] \quad \text{applied BODMAS rule} \\
 &= 18 \div 2 - 6 \quad \text{solved box brackets} \\
 &= 9 - 6 \quad \text{applied BODMAS rule} \\
 &= 3
 \end{aligned}$$



Find Answer

$$12 + 3 - 8 + 3 - 5 - 6 = ?$$



Keep in mind!

When expression has subtraction and addition, then it can be solved from left to right.

Example 2

Simplify: $2.25 \div 8.5 \times [2 \times 1.37 + \{3.2 - (5.2 - 3.2 + 5.2)\}]$

Solution

$$\begin{aligned}
 &2.25 \div 8.5 \times [2 \times 1.37 + \{3.2 - (5.2 - 3.2 + 5.2)\}] \\
 &= 2.25 \div 8.5 \times [2 \times 1.37 + \{3.2 - (5.2 - 3.2 - 5.2)\}] \quad \text{solved bar} \\
 &= 2.25 \div 8.5 \times [2 \times 1.37 + \{3.2 - (-3.2)\}] \\
 &= 2.25 \div 8.5 \times [2 \times 1.37 + \{3.2 + 3.2\}] \quad \text{solved round brackets} \\
 &= 2.25 \div 8.5 \times [2 \times 1.37 + 6.4] \quad \text{solved curly brackets}
 \end{aligned}$$



Solve

$$68 \div 2 \times 2 \div 4 = ?$$



Teachers' Guide

Clear the concept by applying mathematical operations when the expression has subtraction and addition by writing some examples on writing board.

$$\begin{aligned}
 &= 2.25 \div 8.5 \times [2.74 + 6.4] \\
 &= 2.25 \div 8.5 \times 9.14 \quad \text{solved square brackets} \\
 &= 0.265 \times 9.14 \\
 &= 2.4221
 \end{aligned}$$

**Keep in mind**

- When expression has multiplication and division, then solve division first.
- When expression has multiplication and division, then it can be solved from left to right.

Example 3 Simplify:**Solution**

$$\begin{aligned}
 &2\frac{1}{3} \div \left[1\frac{1}{3} - 2\frac{1}{5} - \left\{ 2 \times 5\frac{1}{2} - \left(\frac{1}{2} - \frac{1}{3} + \frac{1}{5} \right) \right\} \right] \\
 &= 2\frac{1}{3} \div \left[1\frac{1}{3} - 2\frac{1}{5} - \left\{ 2 \times 5\frac{1}{2} - \left(\frac{1}{2} - \frac{1}{3} - \frac{1}{5} \right) \right\} \right] \quad \text{solved bar or vinculum} \\
 &= 2\frac{1}{3} \div \left[1\frac{1}{3} - 2\frac{1}{5} - \left\{ 2 \times 5\frac{1}{2} - \left(\frac{15 - 10 - 6}{30} \right) \right\} \right] \\
 &= 2\frac{1}{3} \div \left[1\frac{1}{3} - 2\frac{1}{5} - \left\{ 2 \times 5\frac{1}{2} - \left(-\frac{1}{30} \right) \right\} \right] \\
 &= 2\frac{1}{3} \div \left[1\frac{1}{3} - 2\frac{1}{5} - \left\{ 2 \times 5\frac{1}{2} + \frac{1}{30} \right\} \right] \quad \text{solved round brackets} \\
 &= 2\frac{1}{3} \div \left[1\frac{1}{3} - 2\frac{1}{5} - \left\{ 2 \times \frac{11}{2} + \frac{1}{30} \right\} \right] \\
 &= 2\frac{1}{3} \div \left[1\frac{1}{3} - 2\frac{1}{5} - \left\{ 11 + \frac{1}{30} \right\} \right] \\
 &= 2\frac{1}{3} \div \left[1\frac{1}{3} - 2\frac{1}{5} - \left\{ \frac{330 + 1}{30} \right\} \right] \\
 &= 2\frac{1}{3} \div \left[1\frac{1}{3} - 2\frac{1}{5} - \left\{ \frac{331}{30} \right\} \right] \\
 &= 2\frac{1}{3} \div \left[1\frac{1}{3} - 2\frac{1}{5} - \frac{331}{30} \right] \quad \text{solved curly brackets} \\
 &= 2\frac{1}{3} \div \left[\frac{4}{3} - \frac{11}{5} - \frac{331}{30} \right] \\
 &= 2\frac{1}{3} \div \left[\frac{40 - 66 - 331}{30} \right] = 2\frac{1}{3} \div \left[\frac{-357}{30} \right]
 \end{aligned}$$

**Challenge**

Use each of the digits 1, 2, 3, 4, 5, 6, ..., 9. Place appropriate operation symbol and brackets to drive the given answers.

$4 \times 2 + (1 \times 3 \times 5)$	23
	14
	16
	2
	39

**Teachers' Guide**

Quick mental test by writing some question on mixed operation e.g., $22 \div 11 \times 10$, $50 + 60 \times 5$, $50 \div 5 + 4 \times 3$ etc., on the writing board to clear the concept of BODMAS.

$$\begin{aligned}
 &= 2\frac{1}{3} \times \frac{-30}{357} \\
 &= \frac{\cancel{17}^1}{\cancel{51}^3} \times \frac{-30^{10}}{357^{51}} \\
 &= \frac{-10}{51}
 \end{aligned}$$

solved square brackets



Find Answer

$$12 \times 14 \div 2 = \boxed{?}$$

EXERCISE 1.6

1. Simplify the following:

(i) $70 + [10 + 20 - 2 \{ 8 - 2 + 3 \}]$

(ii) $25 - [5 + \{28 - (16 \div 4 + 12)\}]$

(iii) $12 \times 8 - [64 - \{18 \div (9 - 6 - 3)\}]$

(iv) $9 \times 3 - [28 - \{12 \div (10 - 6 - 2)\}]$

(v) $8 \times 9 - [32 - \{24 \div (8 - 4 - 2)\}]$

(vi) $2\frac{3}{12} \div \left[1\frac{4}{5} \times \left\{ 1\frac{1}{3} + \left(2\frac{1}{2} + 1\frac{1}{3} - 2\frac{1}{6} \right) \right\} \text{ of } 1\frac{2}{3} \right]$

(vii) $1\frac{1}{2} \left[\frac{7}{6} + \left\{ \frac{245}{2} - \left(\frac{4}{3} + 121 \div \frac{11}{8} \right) \right\} \right]$

(viii) $\left[3\frac{2}{3} \times \left\{ 2\frac{1}{4} \div \left(1\frac{1}{8} + 2\frac{1}{4} - 1\frac{1}{2} \right) \right\} \right] - 2\frac{1}{3}$

(ix) $1\frac{3}{5} \div \left[\frac{1}{25} \times \left\{ 1\frac{1}{4} + \left(3\frac{1}{3} \div 2\frac{1}{2} \text{ of } 1\frac{5}{16} \right) \right\} \right] \times \frac{1}{3}$

(x) $\left[3\frac{2}{3} \div \left\{ 1\frac{1}{3} + \left(1\frac{2}{3} + 3\frac{2}{5} - 2\frac{1}{5} \right) \right\} \times 2\frac{3}{5} \right]$

(xi) $[3 + \{1.25 \times 3.85 \div (5.64 - 2.7 + 1.4)\}]$

(xii) $[2.35 + \{12.099 \div (1.45 - 2.1 \times 1.23)\}]$

(xiii) $15.165 \div [2.125 + \{3.04 - (2.75 \times 2.06 - 2.02)\}]$

(xiv) $0.8 \times [4.9 \times \{0.555 \div (0.2 + 0.02 + 0.002)\}]$

(xv) $5.2383 \div [1.026 + \{1.123 \times (9.261 \div 2.345 + 5.432)\}] \times 2.03$

SUMMARY

- The procedure to simplify a mathematical expression is known as simplification.
- When expression has subtraction and addition, it can be solved from left to right.
- When expression has multiplication and division, then it can be solved from left to right.
- BODMAS rule:

B	O	D	M	A	S
Brackets [{ () }]	Orders $\sqrt{x}$, x^2	Division $\div$	Multiplication $\times$	Addition $+$	Subtraction $-$

Students' Learning Outcomes

After studying this sub-domain, students will be able to:

- Use language, notation, and Venn Diagrams to represent different sets and their elements. (natural numbers, whole numbers, integers, even numbers, odd numbers, prime numbers)
- Identify and differentiate between:
 - i. subset and superset
 - ii. proper and improper
 - iii. equal and equivalent
 - iv. disjoint and overlapping.
- Describe and perform operations on sets (union, intersection, difference and complement).
- Verify the following:
 - $A \cap A^c = \emptyset$
 - $A \cup A^c = U$
 - $(A \cup B)^c = A^c \cap B^c$
 - $(A \cap B)^c = A^c \cup B^c$

U



Do you know about the importance of vegetables and fruits in your life?



Which vegetable and fruit do you like the most?



Vegetables and fruits are the excellent source of minerals, vitamins and potassium. Eating vegetables and fruits everyday is important for healthy life.



Introduction

In this world, there are many things or objects which exist in collections or groups form. We use many words in our daily life to explain the collections or groups of things or objects e.g., a hive of bees, bunch of flowers, collection of different kinds of coins, a team of cricket players, group of animals etc. But, in Mathematics, for collection of objects or things, we use the word “Set”.

Set is a fundamental mathematical concept that allows us to group objects. It is an important part of the study of mathematics and computer science as well as other fields like probability and statistics. Let's understand the concept of a set and its types.



History

George Cantor (1845 – 1918) was a German Mathematician. He played an important role in the creation of the set theory. He is known as the founder of the set theory.

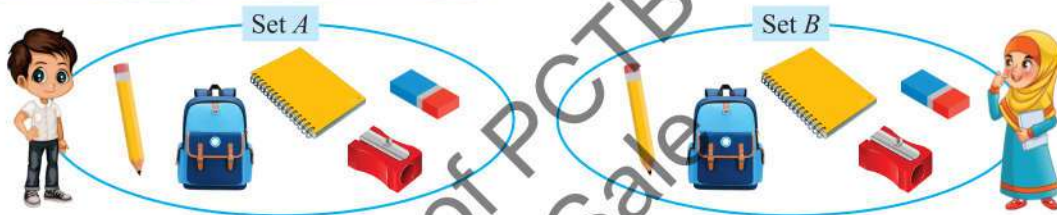


Activity

Visit the nearest market with your parents and find out the different sets of objects/things and note down on your notebooks.



Zain went to the market to buy some items/objects for his 2 children for going to the school.



In set A and set B , all things are well defined and distinct.



Remember!

A set is a collection of well defined and distinct objects.

Well defined: An object is clearly described and there is nothing to describe it more.

Distinct: Each object of the set must be unique.



Keep in mind!

- The objects of a set are called elements or members of the set
- Object may be numbers, names, symbols, geometrical shapes etc.

1.3.1 Notation of a Set

- Sets are represented by capital English alphabet A, B, C etc. The elements of a set are written in curly brackets “{ }” and separated by commas (,) .

For example,

Curly bracket or braces.

$$A = \{a, b, c, d\}$$

It is a set and represented by “ A ”.

The elements a, b, c and d are called the elements of the set.

$$\begin{aligned} a &\in A \\ b &\in A \\ c &\in A \\ d &\in A \end{aligned}$$

but
 $\{a\} \notin A$



Symbol

$\in$ belongs to, $\notin$ does not belong to
... so on or continue on



Teachers' Guide

Clear the concept of set by using the set of real life objects. Also tell them the difference between the well defined and distinct by using real life objects.

Example 1 $A = \{1, 2, 3, 1\}$ is a set or not?

Solution The elements of the set A are not distinct, because “1” comes twice.

So, A is not a set.



Brain Teaser!

Tick (✓) which are sets and cross (✗) which are not sets.

$B = \{a, b, c, d, d\}$ ☐ $C = \{\triangle, \circ, \square, \square\}$ ☐

$D = \{\text{Red, Blue, Green, Blue}\}$ ☐



Skill Practice

If $A = \{2, 7, 8\}$, put $\in$ or $\notin$ in the given boxes.

(i) $2 \square A$ (ii) $6 \square A$ (iii) $8 \square A$



Activity

In the classroom, find out the different sets of objects or things and note down on your notebooks.

1.3.2 Some Families of Sets

- Set of natural numbers = $N = \{1, 2, 3, \dots\}$
- Set of whole numbers = $W = \{0, 1, 2, 3, \dots\}$
- Set of integers = $Z = \{0, \pm 1, \pm 2, \pm 3, \dots\}$
- Set of odd numbers = $O = \{\pm 1, \pm 3, \pm 5, \dots\}$
- Set of even numbers = $E = \{0, \pm 2, \pm 4, \dots\}$
- Set of prime numbers = $P = \{2, 3, 5, 7, 11, \dots\}$



Brain Teaser!

The collection of five months is a set or not? If no then explain.



Remember!

- An empty set has no element and it is represented by $\{\}$ or ϕ . It is also called null set.
- If a set has only one element, then it is called singleton set.

1.3.3 Expressing of a Set

A set can be expressed in three ways.

- (a) Tabular form or Roster form (b) Descriptive form (c) Set builder notation

(a) Tabular Form

When all the elements of a set are written within the curly bracket “ $\{\}$ ” and elements are separated by using commas (,), then it is called tabular form. For example,
 $A = \{0, 1, 3, 5, 7\}$; $B = \{a, b, c, d, e, f\}$

$C = \{10, 5, 15, 25, 20\}$; $D = \{\text{Raza, Hamza, Ayesha}\}$

(b) Descriptive Form

When all the elements of a set are described in words form, then it is called descriptive form. For example,
 $A = \text{Set of multiples of 5}$ $B = \text{Set of vowels in English alphabet}$

$C = \text{Set of days of a week}$ $D = \text{Set of the first five solar months}$



Keep in mind!

Order of the elements in a set does not matter. e.g.,
 $A = \{a, c, d, f, k\}$ or $A = \{f, d, a, c, k\}$
 Both are same.



Skill Practice

Write the descriptive form of sets in the tabular form.

(c) Set Builder Notation

When all the elements of a set are expressed by using mathematical notations, stating all the properties of elements given in a set then it is called set builder notation. For example, $A = \{0, \pm 2, \pm 4, \pm 6, \pm 8, \dots\}$ is expressed in set builder notation as: $A = \{x \mid x \in E\}$.

$B = \{0, 1, 2, 3, 4, 5, \dots, 15\}$ is expressed in set builder notation as: $B = \{x \mid x \in W \wedge x \leq 15\}$

**Remember!**

- The symbol “ $\mid$ ” is read as “such that”
- The symbol “ $\wedge$ ” is read as “and”
- The symbol “ $\vee$ ” is read as “or”
- The symbol “ $\in$ ” is read as “belongs to”

**Skill Practice**

Write the given sets in other form (Descriptive form).

EXERCISE 1.7**1. Write the following sets in tabular form:**

- (i) A = Set of the first five odd numbers. (ii) B = Set of the days of a week.
 (iii) C = Set of the last five English alphabet.
 (iv) D = Set of the even numbers greater than 9 and less than 25.
 (v) $E = \{x \mid x \in Z \wedge -9 \leq x \leq 12\}$ (vi) $F = \{x \mid x \in O\}$
 (vii) $G = \{x \mid x \in N \wedge x < 15\}$ (viii) Set of the first seven prime numbers

2. Write the following sets in descriptive form:

- (i) $A = \{3, 6, 9, 12, 15, 18, 21\}$ (ii) $B = \{a, b, c, d, e, f, g, h, i, j, k, \ell, m\}$
 (iii) $C = \{y \mid y \in E\}$ (iv) $D = \{\pm 4, \pm 5, \pm 6, \dots, \pm 20\}$
 (v) $E = \{x \mid x \in N \wedge x \leq 50\}$ (vi) $F = \{0, 1, 2, 3, 4, 5, \dots\}$
 (vii) $G = \{13, 17, 19, 23, 29, 31, 37\}$ (viii) $H = \{25, 30, 35, 40, 45, 50, 55\}$

3. If $A = \{1, 2, 3, 4, 5, 6\}$, then fill in the blanks by using symbols $\in$ or $\notin$.

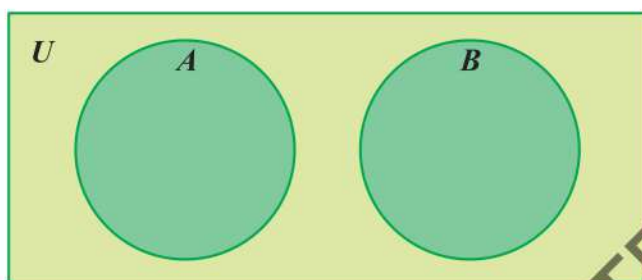
- (i) $2 \underline{\hspace{1cm}} A$ (ii) $7 \underline{\hspace{1cm}} A$ (iii) $4 \underline{\hspace{1cm}} A$ (iv) $6 \underline{\hspace{1cm}} A$
 (v) $3 \underline{\hspace{1cm}} A$ (vi) $-5 \underline{\hspace{1cm}} A$ (vii) $10 \underline{\hspace{1cm}} A$ (viii) $5 \underline{\hspace{1cm}} A$

4. Write the following sets in set builder notation:

- (i) A = Set of natural numbers less than 50 and divisible by 4.
 (ii) B = Set of integers between -5 and 5 .
 (iii) $C = \{1, 3, 5, 7, 9, 11, 13, 15\}$
 (iv) $D = \{10, 11, 12, 13, 14, \dots, 25\}$
 (v) E = Set of the last five solar months.
 (vi) $F = \{-20, -19, -18, -17, \dots, 0, 1, 2, 3, 4, \dots, 20\}$
 (vii) G = Set of odd numbers between 24 and 38.
 (viii) $H = \{10, 20, 30, 40, 50, \dots, 100\}$

1.3.4 Venn Diagram

The Venn diagram is a pictorial way of representing sets and their relationship. Circles, ovals and rectangles are used to make a Venn diagram. Rectangle is used to show the universal set and circles or ovals are used to show the sets under consideration, inside the rectangle. Venn diagram is often used in mathematics, but it can also be used in every day life to represent relationships between things.



Venn diagram

Example 2

Draw a Venn diagram to represent the following sets: $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, $A = \{1, 2, 5\}$ and $B = \{3, 4, 5\}$.

Solution

Step 1

Draw a rectangle and label it U to represent the universal set.

Step 2

Draw circles within the rectangle to represent the other sets. Label the circles and write the relevant elements in each circle.

Step 3

Write the remaining elements outside the circles but within the rectangle.

Search out

[https://www.transum.org/software/sw/starter/oftoday/students/venn diagram.asp](https://www.transum.org/software/sw/starter/oftoday/students/venn%20diagram.asp)

1.3.5 Types of Sets

Sets are classified by how many elements they have and how they relate to other sets.

Subset

If all the elements of a set A are also the elements of a set B , then the set A is called the subset of the set B and we write it as $A \subseteq B$. The symbol ' $\subseteq$ ' stands for 'is a subset of'. For example,

$$A = \{1, 2, 3\} \text{ and } B = \{1, 2, 3, 4, 5, 6, 7, 8\}$$

Here, we can see that all the elements of the set A are elements of the set B . Therefore, $A \subseteq B$



History

John Venn (1834 - 1923) was the inventor of Venn diagram. In 1880, the Venn diagram was introduced by John Venn.



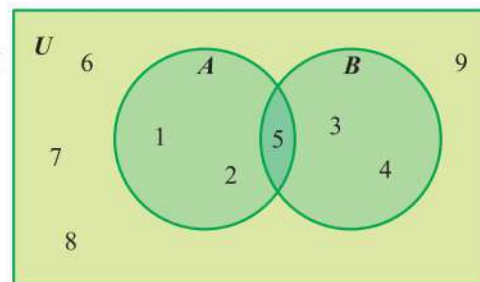
Important Information

Universal Set: A set is called universal set which has all the elements of the sets under consideration. It is denoted by U . For example,

$$\text{If } A = \{2, 4, 6, 8\}$$

$B = \{1, 2, 3, 5, 7, 8\}$, then universal set (U) is

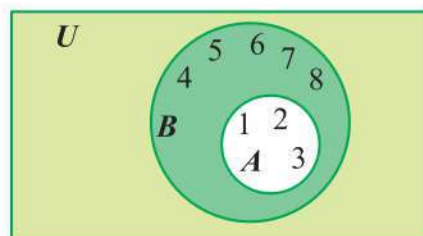
$$U = \{1, 2, 3, 4, 5, 6, 7, 8\}$$



Important Information

A set is called an empty set which has no element. It is written as:

$$\{ \} \text{ or } \phi$$



We can find many examples of subsets in everyday life such as:

1. If we consider school items in a bag as one set, then the set of books is a subset.
2. If all the classes from 1 to 8 in a school form a set, then the sets of class 1 and class 2 are its subsets.



Super Set

If a set A contains all the elements of a set B , then the set A is a superset of a set B and we write it as $A \supseteq B$. For example, $A = \{a, b, c, d\}$ and $B = \{a, b, c\}$

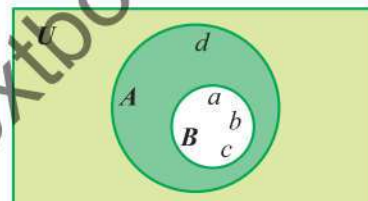
Here, we can see that the set A contains all the elements of the set B . Therefore, $A \supseteq B$ or A is a super set of set B .

There are two types of subset, namely, proper subset and improper subset.



Remember!

- Every set is a subset of itself.
- Empty set is a subset of every set.



Search out

Use the following online video link to explain types of sets: <https://www.youtube.com/watch?v=8dup8yGwBhM>

Proper Subset

A set A is a proper subset of a set B if all the elements of the set A contain in the set B but at least one element of the set B is not an element of set A . It is denoted by $A \subset B$ and read as set A is a proper subset of set B . For example, if $A = \{b, c, d\}$ and $B = \{a, b, c, d, e\}$

Here, we can see that all the elements of the set A contain in the set B but set A does not contain all elements of the set B . So, the set A is a proper subset of the set B or $A \subset B$.

Improper Subset

If set A is a subset of set B and set B is a subset of set A , then A and B are improper subsets of each other. It is denoted by $A \subseteq B$ and read as set A is an improper subset of the set B . For example, if $A = \{0, 2, 4, 6\}$ and $B = \{4, 6, 2, 0\}$, Here, we can see that A and B are subsets of each other because all the elements of the set A are contained in set B and all the elements of the set B are contained in set A . Therefore, A and B are improper subsets of each other.

Example 3

Write three subsets of the set $\{0, 1, 2, 3\}$.

Solution

Three subsets of $\{0, 1, 2, 3\}$ are ϕ , $\{1, 2\}$, $\{0, 1, 3\}$.

Example 4

If $X = \{a, e, i, o, u\}$, then write any four proper subsets and an improper subset.

Solution

Four proper subsets of $\{a, e, i, o, u\}$ are ϕ , $\{a\}$, $\{b\}$ and $\{a, e, i, u\}$

Improper subset of $\{a, e, i, o, u\}$ is $\{a, e, i, o, u\}$.



Note

All the subsets of any set except the set itself are proper subsets. Empty set is a subset of every set. Empty set has no proper subset.



Try yourself

Write the set which has no proper subset? Which set has only one subset? What is the difference between $\{a, b\}$ and $\{\{a, b\}\}$?



Teachers' Guide

Clear the concept between proper subset and improper subset, equivalent and equal sets, disjoint and overlapping sets by using real life objects.

Equivalent Sets

Two sets A and B are said to be equivalent if they have the same number of elements. We write it as $A \leftrightarrow B$. For example, $A = \{a, e, i, o, u\}$ and $B = \{1, 3, 5, 7, 9\}$. We can see that number of elements in set A and set B is 5. So, the sets A and B are equivalent sets or $A \leftrightarrow B$.

**Search out**

[https://www.transum.org/software/sw/starteroftheday/students/venn diagramMatching.asp](https://www.transum.org/software/sw/starteroftheday/students/venn%20diagramMatching.asp)

**Brain Teaser!**

Tick (✓) which are equal sets and cross (✗) which are equivalent sets.

(i) $A = \{\triangle, \circ, \square\}$; $B = \{1, 4, 5\}$ ☐

(ii) $A = \{\text{Amna, Ali}\}$; $B = \{\text{Ali, Amna}\}$ ☐

**Skill Practice**

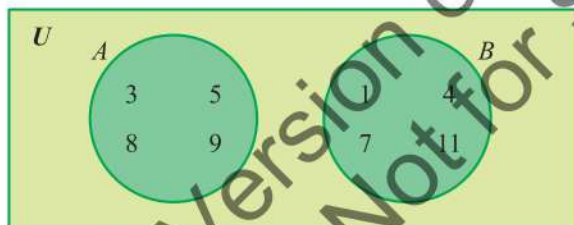
Make at least two pairs of equivalent sets using real life objects.

Equal Sets

If sets A and B contain the same elements, then the sets A and B are equal sets. We write it as $A = B$. e.g., $A = \{2, 3, 5, 7, 9\}$ and $B = \{5, 2, 7, 3, 9\}$. The sets A and B have the same elements. So, the sets A and B are equal sets or $A = B$. Equal sets are always equivalent sets. Equivalent sets may or may not be equal.

Disjoint sets

Two or more sets are disjoint, if they do not have common elements. For example, $A = \{3, 5, 8, 9\}$ and $B = \{1, 4, 7, 11\}$ are disjoint sets because they have no common elements.

**Keep in mind!**

In a set, order of the elements does not matter. It can be changed

e.g., $\{a, e, i, o, u\} = \{o, e, i, a, u\}$

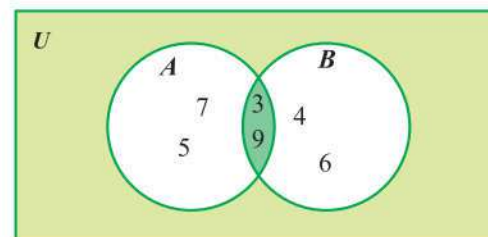
**Skill Practice**

Think any two disjoint sets and write on your notebook.

Overlapping sets

Two or more sets are overlapping sets, if they have at least one common element. For example,

$A = \{3, 5, 7, 9\}$ and $B = \{3, 4, 6, 9\}$ are overlapping sets because they have 3 and 9 common elements.

**EXERCISE 1.8**

1. Write two proper and improper subsets of each of the following sets:

(i) $A = \{a, c, d\}$

(ii) $B = \{2, 4, 6, 8\}$

(iii) $C = \{-1, 0, +1\}$

(iv) $D = \{4, 8, 12, 16\}$

(v) $E = \{\triangle, \circ, \square\}$

(vi) $F = \{\text{Zara, Aslam, Zeeshan}\}$

2. Which is the super set in each of the following pair of sets:

- (i) $A = \{a, c, d\}$, $B = \{a, b, c, d\}$ (ii) $C = \{0, 2, 4, 6, 8\}$, $D = \{2, 4\}$
 (iii) $N = \{1, 2, 3, \dots\}$, $W = \{0, 1, 2, \dots\}$ (iv) $E = \{1, 2, 3, \dots, 10\}$, $F = \{0, \pm 1, \pm 2, \dots, \pm 10\}$
 (v) $L = \{u, v, w, x\}$, $M = \{u, v, w, x, y, z\}$

3. Look at each pair of sets and separate the equal and equivalent sets.

- (i) $A = \{1, 3, 5, 7, 9\}$, $B = \{a, b, c, d, e\}$ (ii) $C = \{0, 2, 4, 6, 8\}$, $D = \{0, 2, 4, 6, 8\}$
 (iii) $E = \{1, 2, 6, 7, 8\}$, $F = \{6, 7, 8, 1, 2\}$ (iv) $G = \{1, 2, 3, \dots, 20\}$, $H = \{1, 2, 3, \dots, 20\}$
 (v) $I = \{\text{Sunday, Monday}\}$, $J = \{\text{Waqar, Ali}\}$

4. Look at each pair of sets and separate the disjoint and overlapping sets.

- (i) $A = \{7, 9, 8, 9\}$, $B = \{a, b, c, d, e\}$ (ii) $C = \{0, 5, 10, 15\}$, $D = \{0, 5, 8, 9, 15\}$
 (iii) $E = \{1, 2, 3, 4, 5, 6\}$, $F = \{1, 3, 2, 5, 6\}$ (iv) $P = \{2, 3, 5, 7, 11\}$, $Q = \{4, 6, 8, 9, 10, 12\}$
 (v) $G = \{x, y, z\}$, $H = \{u, v, w, x, y, z\}$ (vi) $I = \{3, 6, 12\}$, $J = \{\text{January, February, April}\}$

1.3.6 Operations on Sets

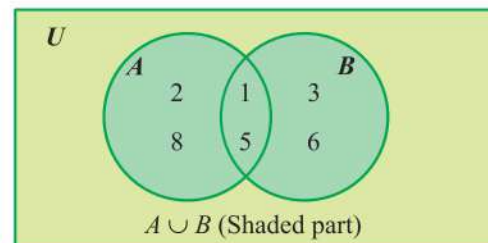
Set operations are the ways to combine sets. They operate on the elements of each set, combining them according to a specific set of rules. The operations are union ($\cup$), intersection ($\cap$) and difference ($-$ or $\setminus$).

i Union of Sets

The union of the sets A and B is the set that contains all elements that are in both sets. The union of two sets is written as $A \cup B$ and read as ' A union B '.

Example 5 If $A = \{1, 2, 5, 8\}$ and $B = \{1, 3, 5, 6\}$, then find the union of sets A and B .

Solution $A \cup B = \{1, 2, 5, 8\} \cup \{1, 3, 5, 6\}$
 $= \{1, 2, 3, 5, 6, 8\}$



Note

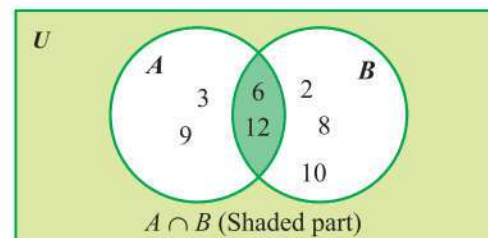
In the union set, all common elements of two sets are written once.

ii Intersections of Sets

The intersection of the sets A and B is the set that contains all common elements of the sets A and B . The intersection of two sets is written as $A \cap B$ and is read as ' A intersection B '.

Example 6 If $A = \{3, 6, 9, 12\}$ and $B = \{2, 6, 8, 10, 12\}$, then find the intersection of sets A and B .

Solution $A \cap B = \{3, 6, 9, 12\} \cap \{2, 6, 8, 10, 12\}$
 $= \{6, 12\}$



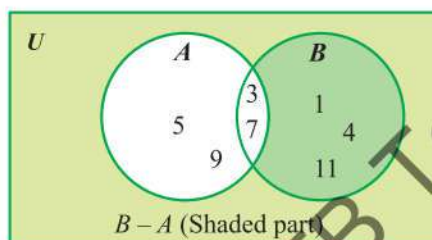
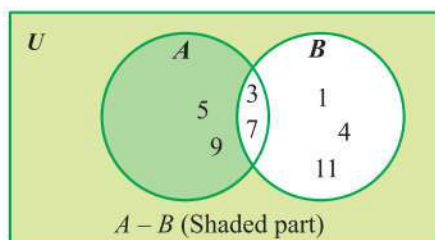
iii Difference of Sets

The difference between the sets A and B is the set that contains all elements of the set A that are not in the set B . It is written as $A - B$ or $A \setminus B$.

Example 7 If $A = \{3, 5, 7, 9\}$ and $B = \{1, 3, 4, 7, 11\}$, then find $A - B$ and $B - A$.

Solution $A - B = \{3, 5, 7, 9\} - \{1, 3, 4, 7, 11\} = \{5, 9\}$.

$B - A = \{1, 3, 4, 7, 11\} - \{3, 5, 7, 9\} = \{1, 4, 11\}$.



Activity

Teacher explain the concept of representing sets using the Venn diagram by this activity. Teacher make 3 sets of hobbies; sports, reading and computer games. Set A is for pupils who like sports, set B is the pupils who like reading and set C is for pupils who like computer games. Teacher write the names of students in each set and construct the Venn diagram on writing board.

iv Universal Set

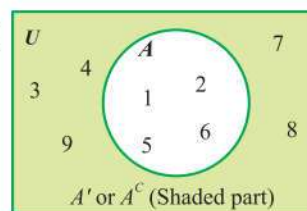
A universal set is a set that contains all elements of sets under consideration and it is denoted by U . For example, $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ is the universal set of the sets $A = \{1, 3, 5, 7\}$ and $A = \{2, 4, 6, 8\}$.

v Complement of a Set

The complement of a set A is defined as a set that contains the elements present in the universal set but not in set A . It is denoted by A' or A^c .

Example 8 If $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ and $A = \{1, 2, 5, 6\}$, then find A^c .

Solution $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ and $A = \{1, 2, 5, 6\}$
 $A^c = U - A$
 $= \{1, 2, 3, 4, 5, 6, 7, 8, 9\} - \{1, 2, 5, 6\}$
 $= \{3, 4, 7, 8, 9\}$



Skill Practice

If $A = \{\text{Monday, Tuesday, Friday}\}$ and $V = \{\text{Tuesday}\}$ then find $A \cap B$. Also draw venn diagram.



Try yourself

Write two sets whose difference is empty set.



Skill Practice

Prove that: $A - B \neq B - A$, If
 $A = \{2, 4, 6, 8, 9\}$ and
 $B = \{2, 8, 11, 12\}$



Keep in mind!

- If the intersection of sets A and B is empty, then the sets A and B are disjoint sets.
- If the intersection of sets A and B is non-empty, then the sets A and B are overlapping sets.



Skill Practice

If $U = \{a, b, c, d, e, f\}$ and $A = \{d, e\}$, then find the complement of the set A .

EXERCISE 1.9

1. Find the union of the following sets:

- (i) $A = \{1, 4, 7, 10, 13\}$, $B = \{2, 3, 5, 8, 9\}$ (ii) $C = \{0, 2, 4, 6, 8, 10\}$, $D = \{0, 1, 2, 3, 4, 6, 8\}$
 (iii) $E = \{a, b, c, d, e\}$, $F = \{a, e, i, o, u\}$ (iv) $G = \{1, 2, 3, \dots, 10\}$, $H = \{0, 1, 2, 3, 4, 6, 8, 9, 10\}$

2 Find the intersection of the following sets:

- (i) $S = \{s, t, u, d, e, n, t\}$, $T = \{t, e, a, c, h, r\}$ (ii) $U = \{0, -1, -2, -3\}$, $V = \{0, 1, 2, 3\}$
 (iii) $W = \{2, 3, 5, 7, 11, 13\}$, $X = \{0, 2, 4, 6, 8, 9, 10\}$ (iv) $Y = \{0, 5, 10, 15, 20\}$, $Z = \{0, 1, 2, \dots\}$

3. If $N = \{1, 2, 3, \dots\}$ and $W = \{0, 1, 2, \dots\}$, then find $N \cup W$ and $N \cap W$.

4. If $E = \{0, 2, 4, \dots\}$ and $O = \{1, 3, 5, \dots\}$, then find $E \cup O$ and $E \cap O$.

5. If $P =$ set of prime numbers and $C =$ set of composite numbers, then find $P \cup C$ and $P \cap C$.

6. If $U = \{1, 2, 3, \dots, 10\}$, $X = \{0, 3, 6, 9\}$, $Y = \{0, 4, 8\}$ and $Z = \{0, 2, 4, 6, 8, 10\}$, then find:

- (i) X^c (ii) Y^c (iii) Z^c (iv) U^c

7. If $X = \{0, 2, 6, 9, 10\}$, $Y = \{1, 2, 3, 4, 5\}$ and $Z = \{1, 3, 5, 7\}$, then find:

- (i) $X \setminus Y$ (ii) $Y \setminus Z$ (iii) $X \setminus Z$ (iv) $Z \setminus Y$

1.3.7 Properties of the Complement of Sets

Properties involving the complement of sets are given below:

- (i) $A \cup A^c = U$ (ii) $A \cap A^c = \phi$ (iii) $(A \cup B)^c = A^c \cap B^c$ (iv) $(A \cap B)^c = A^c \cup B^c$

Example 9

If $U = \{1, 2, 3, \dots, 10\}$, $A = \{2, 4, 6, 8, 10\}$ and $B = \{1, 2, 3, 6, 7, 8, 9\}$, then verify the following properties:

- (i) $A \cup A^c = U$ (ii) $A \cap A^c = \phi$ (iii) $(A \cup B)^c = A^c \cap B^c$ (iv) $(A \cap B)^c = A^c \cup B^c$

Solution

$U = \{1, 2, 3, \dots, 10\}$, $A = \{2, 4, 6, 8, 10\}$ and $B = \{1, 2, 3, 6, 7, 8, 9\}$

First, we find A^c and B^c

$$A^c = U - A$$

$$A^c = \{1, 2, 3, \dots, 10\} - \{2, 4, 6, 8, 10\} = \{1, 3, 5, 7, 9\}$$

$$B^c = U - B$$

$$B^c = \{1, 2, 3, \dots, 10\} - \{1, 2, 3, 6, 7, 8, 9\} = \{4, 5, 9, 10\}$$



Teachers' Guide

Ask the students to choose a random universal set and two subsets and find intersection and complement.



Skill Practice

If $U =$ Set of natural numbers and $A =$ Set of even numbers then prove that:

$$A \cup A^c = U, A \cap A^c = \phi$$



Activity

Explain to students about the complement of sets. Let us say A is a set of all coins which is a subset of a universal set that contains all coins and notes. So, the complement of set A is a set of notes, which does not include coins.

(i) $A \cup A^c = U$

$$\text{L.H.S} = A \cup A^c$$

$$= \{2, 4, 6, 8, 10\} \cup \{1, 3, 5, 7, 9\}$$

$$= \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$$

$$= \{1, 2, 3, \dots, 10\} = U = \text{R.H.S}$$

Hence, L.H.S = R.H.S

(iii) $(A \cup B)^c = A^c \cap B^c$

$$\text{L.H.S} = (A \cup B)^c$$

$$= (\{2, 4, 6, 8, 10\} \cup \{1, 2, 3, 6, 7, 8\})^c$$

$$= (\{1, 2, 3, 4, 6, 7, 8, 10\})^c$$

$$= U - \{1, 2, 3, 4, 6, 7, 8, 10\}$$

$$= \{1, 2, 3, \dots, 10\} - \{1, 2, 3, 4, 6, 7, 8, 10\} = \{5, 9\}$$

$$\text{R.H.S} = A^c \cap B^c$$

$$= \{1, 3, 5, 7, 9\} \cap \{4, 5, 9, 10\} = \{5, 9\}$$

Hence, L.H.S = R.H.S

(ii) $A \cap A^c = \phi$

$$\text{L.H.S} = A \cap A^c$$

$$= \{2, 4, 6, 8, 10\} \cap \{1, 3, 5, 7, 9\}$$

$$= \{\} \text{ or } \phi = \text{R.H.S}$$

Hence, L.H.S = R.H.S

(iv) $(A \cap B)^c = A^c \cup B^c$

$$\text{L.H.S} = (A \cap B)^c$$

$$= (\{2, 4, 6, 8, 10\} \cap \{1, 2, 3, 6, 7, 8, 9\})^c$$

$$= \{2, 6, 8\}^c$$

$$= U - \{2, 6, 8\}$$

$$= \{1, 2, 3, \dots, 10\} - \{2, 6, 8\}$$

$$= \{1, 3, 4, 5, 7, 9, 10\}$$

$$\text{R.H.S} = A^c \cup B^c$$

$$= \{1, 3, 5, 7, 9\} \cup \{4, 5, 9, 10\}$$

$$= \{1, 3, 4, 5, 7, 9, 10\}$$

Hence, L.H.S = R.H.S



History

A British Mathematician Augustus De Morgan (1806 – 1871) introduced De Morgan's law.

$(A \cup B)^c = A^c \cap B^c$ and $(A \cap B)^c = A^c \cup B^c$ are called De Morgan's laws.



Search out

<https://www.w3schools.blog/properties-of-complement-sets>



Skill Practice

If U = Set of the name of months of a year, $A = \{\text{January, June, July}\}$, $B = \{\text{March, July, October, November, December}\}$, then verify:

(i) $(A \cup B)^c = A^c \cap B^c$ (ii) $(A \cap B)^c = A^c \cup B^c$

EXERCISE 1.10

1. If $U = \{0, 1, 2, 3, 4, 5, \dots, 25\}$, $A = \{1, 3, 5, 7, 9, 21, 24\}$, $B = \{5, 10, 15, 20, 25\}$,

$C = \{2, 4, 6, 8, 14, 16, 18, 24\}$ and $D = \{15, 16, 18, 21, 24, 25\}$ then verify the following:

(i) $A \cup A^c = U$ (ii) $A \cap A^c = \phi$ (iii) $B \cup B^c = U$ (iv) $B \cap B^c = \phi$

(v) $C \cup C^c = U$ (vi) $C \cap C^c = \phi$ (vii) $D \cup D^c = U$ (viii) $D \cap D^c = \phi$

2. Verify $(A \cup B)^c = A^c \cap B^c$ and $(A \cap B)^c = A^c \cup B^c$ where $U = \{1, 2, 3, 4, 5, \dots, 20\}$

(i) $A = \{1, 7, 9\}$; $B = \{2, 4, 7, 9, 12, 15\}$

(ii) $A = \{2, 4, 6, 8, 10\}$; $B = \{11, 12, 13, 14, 15, 16\}$

(iii) $A = \{1, 3, 9, 11, 13, 15\}$; $B = \{11, 13, 15, 17\}$

(iv) $A = \{1, 3, 5, 7, 9, 11, 13, 15\}$; $B = \{2, 4, 8, 10, 12, 16, 18, 20\}$

SUMMARY

- A set is a well defined and distinct collection of objects/elements.
- If all the elements of a set A are also the elements of a set B , then the set A is called the subset of the set B .
- If a set A contains all the elements of a set B , then the set A is a super set of the set B .
- A set A is a proper subset of a set B if all the elements of the set A contain in the set B but at least one element of the set B is not an element of set A .
- If set A is a subset of set B and set B is a subset of set A , then A and B are improper subsets of each other.
- Two sets A and B are said to be equivalent, if they have the same number of elements.
- If sets A and B contain the same elements, then the sets A and B are equal.
- Two or more sets are disjoint, if they do not have common elements.
- Two or more sets are overlapping sets, if they have at least one common element.
- Laws of complement:
 - (i) $A \cup A^c = U$ (ii) $A \cap A^c = \phi$ (iii) $(A \cup B)^c = A^c \cap B^c$ (iv) $(A \cap B)^c = A^c \cup B^c$

REVIEW EXERCISE 1(a)

1. Choose the correct option.

- (i) The number that can be expressed in the form of $\frac{p}{q}$, $q \neq 0$ is called:
 (a) mixed number (b) rational number (c) whole number (d) natural number
- (ii) The set of rational number is denoted by:
 (a) W (b) Q (c) N (d) Q'
- (iii) $\frac{1}{3}(18)(-6) =$ _____
 (a) -36 (b) 36 (c) 18 (d) -18
- (iv) $\frac{-(17-17)}{20} =$ _____
 (a) $\frac{-34}{20}$ (b) 0 (c) $\frac{-1}{20}$ (d) $\frac{1}{20}$
- (v) $\frac{-5}{6} + \frac{3}{7} = \frac{3}{7} +$?
 (a) $\frac{1}{2}$ (b) $\frac{6}{7}$ (c) $\frac{5}{6}$ (d) $\frac{-5}{6}$
- (vi) The difference between the actual value and estimated value is called:
 (a) error (b) approximation (c) rounding (d) reasonableness

- (vii) Who introduced BODMAS:
 (a) Achilles Reselfelt (b) Al-Khawarizmi (c) John (d) William
- (viii) In BODMAS rule, "of" stands for:
 (a) addition (b) subtraction (c) division (d) multiplication
- (ix) The set is written in the bracket.
 (a) () (b) { } (c) [] (d) [}
- (x) To write an empty set, we use the symbol:
 (a) $\subset$ (b) ϕ (c) $\cap$ (d) $\{\phi\}$
- (xi) If sets A and B are disjoint sets, then:
 (a) $A \cup B = \phi$ (b) $A \setminus B = B$ (c) $A \cap B = A$ (d) $A \cap B = \phi$
- (xii) If $A = \{1, 2\}$ and $B = \{4, 5\}$, then:
 (a) $A \cup B = \phi$ (b) $A \leftrightarrow B$ (c) $A \cap B = A$ (d) $A = B$
- (xiii) In the Venn diagram, the universal set is represented by:
 (a) rectangle (b) circle (c) square (d) quadrilateral

2. Represent the following on the number line:

- (i) 250, 230, 140, 90, 100, 120 (ii) 50.5, 55.5, 57.6, -55.5, -53.5
 (iii) 2200, 2000, 1800, 2500, 2100, 1500 (iv) -40, -15, -30, -25, -65, -70
 (v) $\frac{1}{5}$ (vi) $\frac{9}{11}$ (vii) $-\frac{2}{5}$ (viii) $-\frac{7}{13}$

3. Compare the given numbers by using symbol $>$ or $<$. Also arrange them in ascending and descending order.

- (i) 326781, 326681, 336281, 336291 (ii) -55451, -55540, -56580, -56508
 (iii) 108.01, 180.08, 111.78, 111.70 (iv) $\frac{7}{10}$, $-\frac{3}{5}$, $-\frac{7}{10}$, $\frac{13}{15}$

4. Convert:

- (i) $1\frac{7}{9}$, $3\frac{1}{5}$, $2\frac{5}{12}$, $8\frac{5}{7}$ into improper fractions
 (ii) $\frac{27}{7}$, $\frac{48}{14}$, $\frac{43}{13}$, $\frac{68}{9}$ into mixed fractions

5. Solve the following rational numbers:

- (i) $\frac{5}{13} + \frac{4}{13}$ (ii) $\frac{13}{8} - \frac{9}{16}$ (iii) $\frac{-7}{11} \times \frac{-3}{21}$ (iv) $\frac{-8}{11} \div \frac{-2}{121}$

6. The product of two numbers is $\frac{77}{56}$. If the first number is $\frac{13}{8}$, then find the other number.

7. Ahmed had to travel $\frac{8}{9}$ km from his house to the library and then to school. The distance between his house and the library was $\frac{1}{3}$ km. How much was the distance between the library and the school?

8. Verify the following properties:

- (i) $\left(\frac{4}{7} + \frac{2}{9}\right) + \frac{3}{5} = \frac{4}{7} + \left(\frac{2}{9} + \frac{3}{5}\right)$ (ii) $\frac{-1}{5} \times \left(\frac{5}{9} + \frac{7}{18}\right) = \left(\frac{-1}{5} \times \frac{5}{9}\right) + \left(\frac{-1}{5} \times \frac{7}{18}\right)$

9. Round the following to the given degree of accuracy:

- (i) 97852 (2 significant figures) (ii) -16.578 (2 significant figures)
 (iii) 0.003757 (3 significant figures) (iv) 0.0657953 (3 significant figures)
 (v) $\frac{113}{8}$ (2 decimal places) (vi) $\frac{27}{83}$ (3 significant figures)

10. Compare your approximated value and accurate value. Also check is the solution reasonable?

- (i) $3.481 \div 1.287$ (2 decimal places) (ii) $10.59 - 7.24$ (2 decimal places)
 (iii) 10.52×5.57 (2 decimal places)

11. Simplify the following:

- (i) $[12 + \{23 \times 75 \div (16 - \overline{22 + 11})\}]$ (ii) $\frac{3}{5} \div \left[\frac{1}{15} \times \left\{ \frac{1}{2} + \left(3\frac{1}{3} \div 2\frac{1}{2} \times 3\frac{7}{16} \right) \right\} \right] \times \frac{1}{3}$
 (iii) $18.3 \div [2.430 + \{3.245 - (2.7 \times \overline{7.06 - 7.02})\}]$

12. State whether each of the following collections is a well defined set. Give a reason for each answer.

- (i) The collection of good students in your class. (ii) The objects that can be worn.
 (iii) Prime numbers between 0 and 10. (iv) The students in your class have one brother.

13. Which of the following are set? Justify your answer.

- (i) The collection of all the months in a year starting with the letter "A".
 (ii) The collection of good stories books.
 (iii) $A = \{s, c, h, o, o, l\}$. (iv) $B = \{b, a, g\}$ (v) $C = \{1, 2, 3, 4, 5\}$

14. Write the following sets in other two forms.

- (i) $A = \{4, 8, 12, 16, 20, 24, 28, 32\}$ (ii) $B = \{x \mid x \in N \wedge 10 \leq x \leq 20\}$
 (iii) $C = \{0, \pm 1, \pm 2, \pm 3, \dots\}$ (iv) $D = \text{Set of all perfect square numbers between 5 and 100.}$
 (v) $F = \{y \mid y \in N \wedge y \leq 15\}$ (vi) $G = \{x \mid x \in W \wedge x < 10\}$

15. If $U = \{0, \pm 1, \pm 2, \dots\}$, then represent the following sets in Venn diagram:

- (i) $X = \{-1, -2, -3, -4\}$, $Y = \{1, 2, 3, 4, 5\}$ (ii) $A = \{-1, 2, -3, 4\}$, $B = \{1, -2, -3, 4, 5\}$

16. Find the union and intersection of the following sets:

- (i) $U = \{-1, -3, -5, -7, -9\}$, $V = \{-1, -2, -3, \dots, -10\}$ (ii) $X = \{-1, -2, -3, -4\}$, $Y = \{1, 2, 3, 4, 5\}$

17. If $Y = \{a, b, c, \dots, z\}$, $Z = \{a, e, i, o, u\}$, then find: (i) $Y - Z$ (ii) $Z \setminus Y$

18. If $U = \{21, 22, 23, 24, \dots, 40\}$, $A = \{21, 23, 24, 26, 27\}$ and $B = \{25, 26, 27, 28, 29, 30, 35, 36\}$ then verify the following:

- (i) $A \cup A^c = U$ (ii) $A \cap A^c = \phi$ (iii) $B \cup B^c = U$ (iv) $B \cap B^c = \phi$

19. Verify $(A \cup B)^c - (A^c \cap B^c) = \phi$ and $(A \cap B)^c - (A^c \cup B^c) = \phi$ where $U = \{0, 1, 2, 3, 4, 5, \dots, 18\}$

- (i) $A = \{2, 4, 6, 7, 12, 14, 15, 17\}$; $B = \{3, 9, 13, 16, 18\}$
 (ii) $A = \{1, 2, 3, 4, 5, 10, 12, 14\}$; $B = \{5, 10, 15\}$

Sub-domain

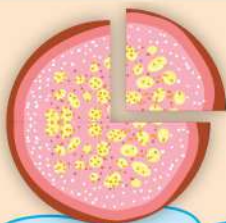
(iv)

Rate, Ratio and Proportion

Students' Learning Outcomes

After studying this sub-domain, students will be able to:

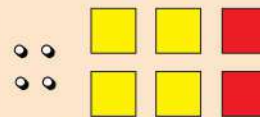
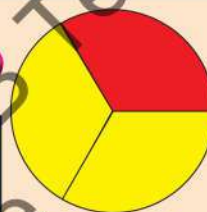
- Calculate rate and average rate of quantities.
- Calculate increase and decrease in a ratio based on change in quantities.
- Explain and calculate direct and inverse proportion and solve real-world word problems related to direct and inverse proportion.



1 : 4 (Ratio Form)
 $\frac{1}{4}$ (Fraction Form)
 1 is to 4 (Word Form)

THINK!

Are the ratio of coloured part of circle and the coloured parts of square are in proportion?

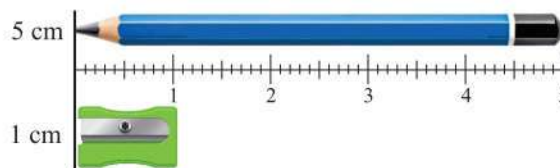


$$\frac{1}{3} :: \frac{2}{6}$$

Here, we can compare the first ratio to the second ratio.

1.4.1 Ratio

In our daily life, there are many situations in which we compare two quantities. For example, The length of pencil is 5 times of the length of sharpener. In other words, we can also say that the length of sharpener is $\frac{1}{5}$ th of the length of pencil.



The length of pencil = 5 (Length of sharpener) OR The length of sharpener = $\frac{1}{5}$ (Length of pencil)
 We can also compare the above objects by using ratio.

In the given case, we write the ratio of the length of sharpener to the length of pencil as:

Length of sharpener : Length of pencil
 1 : 5

Or the ratio of the length of pencil to the length of sharpener is as:

Length of pencil : Length of sharpener
 5 : 1



Remember!

Ratio is changed when we change the order of quantities e.g., 1 : 5 and 5 : 1 are different from each other.

The first term in the ratio is known as antecedent and the second term is known as consequent. e.g., In $5 : 1$ ($5 =$ antecedent and $1 =$ consequent). To compare two quantities in the ratio, the units of the quantities must be same. A ratio has no units. Ratio is a dimension less number or quantity.

**Skill Practice**

Find ratio of:

- 5 km to 950 m
- 2 kg to 800 g
- 7 weeks to 21 days

**Keep in mind!**

When the ratio is multiplied or divided by the same (non zero) number, then the ratio is not changed.

**Remember!**

In general, the ratio of a to b is written as:

$$a : b \text{ or } \frac{a}{b}; (b \neq 0)$$

Example 1

The price of a mobile is Rs. 21000 and the price of a LED is Rs. 42000. Find the ratio of the price of mobile to LED and LED to price of mobile.

Solution

The ratio of the price of mobile to the price of LED is written as:

$$21000 : 42000$$

After simplifying to the lowest form:

$$3 : 6$$

$$1 : 2$$

The ratio of the price of LED to the price of mobile is written as:

$$42000 : 21000$$

$$6 : 3$$

$$2 : 1$$

Example 2

If $a : b = 3 : 5$ and $b : c = 6 : 7$, then find $a : c$

Solution

$$a : b = 3 : 5$$

$$\frac{a}{b} = \frac{3}{5}$$

$$b : c = 6 : 7$$

$$\frac{b}{c} = \frac{6}{7}$$

$$\begin{aligned} \text{Now, } \frac{a}{c} &= \frac{a}{b} \times \frac{b}{c} \\ &= \frac{3}{5} \times \frac{6}{7} = \frac{18}{35} \end{aligned}$$

$$a : c = 18 : 35$$

**Skill Practice**

Can you tell the price of LED how many times greater than the price of mobile?

**Challenge**

Find ratio of $\frac{1}{5}$ to $\frac{3}{15}$

**Skill Practice**

If $x : y = 10 : 8$ and $y : z = 5 : 9$, then find $x : z$.

i

Increase and Decrease in Ratio

Sara went to the market. She observed that the price of 1 kg rice was Rs. 120 but now the price of 1 kg rice is Rs. 180 of the same kind while purchasing.



Can you tell how much price is increased in ratio? In such situation, increase and decrease in ratio play significant role to solve this problem.



(a) Increase in Ratio

Example 3

If the height of Umer is increased from 40 inches to 65 inches in 13 years, then find the ratio of increased height.

Solution

The required ratio = New height : Old height
 $= 65 : 40$
 $= 13 : 8$

Thus, the height of Umer increased in the ratio of 13 : 8.

Example 4

Increase 1200 in the ratio of 8 : 5

Solution

First of all convert the ratio into a fraction, then multiply it by 1200.

$$\begin{aligned} &= \frac{8}{1.5} \times 1200 \\ &= 1920 \end{aligned}$$

Hence, we can say that increase of 1200 in the ratio of 8 : 5 is 1920.

(b) Decrease in Ratio

Example 5

If the price of wheat is decreased from Rs. 180 to 150, then find the ratio of decreased price.

Solution

The required ratio can be calculated by this method.

The required ratio = New value : Old value
 $= 150 : 180$
 $= 5 : 6$

Thus, price of wheat decrease in the ratio of 5 : 6.

40 inches 65 inches

Skill Practice

If old price of an object is Rs. 800 and new price is Rs. 1100. Can you find out the ratio of increased price?

Keep in mind!

The first term of the ratio (antecedent) is always greater than consequent in increase in ratio.

Keep in mind!

The first term of the ratio (antecedent) is always less than consequent in decrease in ratio.



Skill Practice

- The length of the ribbon was originally 30 cm. It was reduced in the ratio of 3 : 5. What is its length now?
- If old price of an item was Rs. 750 and new price of the item is Rs. 550. Find out the ratio of decreased prices.

Skill Practice

Decrease: • 2 litres in the ratio of 3 : 5 • 500 days in the ratio of 7 : 15

Example 6 Decrease 800 in the ratio of 7 : 10.

Solution First convert 7 : 10 into fraction and multiply it by 800.

$$= \frac{7}{10} \times 800 = 560$$

Hence, we can say that decrease in the ratio of 7 : 10 is 560.



Remember!

In increasing ratio, the numerator is greater than denominator. In other words, the fraction is always greater than 1.
In decreasing ratio, the numerator is less than the denominator. In other words, the fraction is less than 1.

EXERCISE 1.11

1. Write the following in the simplest form of ratio:

- | | | |
|-----------------------------------|-----------------------------------|-------------------------------------|
| (i) 15 : 75 | (ii) $\frac{2}{15} : \frac{3}{5}$ | (iii) $\frac{1}{16} : \frac{25}{8}$ |
| (iv) $\frac{2}{15} : \frac{3}{5}$ | (v) 5 kg and 500 g | (vi) 2 km and 700 m |
| (vii) 8 weeks and 14 days | (viii) 240 minutes and 2 hours | |
| (ix) 180 days and 1 month | (x) 5 years and 24 months. | |

2. Find increase:

- | | | |
|------------------------------|-----------------------------|-------------------------------|
| (i) 40 in the ratio 5 : 4 | (ii) 36 in the ratio 8 : 3 | (iii) 175 in the ratio 11 : 5 |
| (iv) 48 in the ratio 17 : 12 | (v) 360 in the ratio 14 : 9 | (vi) 425 in the ratio 7 : 5 |

3. Find decrease:

- | | | |
|----------------------------|----------------------------|------------------------------|
| (i) 30 in the ratio 2 : 3 | (ii) 35 in the ratio 5 : 7 | (iii) 50 in the ratio 2 : 5 |
| (iv) 70 in the ratio 2 : 7 | (v) 160 in the ratio 5 : 8 | (vi) 420 in the ratio 7 : 15 |

- If old price was Rs. 990 and new price is Rs. 1200, then find the ratio of increased price.
- If old price was Rs. 1500 and new price is Rs. 1300, then find the ratio of decreased prices.
- If new mass of Raza is 80 kg and old mass was 120 kg, then find the ratio of decreased masses.
- The price of cotton box has increased in ratio of 7 : 6. If the old price of the cotton box was Rs. 1836, then find the new price of cotton box.
- The height of Ali increased in ratio of 11 : 6 and the height of Ali was 1220 cm, then what will be the new height of Ali?
- Mohsin's mass was 140 kilograms. If he reduces his mass in the ratio of 6 : 7, find his new mass.
- Ahmad mass was 140 kilograms and reduced his mass to 112 kilograms. In what ratio did Ahmad reduce his mass?

1.4.2 Rate and Average Rate

i Rate

We have already discussed application of rate in our daily life in previous grade. We compare two quantities with different units. e.g.,



The speed of tiger is 60 km/hour

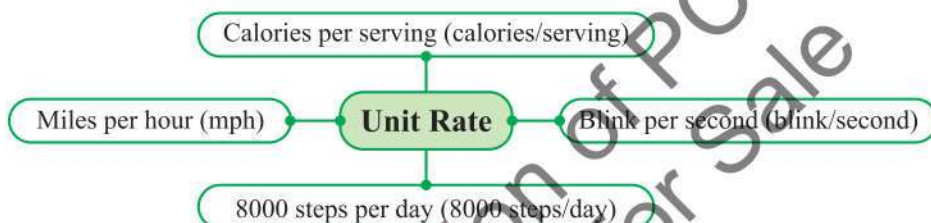


The speed of a bus is 80 km/hour

We normally read these as:

- (i) the speed of tiger is 60 kilometres per hour
- (ii) the speed of bus is 80 kilometres per hour

All the above examples are of unit rate.



Remember!

Rate is a ratio of two quantities which have different units.



Important Information

Unit rate is a rate that is reduced to 1 unit. In other words, the 2nd term is always 1.



Search out

Following Online game links can be shared with students for practice of unit rate:

https://www.brainpop.com/games/unitrates/?topic_id=

ii Average Rate

Average rate is a rate in which an object changes its rate according to the period of time. Let us consider some examples.

Example 7 Suppose a truck travels 90 km in the first hour and 80 km in the second hour. What is average speed of the truck?

Solution

Total distance covered by the truck = $90 + 80 = 170$ km

Total time taken by the truck = $1 + 1 = 2$ hours

$$\text{Average speed} = \frac{\text{Total distance covered}}{\text{Total time taken}}$$

$$= \frac{170 \text{ km}}{2 \text{ h}}$$

$$= 85 \text{ km/h}$$

Hence, the average speed of the truck is 85 km/h.



Example 8

A bus is to cover 630 km. It covers first 180 km of the journey at speed of 60 km/h and covers next 450 km at speed of 90 km/h. What is its average speed?

Solution

For first 180 km

As, we know that:

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

$$60 = \frac{180}{\text{Time}}$$

$$\text{Time} = \frac{180}{60}$$

$$\text{Time} = 3 \text{ hours}$$

For next 450 km

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

$$90 = \frac{450}{\text{Time}}$$

$$\text{Time} = \frac{450}{90}$$

$$\text{Time} = 5 \text{ hours}$$



Total distance covered by the bus = 630 km

Total time taken by the bus = 3 + 5 = 8 hours

$$\text{Average speed} = \frac{\text{Total distance covered}}{\text{Total time taken}}$$

$$= \frac{630 \text{ km}}{8 \text{ hours}}$$

$$= 78.75 \text{ km/h}$$

Hence, the average speed of the bus is 78.75 km/h.

EXERCISE 1.12

- Hassan earns Rs. 3500 in 5 days. What is his pay for one day?
- Zain bought one dozen oranges for Rs. 240. What is the price of one orange?
- A car covers 220 km in 11 litres. Find the rate of kilometre per litre. If car travels 400 km, then what will be the fuel consumption in litres?
- Moeen cooked 10 dishes in 2 hours. What will be the speed of cooking dishes per hour? How much dishes will he cook in 8 hours?
- The cost of 25 units of electricity is Rs. 375. Find the cost of 75 units of electricity.
- Hamza covers the distance of 225 km in 3 hours by car. At this rate, how far can he drive in 8 hours?
- The cost of 20 kg apples is Rs. 4000. What is the cost of 25 kg apples?
- In the first cricket match, Hamza made 52 runs in 6 overs and in the second cricket match he made 68 runs in 8 overs. What is the average run rate per over?

9. Atif sold 40 pens in 15 minutes on Monday and 95 pens in 30 minutes on Tuesday. What is the average sale of pens per minute?
10. A train is to cover 1180 km. If it covers first 550 km at the speed of 100 km/hour and the remaining distance at the speed of 90 km/hour. Find the average speed of the train.

1.4.3 Proportion



Let us consider the given situation.



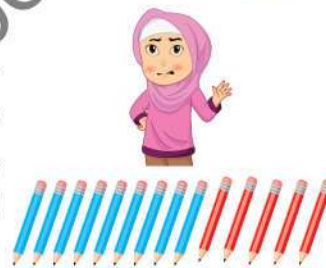
Zainab has 10 red pencils



Tabassum has 18 blue pencils



They want to distribute these pencils between themselves. Zainab gave 5 red pencils to her sister Tabassum. Tabassum gave 9 blue pencils to Zainab. Tabassum was not happy. She felt that she gave more blue pencils to Zainab than the red pencils gave by Zainab to her.



What do you think? Is Tabassum right?

To solve this problem, she went to her mother. Tabassum's mother explained that Zainab gave 5 red pencils out of 10 red pencils to Tabassum. The ratio is 5 : 10 or 1 : 2.

Tabassum gave 9 blue pencils out of 18 blue pencils to Zainab. The ratio is 9 : 18 or 1 : 2.

As, both the ratios are same, so the distribution is fair.

If two ratios are equal, then they are in proportion.



A proportion is a mathematical comparison between two ratios. In other words "The equality of two ratios originates proportion. It is denoted by "::"



Skill Practice

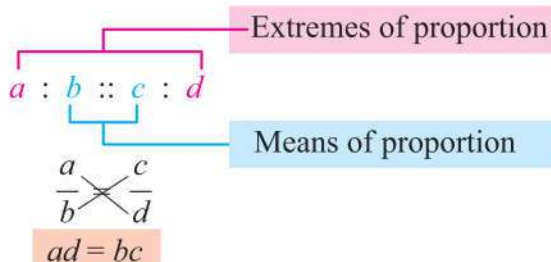
Whether the given ratios are in proportion. If "yes" then write them in proportion form.

(i) 6, 8, 9, 12

(ii) 5, 4, 10, 4

In the above example, we say 5, 10, 9 and 18 are in proportion which is written as: $5 : 10 :: 9 : 18$

If $a : b$ and $c : d$ are two ratios, then its proportion is written as:



- "a" is called the 1st element/term of proportion.
- "b" is called the 2nd element/term of proportion.
- "c" is called the 3rd element/term of proportion.
- "d" is called the 4th element/term of proportion.

**Skill Practice**

Separate means and extremes from the following:

- (i) $5 : 10 :: 25 : 50$ (ii) $6 : 48 :: 10 : 80$

**Remember!**

In proportion,

Product of extremes = Product of means

Example 9

Whether the following are in proportion?

(i) $16 : 10 :: 32 : 20$

(ii) $4 : 8 :: 10 : 2$

Solution

(i) $16 : 10 :: 32 : 20$

$$(16)(20) = (10)(32)$$

$$320 = 320$$

As, Product of means = Product of extremes
Hence, these ratios are in proportion.

(ii) $4 : 8 :: 10 : 2$

$$(4)(2) = (8)(10)$$

$$8 \neq 80$$

As, Product of means $\neq$ Product of extremes
Hence, these ratios are not in proportion.

Example 10

Find the value of the x

$$12 : x :: 3 : 6$$

Solution

As, Product of extremes = Product of means

$$(12)(6) = 3x$$

$$3x = 72$$

$$x = \frac{72}{3}$$

$$x = 24$$

**Skill Practice**

Find the 3rd term in the given proportion:

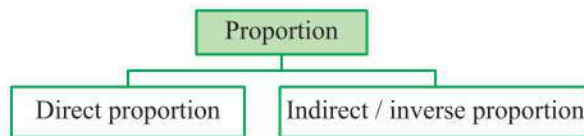
$$12 : 32 :: x : 8$$

Kinds of Proportion

There are two kinds of proportion.

(a) Direct Proportion

Two quantities are said to be in direct proportion if increase or decrease in one quantity causes the increase or decrease in the other quantity in the same ratio. The concept of direct proportion is widely used in our daily life. e.g., as, the quantity of books is increased, the price of books will also be increased. As, the quantity of envelopes is increased, the cost of envelopes will also be increased.

**Example 11**

If the cost of 15 pens is Rs. 150, then find the cost of 20 such pens.

Solution

Let the cost of 20 pens = x

Cost of pens (Rs.) : Number of pens

$$\begin{array}{ccc} 150 & : & 15 \\ x & : & 20 \end{array}$$

**Teachers' Guide**

Explain to the students that a change in the proportion of one quantity means a change in the proportion of the other. For example, when you buy more apples, you will have to pay more money.

As, if the number of pens increased, then the price will also be increased.

Therefore, it is a direct proportion

$$\begin{aligned}\frac{150}{x} &= \frac{15}{20} \\ (15)(x) &= (150)(20) \\ (15x) &= 3000 \\ x &= \frac{3000}{15} = 200\end{aligned}$$

So, the cost of 20 such pens is Rs. 200.

By Using Proportion Method

$$\begin{array}{ccccccc} \text{Rs.} & : & \text{Rs.} & :: & \text{No. of pens} & : & \text{No. of pens} \\ 150 & : & x & :: & 15 & : & 20 \end{array}$$

In proportion, Product of means = Product of extremes

$$\begin{aligned}(x)(15) &= (150)(20) \\ x &= \frac{3000}{15} = 200\end{aligned}$$

(b) Indirect / Inverse Proportion

Two quantities are said to be indirect proportion if the increase or decrease in one quantity causes the decrease or increase in the other quantity in the same ratio.

This concept is also widely used in our daily life. e.g.,

- If the speed of car is increased, then the travelling time will be decreased.
- If the number of workers is increased, then less time will be required to complete the same task.

An indirect proportion is written as:

$$a : b :: \frac{1}{c} : \frac{1}{d}$$

$$a : b :: \frac{1}{c}cd : \frac{1}{d}cd$$

$$a : b :: d : c$$

Example 12

If 10 men do a work in 24 days. In how many days 15 men can do the same task?

Solution

Let the required number of days = x

No. of days	:	No. of men
24	:	10
x	:	15

As, the number of men is increased then less time will be required to complete the same task.

So, it is an indirect proportion.

$$\begin{aligned}\frac{24}{x} &= \frac{15}{10} \\ (x)(15) &= (24)(10) \\ (15x) &= 240 \\ x &= 16\end{aligned}$$

Thus, 15 men will complete the same task in 16 days.



Skill Practice

Solve the example by using proportion method.

EXERCISE 1.13

1. Which of the following are in proportion?

(i) $7 : 2 :: 35 : 10$

(ii) $2 : 6 :: 5 : 20$

(iii) $16 : 12 :: 4 : 3$

(iv) $10 : 8 :: 70 : 56$

(v) $15 : 50 :: 9 : 30$

(vi) $20 : 4 :: 3 : 40$

2. Find the value of variable in the following proportions.

(i) $7 : 6 :: x : 50$

(ii) $y : 8 :: 45 : 20$

(iii) $z : 9 :: 15 : 35$

(iv) $w : 2 :: 24 : 18$

3. If 7 kg potatoes cost 140 rupees, how much we pay for 12 kg potatoes?

4. For 10 horses, 12 kg 400 g of food is required daily. In the same proportion, how much will be needed for 16 horses?

5. The cost of 16 quintals of bean is 40,400 rupees. How much will 4 quintals cost?

6. If a box of sweets is divided among 24 children, they will get 5 sweets each. How many each will get if the number of children is reduced by 4?

7. If 2 men take 10 days to cut a tree, how many days will take 4 men to do the same job?

8. If 1 person takes 10 days to pick the apples from a garden, how many days will take 5 people to pick the apples from a garden?

9. If 1 person takes 7 days to clean a house from bugs, how many days will take 7 people take to clean the house from bugs?

SUMMARY

- The rate is a ratio of two quantities having different units.
- An equality of two ratios is called a proportion.
- Two quantities are said to be in direct proportion if increase or decrease in one quantity causes the increase or decrease in the other quantity in the same ratio.
- Two quantities are said to be indirect proportion if the increase or decrease in one quantity causes the decrease or increase in the other quantity in the same ratio.

Sub-domain

(v)

Financial Arithmetic

Students' Learning Outcomes

After studying this sub-domain, students will be able to:

- Identify and differentiate between selling price, cost price, loss, discount, profit percentage and loss percentage.
- Explain income tax, property tax, general sales tax, value-added tax, zakat and ushr.
- Solve real world word problems involving profit, loss, discount, commission, tax, zakat and ushr.



INTRODUCTION

In our daily life, we are familiar with the words profit and loss. Mostly, we use these words in business and while selling and purchasing the objects or items.



Zain purchased two laptops of Rs. 65000 each. He sold one laptop for Rs. 66000 and other for Rs. 63000. How can we find out that Zain is in profit or loss?



Remember!

If the S.P. is greater than the cost price, then Zain will be in profit.

If the S.P. is less than the cost price, then Zain will be in loss.

1.5.1 Selling Price

The price at which an item is sold is called its selling price. It is denoted by S.P. e.g., the selling price of the first laptop is Rs. 66000. The selling price of the second laptop is Rs. 63000.

1.5.2 Cost Price

The price at which an item is bought is called its cost price. It is denoted by C.P. e.g., the cost price of a laptop is Rs. 65000



Activity

Write C.P. and S.P. of any object on the writing board. Call a student in front of writing board and ask the student to find profit or loss.

1.5.3 Profit

The cost price of a laptop is Rs. 65000 and the selling price of the first laptop is Rs. 66000.

As, while selling the first laptop, selling price (S.P.) is greater than cost price (C.P.)

Zain is in profit while selling the first laptop. The profit of Zain can be calculated by their difference.

$$\text{Profit} = \text{Selling price} - \text{Cost price}$$

$$= \text{S.P.} - \text{C.P.}$$

$$= 66000 - 65000$$

$$\text{Profit} = \text{Rs. } 1000$$

So, the profit of Zain is Rs. 1000



Keep in mind!

In profit, S.P. > C.P.

1.5.4 Loss

The cost price of the other laptop is also Rs. 65000, and the selling price of the second laptop is Rs. 63000. As, while selling the second laptop, cost price is greater than selling price.

So, Zain is in loss while selling the second laptop. The loss of Zain can also be calculated by their differences.

$$\text{Loss} = \text{Cost price} - \text{Selling price}$$

$$= \text{C.P.} - \text{S.P.}$$

$$= 65000 - 63000$$

$$\text{Loss} = \text{Rs. } 2000$$

So, the loss of Zain is Rs. 2000



Keep in mind!

In loss, S.P. < C.P.

Hence, Zain is in profit while selling the first laptop and he is in loss while selling the second laptop.

1.5.5 Profit Percentage and Loss Percentage

Recall

Percentage is a mixture of two words i.e., “per” and “centum”. It means “out of hundred”. It is denoted by %. We can say that 80% flower is coloured.



$$\text{Percentage of coloured flower} = \frac{4}{5} \times 100 = 80\%$$

$$\text{Percentage of uncoloured flower} = \frac{1}{5} \times 100 = 20\%$$



Skill Practice

Solve the following:

(i) $\frac{15}{20} \times 100 =$

(ii) $\frac{200}{300} \times 100 =$

(iii) 5% of 50 kg =

(iv) 10% of 100 ₹ =

Profit percentage and loss percentage are widely used in comparison of objects. For example,



A shopkeeper purchases a bag for Rs. 500 and sells it for Rs. 550. He buys a book for Rs. 250 and sells it for Rs. 300. On which product does the shopkeeper get more profit?



To solve this problem, we will find the profit percentage of both the items. Which item has greater profit percentage, the shopkeeper will get more profit on that item.



Skill Practice

A shopkeeper bought 200 kg onion and 180 kg potatoes. He found 5% of onion and 10% of potatoes were rotten. How many vegetables are in good condition?



Teachers' Guide

Ask the students to write some examples on their notebooks regarding selling price and cost price of any object and calculate profit or loss of these objects.

i Profit Percentage

The profit percentage is calculated by the formula:

$$\text{Profit \%} = \frac{\text{Profit}}{\text{Cost price}} \times 100\%$$

$$\begin{aligned}\text{Profit on bag} &= \text{S.P.} - \text{C.P.} \\ &= 550 - 500 = \text{Rs. } 50\end{aligned}$$

$$\text{Profit \% on bag} = \frac{50}{500} \times 100 = 10\%$$

$$\begin{aligned}\text{Profit on book} &= \text{S.P.} - \text{C.P.} \\ &= 300 - 250 = \text{Rs. } 50\end{aligned}$$

$$\text{Profit \% on book} = \frac{50}{250} \times 100 = 20\%$$

As, the profit percentage of book is greater than the bag. So, the shopkeeper gets more profit on book.

Example 1

A women purchased a dress for Rs. 4300. She sold it at a profit of 5%. Find the selling price of the dress.

Solution

The cost price of a dress = Rs. 4300

$$\begin{aligned}\text{Profit} &= 5\% \text{ of Rs. } 4300 \\ &= \frac{5}{100} \times 4300 = \text{Rs. } 215\end{aligned}$$

$$\begin{aligned}\text{Selling price} &= \text{C.P.} + \text{Profit} \\ &= 4300 + 215 = \text{Rs. } 4515\end{aligned}$$

So, the selling price of the dress is Rs. 4515.

ii Loss Percentage

The loss percentage is calculated by the formula:

$$\text{Loss \%} = \frac{\text{Loss}}{\text{Cost price}} \times 100\%$$



Saleem bought a watch for Rs. 1500. After few months he sold his watch for Rs. 1200. Find his loss percentage.

The cost price of the watch (C.P.) = Rs. 1500

The selling price of watch (S.P.) = Rs. 1200

$$\begin{aligned}\text{Loss} &= \text{Cost price} - \text{Selling price} \\ &= 1500 - 1200 = \text{Rs. } 300\end{aligned}$$

$$\begin{aligned}\text{Loss \%} &= \frac{\text{Loss}}{\text{Cost price}} \times 100\% \\ &= \frac{300}{1500} \times 100\% = 20\%\end{aligned}$$

Hence, the loss percentage of the watch is 20%.



Keep in mind!

The profit in terms of percentage is called profit percentage.



Skill Practice

A LED was bought for Rs. 67000 and sold at a profit of 10%. Find the selling price of the LED.



Skill Practice

In a furniture shop, 15 chairs were bought at the rate of Rs. 280 per chair. The shopkeeper sold 10 of them at rate of Rs. 400 per chair and the remaining at the rate of Rs. 250 per chair. Find his profit or loss percentage.



1.5.6 Discount

The reduction or cut offered on the marked price is called discount.



Zaheer went to an electronic shop and bought an oven. The marked price of the oven was Rs. 14000. The shopkeeper sold the oven for Rs. 12500. How much discount did the shopkeeper give to Zaheer? To solve this problem, we will find out the difference of marked price and selling price.



$$\begin{aligned}\text{Discount} &= \text{Marked price} - \text{Selling price} \\ &= 14000 - 12500 = \text{Rs. } 1500\end{aligned}$$

So, the shopkeeper gave the discount of Rs. 1500 to Zaheer.



Can you find out how much discount did the shopkeeper give to Zaheer in percentage?

Remember! The difference between the marked price and selling price is known as discount. In discount, selling price is always less than the marked price.

To answer the given question, we will have to find out the discount percentage.

i Discount Percentage

$$\begin{aligned}\text{Discount percentage} &= \frac{\text{Discount}}{\text{Marked price}} \times 100\% \\ &= \frac{1500}{14000} \times 100\%\end{aligned}$$

$$\text{Discount \%} = 10.71\%$$

Hence, the shopkeeper gave 10.71% discount to Zaheer.

Example 2 The marked price of an electric heater is Rs. 4800. A discount of 15% is announced on sale. What is the amount of discount and its selling price?

Solution Marked price of an electric heater = Rs. 4800

$$\begin{aligned}\text{Discount} &= 15\% \text{ of Rs. } 4800 \\ &= \frac{15}{100} \times 4800 = \text{Rs. } 720\end{aligned}$$

$$\begin{aligned}\text{Selling price of the electric heater} &= \text{Marked price} - \text{Discount amount} \\ &= 4800 - 720 = \text{Rs. } 4080\end{aligned}$$

Hence, the amount of discount is Rs. 720 and its selling price is Rs. 4080.



Skill Practice



Can you find the selling price of the shirt?



Skill Practice

Aslam purchased the following items whose marked price and discount % are given below:

Item	Quantity	Marked price	Discount %	Selling Price
Oven	1	4700	5%	
Iron	1	2400	10%	
Food factory	1	8500	17%	
Washing machine	2	12000	12%	

Find the total amount of the bill he has to pay.

EXERCISE 1.14

1. Fill in the boxes.

- (i) Cost Price = 450, Selling Price = 560, Profit =
- (ii) Cost Price = 1230, Selling Price = 1180, Loss =
- (iii) Marked Price = 4550, Selling Price = 3950, Discount =

2. A bakery sold 1 kg of sweets for Rs. 1250. 1kg of sweets costs Rs. 1100 to the owner. Did the owner make a profit or loss? Calculate the amount of profit or loss.
3. Majid bought a book for Rs. 2500 from a bookstore. The bookstore bought this book for Rs. 2200. Did the bookstore make a profit or loss? Calculate the amount of profit or loss.
4. The Price of one bicycle is Rs. 7000. Adnan bought two bicycles and sold them for Rs. 13000. Calculate the amount of profit or loss.
5. Hamza bought a sack of rice for Rs. 12000. Find selling price if he got a profit of 4%.
6. Atif purchased a sofa set for Rs. 48000. Find selling price if he bear a loss of 10%.
7. A shopkeeper bought an aquarium for Rs. 7000 and sold it for Rs. 10000. Find his profit percentage.
8. Samar bought an old motorbike for Rs. 35000 and spent Rs. 15000 on its repairing. After few months he sold this bike for Rs. 60000. Find his profit percentage.
9. Salman bought a sofa set for Rs. 25000 and sold it for Rs. 20000. Find his loss percentage.
10. The price of a cycle is Rs. 12000 in market. Ahmad ordered this online and got this cycle for Rs. 13000. Find his loss percentage.
11. If the marked price of an item is Rs. 7690 and selling price is Rs. 7000. Find the discount offered on this item.
12. Ramaisa sells an item having marked price Rs. 370. Find its selling price if the profit is 15%.
13. Minahil bought a dress for Rs. 3200. If the loss is 8%, then find the selling price of the dress.
14. Find marked price of a LED at a discount of 13% having selling price Rs. 39000.
15. Eman purchases a heater for Rs. 4600 and sells it for Rs. 4500. She buys an oven for Rs. 14500 and sells it for Rs. 14000. On which item does Eman bear less loss?
16. The marked price of a computer table is Rs. 6700. It is sold at a discount of 12%. Find the selling price of the computer table.

1.5.7 Tax

In the 14th century, the word "tax" is appeared first time in English language. It is derived from a Latin word taxare and its meaning is "to assess".

A tax is an amount that government imposes on public to give them facilities, like education, health, security, justice, roads, electricity etc.

Tax is the most important source of government income. Some taxes are paid directly and some indirectly by the public.



Need to Know!

In Pakistan, Federal Board of Revenue (FBR) is a main department which is responsible for tax collection.



i Direct Tax

A tax in which the tax payer pays directly to the government. For example income tax, property tax, etc. Direct tax is different for everyone.

ii Indirect Tax

In such taxes, taxes are charged on goods and services or on commodities. Indirect tax is same for everyone.

The following different types of taxes, we will discuss and learn one by one.

(a) Income Tax

Income tax is the tax imposed by the government on the income of individuals exceeding a certain amount. It means Federal Government fixes a certain limit of income above which tax has to be paid by the individual. Federal Government announces the rates of income taxes annually. The rates of income tax for financial year 2022 – 2023 are given below:

Sr. #	Taxable Income	Income Tax
1	Taxable income exceeds Rs. 600,000 but does not exceed Rs. 1,200,000	2.5% of the amount exceeding Rs. 600,000
2	Taxable income exceeds Rs. 1,200,000 but does not exceed Rs. 2,400,000	Rs. 15,000 + 12.5% of the amount exceeding Rs. 1,200,000
3	Taxable income exceeds Rs. 2,400,000 but does not exceed Rs. 3,600,000	Rs 165,000 + 20% of the amount exceeding Rs. 2,400,000
4	Taxable income exceeds Rs. 3,600,000 but does not exceed Rs. 6,000,000	Rs. 405,000 + 25% of the amount exceeding Rs. 3,600,000
5	Taxable income exceeds Rs. 6,000,000 but does not exceed Rs. 12,000,000.	Tax rate Rs. 1,005,000 + 32.5% of the amount exceeding Rs. 6,000,000
6	Taxable income exceeds Rs. 12,000,000	Rs. 2,955,000 + 35% of the amount exceeding Rs. 12,000,000



Think!

On which commodities, indirect tax is paid by us? Give at least five examples.



Remember!

Exempt Income: The amount which is exempted from income tax is called exempt income.

Taxable Income: The income above a certain limit for which tax has to be paid is called the taxable income.



Think!

How much amount did your parents pay as income tax in the year 2022?



Keep in mind!

For paying tax, financial year starts from the 1st July and ends on 30th June of next year.

Example 3 Ahmad earns Rs. 80000 per month. Calculate the income tax on Ahmad's annual income.

Solution

Ahmad's monthly income = Rs. 80000

Ahmad's annual income = 80000×12

= Rs. 960000

Exempted amount = Rs. 600000

According to the given income slab

Taxable income = Gross Income – Exempted amount

Taxable income = $960000 - 600000$

Taxable income = Rs. 360000

Rate of income tax = 2.5%

According to the given taxable income slab

$$\text{Income tax} = \frac{25}{1000} \times 360000$$

= Rs. 9000

Therefore, Ahmad has to pay Rs. 9000 as income tax at the end of the financial year.

(b) Property Tax

Property Tax is a tax imposed by Government on the properties such as house, land and shops. Government imposes property tax on the annual value of a property. The amount of this tax depends on the location of the property and varies from location to location. The value of the property is assessed by the Government Departments.

Example 4

Ahmad owns a house of worth Rs. 4500000. Calculate the amount of property tax at the rate of 4.5%.

Solution

Total value of the house = Rs. 4500000

Rate of tax = 4.5%

$$\text{Amount of tax} = \frac{4.5}{100} \times 4500000$$

= Rs 202, 500



Activity

Shows the sample of utility bills and ask the students to check the rate of tax which is charged on the amount of bill.



Skill Practice

Discuss with your parents about the income tax. Ask your parents how much income tax did they pay last year and calculate the rate of income tax.

Example 5

Aslam owns a property. If he has to pay property tax of Rs. 22000 at the rate of 2%. Find the total worth of the property.

Solution

Let the rate of property = x

Amount of tax = Rs. 22000

Rate of tax = 2%

2% of x = Rs. 22000

$$\frac{2}{100} \cdot x = \text{Rs. } 22000$$

$$x = \text{Rs. } 22000 \times \frac{100}{2}$$

x = Rs. 1100000

Hence, the total worth of the property is Rs. 1100000.

(c) General Sales Tax

When a customer purchases an item, he pays an extra amount in addition to the original price of the item. This extra amount is called General Sales Tax.

Usually, Government imposes this tax on expensive items. In Pakistan, rate of GST varies from 0% to 25% depending on the type of items.

Example 6 The price of a motorcycle is Rs. 110000. Find the GST on it at the rate of 17%.

Solution

$$\begin{aligned}\text{Price of the motorcycle} &= \text{Rs. } 110000 \\ \text{Rate of GST} &= 17\% \\ \text{Amount of GST} &= \frac{17}{100} \times 110000 \\ &= \text{Rs. } 18700\end{aligned}$$

Hence, the GST on this motorcycle is Rs. 18700.

**Skill Practice**

The marked price of a packed tea bag is Rs. 350 including 5% GST. What will be the original price of tea bag?

**Brain Teaser!**

Think and tell the name of any five products, in which we pay GST while purchasing commodities. Can you guess whether GST is direct or indirect?

(d) Value Added Tax

A tax in which the price of an object is increased during production or distribution at each stage. Mostly, we write it as VAT. In Pakistan, GST and VAT are normally same. VAT is normally known as goods and services tax.



The price of a packet of cakes is Rs. 175. Find value added tax on it at the rate of 17%.

To calculate the value added tax (VAT), we will have to use the following formula:

$$\begin{aligned}\text{Amount of VAT} &= \text{Rate of VAT} \times \text{The value of an item} \\ \text{Amount of VAT} &= 17\% \times 175 \\ &= \frac{17}{100} \times 175 = \text{Rs. } 29.75\end{aligned}$$

Hence, the value added tax on the packet of cakes is Rs. 29.75

**Teachers' Guide**

By using the examples of real life objects, clear the concept of GST and explain them, is it direct or indirect?

**Key fact!**

- General sales tax is normally read as (GST).
- Usually we pay 17% GST on our daily purchasing.
- The value of GST varies from (0 – 25)%.
- GST on any item is same for everyone.

Example 7

The price of 1 fan is Rs. 6000. Find the price of 10 fans including GST.

Solution

$$\begin{aligned}\text{The price of 1 fan} &= \text{Rs. } 6000 \\ \text{The price of 10 fans} &= 6000 \times 10 \\ \text{The price of 10 fans} &= 60000 \\ \text{Rate of GST} &= 17\% \\ \text{Amount of GST} &= \frac{17}{100} \times 60000 \\ &= \text{Rs. } 10,200 \\ \text{Total Price Payable} &= 60000 + 10200 \\ &= \text{Rs. } 70200\end{aligned}$$

**Do you know?**

In Pakistan, the standard rate of VAT is 17% for goods and (13% – 18%) for services.

**Think!**

Can you guess value added tax whether a direct tax or indirect tax?

1.5.8 Commission

Commission is an amount of money which is paid by the seller or purchaser to the agent for his services. In other words, commission is an amount of money paid to an employee or agent by the seller or purchaser for selling or purchasing something.

i Rate of Commission

The rate at which an agent or person gets the commission for doing a service is called rate of commission. Rate of commission is always expressed in percentage form i.e., 5%, 2%, 10% etc. Commission is calculated by the given formula:

$$\text{Commission} = \text{Rate of commission} \times \text{Selling price}$$



If an agent gets 5% commission on selling a property of worth Rs. 3500000, then how much amount does he receive from the seller as a commission? We can find out his commission by using the following formula:

$$\begin{aligned} \text{Commission} &= \text{Rate of commission} \times \text{Selling price} \\ &= 5\% \times 3500000 \\ &= \frac{5}{100} \times 3500000 \\ &= \text{Rs. } 175,000 \end{aligned}$$

Hence, the agent received commission of Rs. 175,000 from the seller.

Example 8 If a property dealer gets Rs. 100000 as commission on the sale of a shop for Rs. 4000000. Find the rate of commission.

Solution

$$\text{Selling price of shop} = \text{Rs. } 4000000$$

$$\text{Amount of commission} = \text{Rs. } 100000$$

$$\text{Commission} = \text{Rate of commission} \times \text{selling price}$$

$$100000 = \text{Rate of commission} \times 4000000$$

$$\text{Rate of commission} = \frac{100000}{4000000}$$

$$\begin{aligned} \text{Rate of commission} &= 0.025 \\ &= 2.5\% \end{aligned}$$



Keep in mind!

Rate of commission is decided between the agent / person and the buyer / seller that's why it can vary.



Skill Practice

A real estate agent receive Rs. 75000 as commission. Which is 6% of the selling price. At what price does the agent sell the property.

EXERCISE 1.15

1. Zawar has annual income of Rs. 1000000. Find the amount of income tax.
2. If the income tax paid by Ali is Rs. 10000, then Find the total annual income of Ali.
3. Asra owns a plot of worth Rs. 400000. Find the amount of property tax at the rate of 3%.

4. Faheem owns one plot and a house of values Rs. 4000000 and Rs. 7000000 respectively. Find the amount of property tax at the rate of 4%.
5. A man paid Rs. 17000 as property tax at the rate of 1%. Find the total value of the property.
6. Ammar wants to buy an air conditioner of price Rs. 200000. If the rate of GST is 17%, then find how much money Ammar has to pay?
7. The price of one chair is Rs. 5000. Find the amount of GST on 50 such chairs at the rate of 17%.
8. Waqar purchased ten packets of biscuits for Rs. 350. He paid VAT of Rs. 42 on ten packets. Find the rate of GST on these packets.
9. Naveed purchased a shop of worth Rs. 6500000. He paid property tax worth Rs. 520000. What was the rate of property tax?
10. Ali sold a plot for Rs. 5000000 by an agent who received 2.5% commission. Find amount of commission.
11. Salman sold 50 articles of a company. If each article is sold for Rs. 2500. Find the amount of commission earned by Salman at the rate of 2%.
12. An online car service charges 20% commission from its drivers on each ride. If a driver got Rs. 5000 from all rides in a day. Calculate how much commission did he pay to car service?
13. A travel agency got 5% commission on the sale of air tickets. If he sold tickets for Rs. 2000000 in a day. Find the amount of commission.
14. An online market place charges 7% on all the sale from its sellers. If the sale of 1000000 rupees is made on Blessed Friday Sale. Calculate the amount of commission earned by online market place owner.
15. A consultant charges 10% of the visa fee for its consultancy. If visa fee is 1000000 rupees. Calculate the commission of consultant.

1.5.9 Zakat

There are five pillars of Islam. Zakat is one of the pillars of Islam. It imposes on those muslims who have certain amount of wealth the whole year. There are eight types of recipients of zakat. The purpose of zakat is to help the poor and needy among the Muslims to create a welfare muslim state. The muslims pay zakat if their annual savings reaches a certain level(Nisab).

Nisab of Zakat

Nisab is the minimum amount of annual savings on which zakat has to be paid. Nisab for zakat is 7.5 tola (87.48 grams) gold or 52.5 tola (612.36 grams) silver or equivalent amount.

Rate of Zakat

The rate of zakat is 2.5% of the total wealth.

The amount of zakat is calculated by the following formula:

$$\text{Amount of zakat} = \text{Rate of zakat} \times \text{Total amount}$$

Example 9

Aslam saved Rs. 2000000 for one year in his account. Calculate the amount of zakat Aslam has to pay.

Solution

Total Amount = Rs. 2000000

Rate of zakat = 2.5%

Amount of zakat = Rate of zakat $\times$ Total amount

Amount of zakat = $\frac{2.5}{100} \times 2000000 = \text{Rs. } 50000$

Hence, Aslam has to pay Rs. 50000 as a zakat

**Remember!**

Mostly, people pay zakat during the month of Ramadan but it is an annual duty and can be paid any time of the year when your wealth exceeded from the amount of Nisab.

1.5.10 Ushr

Ushr is paid on agricultural yield (vegetable, fruits, grains etc.).

Rate of Ushr

- The rate of ushr is 10%, when the land is irrigated by natural sources like rain, river and streams etc.
- The rate of ushr is 5%, when the land is irrigated by artificial resources like wells, tube-wells and artificial canals etc.

Example 11

Aslam has crop of worth Rs. 700000 irrigated by natural sources. Find the amount of ushr on it.

Solution

Total worth of the crop = Rs. 700000

Rate of ushr = 10%

We can find the amount of ushr by using the given formula:

Amount of ushr = Rate of ushr $\times$ Total worth of crop

$$= 10\% \times 700000$$

$$\text{Amount of ushr} = \frac{10}{100} \times 700000 = \text{Rs. } 70000$$

Hence, the amount of ushr is Rs. 70000.

**Teachers' Guide**

Gather students in the middle of the room, and read multiple-choice questions and their possible answers aloud. Students then move to the corner that represents what they believe is the correct answer. The top left room corner can be option A, the bottom-left can be B and so on.

Example 10

Sadia paid zakat of Rs. 15000 on gold. Find the total price of the gold.

Solution

Let the total price of the gold = x

Amount of zakat paid = Rs. 15000

Amount of zakat = Rate of zakat $\times$ Total amount

$$\text{Rs. } 15000 = 2.5\% \times \text{Total amount}$$

$$\text{Rs. } 15000 = \frac{2.5}{100} \times \text{Total amount}$$

$$\text{Total amount} = 15000 \times \frac{100}{2.5} = \text{Rs. } 600000$$

Hence, the total price of the gold is Rs. 600000

**Important Information**

Zakat helps to reduce the financial gap between rich and poor, it reduces poverty and purifies or cleans the Muslim's wealth.



Example 12 Find ushr on 2300 kg onion irrigated by natural source at the rate of Rs. 140 per kg and 1400 kg ginger irrigated by artificial source at the rate of Rs. 220 per kg.

Solution

The total quantity of onion = 2300 kg

Price of onion per kg = Rs. 140

The total amount of onion = 2300×140
= Rs. 322000

Ushr on 2300 kg onion = Rate of ushr $\times$ Total worth of onion

= $10\% \times 322000$

= $\frac{10}{100} \times 322000$

Ushr on 2300 kg onion = Rs. 32200

The total quantity of ginger = 1400 kg

The amount of ginger = 1400×220 = Rs. 308000

Ushr on 2300 kg ginger = Rate of ushr $\times$ Total worth of ginger

= $5\% \times 308000$

= $\frac{5}{100} \times 308000$

Ushr on 1400 kg ginger = Rs. 15400

Total ushr on both the items = $32200 + 15400$
= Rs. 47600

Hence, total ushr on both the items is Rs. 47600.



As, onion is irrigated by natural source, that's why rate of ushr is 10%.



As, ginger is irrigated by artificial resource, that's why rate of ushr is 5%.



Skill Practice

A farmer produces 18000 kg corn. Find his amount of ushr at the rate of 5%.
The rate of corn is Rs. 110 per kg.

EXERCISE 1.16

- Find the amount of the zakat on 15 tola gold if value of 1 tola gold is Rs. 130000.
- Find the amount of zakat on 80 tola silver if value of 1 tola silver is Rs. 1500.
- Raheel paid zakat worth Rs. 47000, on gold and his savings. Find the price of gold if his savings is Rs. 1000000.
- Find the amount of zakat on total of 9 tola gold and 50 tola silver if the values of 1 tola gold and 1 tola silver are Rs. 130000 and Rs. 1500 respectively.
- Hina has jewelry of 11 tola gold. Find the amount of zakat on it if the value of 1 tola of gold is Rs. 130000.



6. Zahid paid zakat of Rs. 23500. Find his savings.
7. Ahsan has a crop wheat of worth Rs. 400000 irrigated by natural sources. Find the amount of ushr on it.
8. Hamid has rice crop of worth Rs. 800000 irrigated by artificial resources. Find the amount of ushr on it.
9. Suleman paid ushr of Rs. 6500 on cotton. If the land is irrigated by rain, then find the worth of the cotton.
10. Find ushr on 850 kg wheat irrigated by natural source, the rate is Rs. 110 per kg and 1250 kg potatoes irrigated by artificial resource, the rate is Rs. 60 per kg.



SUMMARY

- The price at which an item is bought is called its cost price.
- The price at which an item is sold is called its selling price.
- Profit = Selling Price – Cost Price
- Loss = Cost Price – Selling Price
- Profit % = $\frac{\text{Profit}}{\text{Cost Price}} \times 100\%$
- Loss % = $\frac{\text{Loss}}{\text{Cost Price}} \times 100\%$
- The reduction or cut offered on the marked price is called the discount.
- Income tax is the tax imposed by the government on the income of individuals exceeding a certain amount.
- Property tax is a tax imposed by government on the properties such as house, land and shops.
- When a customer purchases an item, he pays an extra amount in addition to the original price of the item. This extra amount is called General Sales Tax.
- The rate of zakat is 2.5% of the total wealth.
- The rate of ushr is 5% and 10% depending on the land irrigated by natural sources or artificial resources respectively.

Sub-domain

(vi)

Squares and Square Roots

Students' Learning Outcomes

After studying this sub-domain, students will be able to:

- Recognise and calculate squares of numbers up to 3-digits.
- Find the square roots of perfect squares of (up to 3-digits) natural numbers, fractions and decimals.
- Solve real-world word problems involving squares and square roots.

Can you find out the area of this room if the length of the room is 4 metres?



Yes, by taking square of 4, we can find out the area of this room.



1.6.1 Square of a Number

As we know that the area of a square shape is calculated by multiplying its length of a side by itself as given below:

$$\begin{aligned}\text{Area of a square} &= \text{length} \times \text{length} \\ &= a \times a = a^2\end{aligned}$$

Here a^2 is the square of a . That is, the square of $a = a^2$

When a number is multiplied by itself, the resultant number is known as its square. For example, when we multiply $6 \times 6 = 6^2$, we get 36. Here, 36 is a squared number.

Example 1 Write 25 in terms of square of a number.

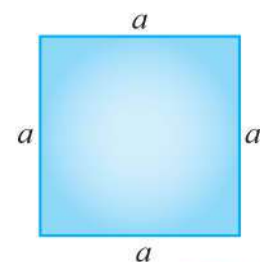
Solution Recall the table of 5, we can write

$$5 \times 5 = 25$$

Applying the law of exponent, $a \times a = a^2$

$$5^2 = 25$$

Therefore 25 is the square of 5.



Remember!

- $6^2 = 36$ can be read as:
- The square of 6 is 36.
 - 6 squared is 36.
 - 6 to the power of 2 is 36.

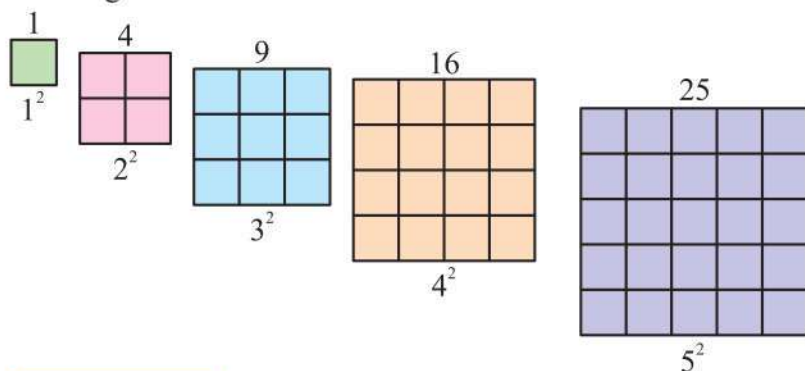


Note

Perfect squares are used frequently in math. Try to remember these familiar numbers so that you can recognize them as they are used in most of the math problems.

1.6.2 Perfect Square of a Number

Perfect squares are the natural numbers that are squares of natural numbers. The first five squares of the counting numbers are shown below:



Square Notation	Perfect Square
1^2 (1-Squared)	1
2^2 (2-Squared)	4
3^2 (3-Squared)	9
4^2 (4-Squared)	16
5^2 (5-Squared)	25

Example 2

Find the perfect squares of the numbers: (i) 23 (ii) 331

Solution



Skill Practice

Find the perfect squares of the following numbers:

- (i) 37 (ii) 58
(iii) 429 (iv) 900

$$\begin{array}{r} \text{i} \quad 23 \\ \times 23 \\ \hline 69 \\ 460 \\ \hline 529 \end{array} \quad (23)^2 = 23 \times 23 = 529$$

$$\begin{array}{r} \text{ii} \quad 331 \\ \times 331 \\ \hline 331 \\ 9930 \\ 99300 \\ \hline 109561 \end{array} \quad (331)^2 = 331 \times 331 = 109561$$

Example 3

Check whether the numbers are perfect squares or not:

- (i) 25 (ii) 105 (iii) 1225

Solution

(i) 25

Factorize 25

The prime factors of $25 = 5 \times 5$

Prime factors of 25 form a pair.

So, 25 is a perfect square.

5	25
	5

(ii) 105

Factorize 105

The prime factors of $105 = 3 \times 5 \times 7$

We can see factors of 105 cannot be paired.

So, 105 is not a perfect square.

3	105
5	35
	7

(iii) 1225

Factorize 1225

The prime factors of $1225 = 5 \times 5 \times 7 \times 7$

Each prime factor of 1225 forms a pair.

So, 1225 is a perfect square.

5	1225
5	245
7	49
	7



Remember!

If each prime factor of a natural number forms a pair of prime numbers then natural number is a perfect square.



Teachers' Guide

Explain the concept of square with the help of examples. Ask students to draw multiplication table chart and highlight perfect squares.

Properties about Perfect Squares

- The square of an even number is always even. For example, the square of 12 is 144.
- The square of an odd number is always odd. For example, the square of 25 is 625.
- A number ends with 1 or 9, its square will end with 1. For example, the square of 11 is 121 and the square of 9 is 81.
- A number ends with 4 or 6, its square will end with 6. For example, the square of 14 is 196 and the square of 26 is 676.
- The square of a proper fraction is less than itself. For example, $\left(\frac{1}{2}\right)^2 = \frac{1}{4}$. Now, $\frac{1}{4} < \frac{1}{2}$.
Hence, the square of a proper fraction is less than the given fraction.
- The square of decimal less than 1 is smaller than itself. For example, the square of 0.15 is 0.0225.

Example 4 Find the square of given numbers and write the property of perfect squares of a number which it satisfy: (i) 325 (ii) 1026 (iii) $\frac{12}{15}$ (iv) 0.25

Solution

(i) 325

Number 325 is an odd number

Square of 325 = 325×325

= 105625 is an odd number

Therefore, square of odd number is odd.

(iii) $\frac{12}{15}$

$\frac{12}{15}$ is a proper fraction

Square of $\frac{12}{15} = \frac{12}{15} \times \frac{12}{15} = \frac{12 \times 12}{15 \times 15} = \frac{144}{225}$

To compare $\frac{12}{15}$ and $\frac{144}{225}$,

by cross multiplication

$12 \times 225 = 2700$ and $15 \times 144 = 2160$

As $2700 > 2160$

Therefore, square of proper fraction is less than itself, i.e. $\frac{144}{225} < \frac{12}{15}$

(ii) 1026

Number 1026 is an even number

Square of 1026 = 1026×1026

= 1052676 is an even number

Therefore, square of even number is even and 1026 ends with 6 so its square 1052676 ends with 6.

(iv) 0.25

0.25 is a decimal less than 1

Square of 0.25 = $0.25 \times 0.25 = \frac{25}{100} \times \frac{25}{100} = \frac{625}{10000}$

Hence, 0.0625 is smaller than 0.25, i.e. $0.0625 < 0.25$



Challenge

Write five numbers which you can decide by looking at their ones digit that they are not square numbers.



Teachers' Guide

Ask students whether strength of classroom students and classroom furniture represents perfect squares or not.



Skill Practice

- Which of 123^2 , 77^2 , 83^2 , 161^2 , 109^2 would end with digit 1?
- Check whether the square of proper fraction $\frac{21}{35}$ is less than itself or not.
- Check whether the square of decimal 0.35 is smaller than itself or not.

EXERCISE 1.17

1. Find the squares of the following natural numbers:

- | | | | |
|---------|----------|-----------|------------|
| (i) 49 | (ii) 66 | (iii) 75 | (iv) 111 |
| (v) 230 | (vi) 313 | (vii) 279 | (viii) 400 |

2. Check whether the following numbers are perfect squares or not:

- | | | | |
|----------|-----------|------------|------------|
| (i) 900 | (ii) 912 | (iii) 1764 | (iv) 1600 |
| (v) 1650 | (vi) 2100 | (vii) 3025 | (viii) 710 |

3. Find the square of following numbers by using the property of perfect square:

- | | | | |
|------------------------|----------------------|----------------------|-----------------------|
| (i) 72 | (ii) 80 | (iii) 63 | (iv) 355 |
| (v) 524 | (vi) 4216 | (vii) 821 | (viii) 629 |
| (ix) 0.33 | (x) 0.18 | (xi) 0.23 | (xii) 0.04 |
| (xiii) $\frac{14}{16}$ | (xiv) $\frac{2}{18}$ | (xv) $\frac{19}{28}$ | (xvi) $\frac{25}{28}$ |

1.6.3 Square Root

The power “ $\frac{1}{2}$ ” of a perfect square of any positive number is called square root of that positive number.

Square root symbol is denoted by radical sign $\sqrt{\quad}$, it is equivalent to power $\frac{1}{2}$.

We know that: $4^2 = 4 \times 4 = 16$. We say square root of 16 is 4. This is written as: $\sqrt{16} = 4$

Let us see some more examples $7^2 = 49 \longrightarrow \sqrt{49} = 7$; $5^2 = 25 \longrightarrow \sqrt{25} = 5$

i Finding Square Root by Prime Factorization Method

To find the square root of a perfect square by prime factorization, we follow the below steps:

- | | |
|---|--------------------------------------|
| (i) Find the prime factors of the given number. | (ii) Make pairs of prime factors. |
| (iii) Write each pair of prime factors in square form . | |
| (iv) Take square root or power $\frac{1}{2}$ of each square term. | (v) Simplify/cancel powers of terms. |
| (vi) Find the product of terms, which will be the required square root. | |



Teachers' Guide

Explain the concept of square root by using real life situation.

Let us take an example to find the square root by prime factorization method.

Example 5 Find the square root of 784 by prime factorization method.

Solution Factorize 784

$$\begin{aligned}\text{The prime factors of } 784 &= \overline{2 \times 2} \times \overline{2 \times 2} \times \overline{7 \times 7} \\ &= 2^2 \times 2^2 \times 7^2\end{aligned}$$

$$\begin{aligned}\text{Square root of } 784 &= \sqrt{784} \\ &= (784)^{\frac{1}{2}} \\ &= (2^2 \times 2^2 \times 7^2)^{\frac{1}{2}} \\ &= (2^2)^{\frac{1}{2}} \times (2^2)^{\frac{1}{2}} \times (7^2)^{\frac{1}{2}} \\ &= 2^{2 \times \frac{1}{2}} \times 2^{2 \times \frac{1}{2}} \times 7^{2 \times \frac{1}{2}} \\ &= 2 \times 2 \times 7 = 28\end{aligned}$$

(Take power $\frac{1}{2}$ of each square term separately)
(Cancel the powers)

2	784
2	392
2	196
2	98
7	49
	7

Here, 28 is the square root of 784.

ii Finding Square Root by Long Division Method

We use long division method, when the numbers are very large. Because the method to find out their square root by prime factorization becomes very lengthy.

Let us find the square root of 484 by long division method.

Step 1 Place a bar over the pair of numbers starting from the ones place or right hand side of the number.

$\overline{4 \ 84}$

Step 3 Bring down the number, which is under the bar, to the right side of the remainder.

2
2 $\overline{4 \ 84}$
$\underline{4}$
0 84

Step 2 Take the largest number as the divisor whose square is less than or equal to the number on the extreme left of the number.

2
2 $\overline{4 \ 84}$
$\underline{4}$
0

Step 4 Double the quotient and enter it with a blank space on the right side.

2
2 $\overline{4 \ 84}$
$\underline{4}$
4 $\underline{\quad}$ 0 84

Step 5 Next, we have to select the largest digit for the ones place of the divisor (4_) such that new number 2, when multiplied by the new digit at the ones place, is equal to or less than dividend (84).

The remainder is 0 and we have no number left for division.

$$\text{Therefore, } \sqrt{484} = 22$$

22
2 $\overline{4 \ 84}$
$\underline{4}$
42 $\overline{0 \ 84}$
$\underline{84}$
0



Challenge

What will be the square root of 0?



Skill Practice

Find the square root of 225 by long division method.

iii Square Root of Fractions

Let us find the square root of fraction $\frac{4}{9}$

$$\begin{aligned}\sqrt{\frac{4}{9}} &= \left(\frac{4}{9}\right)^{\frac{1}{2}} \\ &= \left(\frac{2 \times 2}{3 \times 3}\right)^{\frac{1}{2}} \\ &= \left(\frac{2^2}{3^2}\right)^{\frac{1}{2}} = \frac{2^{2 \times \frac{1}{2}}}{3^{2 \times \frac{1}{2}}} = \frac{2}{3}\end{aligned}$$

Here, $\frac{2}{3}$ is the square root of $\frac{4}{9}$

Example 6

Find the square root of $\frac{196}{400}$ by prime factorization method.

Solution

To find the square root of $\frac{196}{400}$, we will find square roots of numerator and denominator separately.

Factorize 196

Prime factors of 196 = $\overline{2 \times 2} \times \overline{7 \times 7}$

$$= 2^2 \times 7^2$$

Square root of 196 = $\sqrt{196}$

$$= (196)^{\frac{1}{2}}$$

$$= (2^2 \times 7^2)^{\frac{1}{2}}$$

$$= (2^2)^{\frac{1}{2}} \times (7^2)^{\frac{1}{2}}$$

$$= 2^{2 \times \frac{1}{2}} \times 7^{2 \times \frac{1}{2}}$$

$$= 2 \times 7 = 14$$

Here, 14 is the square root of 196.

So, square root of $\frac{196}{400} = \sqrt{\frac{196}{400}} = \frac{14}{20}$ or $\frac{7}{10}$ (Put values of $\sqrt{196} = 14$ and $\sqrt{400} = 20$)

Also factorize 400

Prime factors of 400 = $\overline{2 \times 2} \times \overline{2 \times 2} \times \overline{5 \times 5}$

$$= 2^2 \times 2^2 \times 5^2$$

Square root of 400 = $\sqrt{400}$

$$= (400)^{\frac{1}{2}}$$

$$= (2^2 \times 2^2 \times 5^2)^{\frac{1}{2}}$$

$$= (2^2)^{\frac{1}{2}} \times (2^2)^{\frac{1}{2}} \times (5^2)^{\frac{1}{2}}$$

$$= 2^{2 \times \frac{1}{2}} \times 2^{2 \times \frac{1}{2}} \times 5^{2 \times \frac{1}{2}}$$

$$= 2 \times 2 \times 5 = 20$$

Here, 20 is the square root of 400. So,

2	400
2	200
2	100
2	50
5	25
	5

**Teachers' Guide**

Provide steps for finding square root of fractions.

Example 7

Find the square root of $\frac{289}{625}$ by division method.

Solution

To find the square root of $\frac{289}{625}$, we will find square roots of numerator and denominator separately.

First we find square root of numerator 289.

Consider 1 as a divisor and quotient

$\because 2 \times 2 = 4$ is greater than 2

Sum of divisor and quotient is 2,

write 2 at tens place of new divisor,

whereas 7 is at ones place of new

divisor

$\because 27 \times 7 = 189$

$$\begin{array}{r} 17 \\ 1 \overline{) 289} \\ \underline{1} \\ 27 \\ \underline{189} \\ 0 \end{array}$$

Here, 17 is the square root of 289.

$$\begin{aligned} \text{Square root of } \frac{289}{625} &= \sqrt{\frac{289}{625}} \\ &= \frac{17}{25} \quad (\text{Put values of } \sqrt{289} = 17 \text{ and } \sqrt{625} = 25) \end{aligned}$$

iv Square Root of Decimal**Example 8**

Find the square root of 0.49 by prime factorization method.

Solution

To find the square root of 0.49, we shall find square root of $\frac{49}{100}$.

Factorize 49

The prime factors of 49 = 7×7

$$= 7^2$$

The square root of 49 = $\sqrt{49}$

$$= (49)^{\frac{1}{2}}$$

$$= (7^2)^{\frac{1}{2}}$$

$$= 7^{2 \times \frac{1}{2}}$$

$$= 7$$

(Cancel the powers)

$$\begin{array}{r} 7 \overline{) 49} \\ \underline{49} \\ 0 \end{array}$$

Here, 7 is the square root of 49.

$$\text{So, the square root of } 0.49 = \sqrt{0.49} = \frac{\sqrt{49}}{\sqrt{100}} = \frac{7}{10} \text{ or } 0.7 \quad (\text{Put values of } \sqrt{49} = 7 \text{ and } \sqrt{100} = 10)$$

Now, we find square root of denominator 625.

Consider 2 as a divisor and quotient

$\because 3 \times 3 = 9$ is greater than 6

Sum of divisor and quotient is 4,

write 4 at tens place of new

divisor, whereas 5 is at ones place

of new divisor.

$\because 45 \times 5 = 225$

Here, 25 is the square root of 625.

$$\begin{array}{r} 25 \\ 2 \overline{) 625} \\ \underline{4} \\ 45 \\ \underline{225} \\ 0 \end{array}$$

**Skill Practice**

Find square root of following decimals by prime factorization and long division method.

- (i) 17.64 (ii) 1.21

$$\begin{array}{r} 10 \\ 2 \overline{) 100} \\ \underline{40} \\ 20 \\ \underline{20} \\ 0 \end{array}$$

Also factorize 100

The prime factors of 100 = $2 \times 2 \times 5 \times 5$

$$= 2^2 \times 5^2$$

Square root of 100 = $\sqrt{100}$

$$= (100)^{\frac{1}{2}}$$

$$= (2^2 \times 5^2)^{\frac{1}{2}}$$

$$= (2^2)^{\frac{1}{2}} \times (5^2)^{\frac{1}{2}}$$

$$= 2^{2 \times \frac{1}{2}} \times 5^{2 \times \frac{1}{2}}$$

$$= 2 \times 5$$

$$= 10$$

Here, 10 is the square root of 100.

Example 9 Find the square root of 2.56 by division method.

Solution To find the square root of 2.56

Consider 1 as a divisor and quotient

$\because 2 \times 2 = 4$ is greater than 2

Sum of divisor and quotient is 2,

write 2 at tens place of new divisor,

whereas, 6 is at ones place of new divisor

$\because 26 \times 6 = 156$

Here, 1.6 is the square root of 2.56.

$$\begin{array}{r} 1.6 \\ 1 \overline{) 2.56} \\ \underline{1} \\ 26 \\ \underline{156} \\ 0 \end{array}$$

Place the bars

You can check $25 \times 5 = 125$ is less than 156 and $27 \times 7 = 189$ is greater than 156.

EXERCISE 1.18

1. Find the square roots of the following natural numbers:

(i) 49

(ii) 100

(iii) 144

(iv) 196

(v) 324

(vi) 784

(vii) 529

(viii) 676

2. Find the square roots of following numbers by prime factorization method:

(i) 900

(ii) 2500

(iii) 1849

(iv) 1444

(v) $\frac{49}{144}$

(vi) $\frac{64}{225}$

(vii) 7.29

(viii) 6.76

3. Find the square roots of following numbers by division method:

(i) 576

(ii) 2304

(iii) 1521

(iv) 0.0441

(v) $\frac{256}{1089}$

(vi) $\frac{81}{289}$

(vii) $\frac{676}{841}$

(viii) 26.01

1.6.4 Real-World Word Problems Involving Squares and Square Roots

Now, we discuss real life problems involving square and square root of numbers.

Example 10 Find the length of side of a square shaped park. If its area is 1296 m^2 .

Solution Area of square shaped park = 1296 m^2

$$\text{Length of side} = \sqrt{1296}$$

Consider 3 as a divisor and quotient

$\because 4 \times 4 = 16$ is greater than 12.

Sum of divisor and quotient is 6,

write 6 at tens place of new divisor,

whereas, 6 is at ones place of new divisor

$\because 66 \times 6 = 396$

$$\begin{array}{r} 36 \\ 3 \overline{) 1296} \\ \underline{9} \\ 66 \\ \underline{396} \\ 0 \end{array}$$

$$\because \text{Area} = x^2 \text{ and Length} = \sqrt{x^2} = x$$

Place the bars

You can check $65 \times 5 = 325$ is less than 396 and $67 \times 7 = 469$ is greater than 396.

Here, 36 is the square root of 1296, so length of side of a square shaped park is 36 m.

Example 11

Find the length of a rectangular field if area of a rectangular field is 19.36 m^2 , where length of a rectangular field is four times of its width.

Solution

Area of a rectangular field = 19.36 m^2

Length of side = 4 (Width)

Let width of rectangular field is x ,

Area of a rectangular field = 19.36 m^2

Length $\times$ Width = 19.36 m^2

$$(4x) \times (x) = 19.36 \text{ m}^2$$

$$4x^2 = 19.36 \text{ m}^2$$

$$x^2 = \frac{19.36}{4} \text{ m}^2$$

$$x = \sqrt{\frac{19.36}{4}} \text{ m}$$

$$\text{Width} = \sqrt{\frac{19.36}{4}} \text{ m}$$

$$= \frac{\sqrt{19.36}}{2} \text{ m}$$

$$= \frac{4.4}{2} \text{ m}$$

$$\text{Width} = 2.2 \text{ m}$$

Length of side = 4 (Width)

$$= 4(2.2 \text{ m})$$

$$= 8.8 \text{ m}$$

So, the length of a side of the rectangular field is 8.8 m.

Example 12

Find the area of square shaped shop if length of one side of shop is 33 m.

Solution

Area of square shaped shop = Square of side length of shop

$$= (33 \text{ m})^2$$

$$= 33^2 \text{ m}^2$$

$$= 33 \times 33 \text{ m}^2$$

Area of square shaped shop = 1089 m^2

	4.4
4	<u>19.36</u>
	<u>16</u>
84	<u>336</u>
	<u>336</u>
	0

**Teachers' Guide**

Ask the students to find square root of 1444 by prime factorization method and long division method. Also check whether the answers are same.

EXERCISE 1.19

1. In a lecture hall, 8649 students are sitting in such a manner that there are as many students in a row as there are rows in the lecture hall. How many students are there in each row of the lecture hall?
2. The students of class-VII of a school donated Rs. 2304 for the Prime Minister's Relief Fund. Each student donated as many rupees as the number of students in the class. Find the number of students in class.
3. Kiran wants to wish her teacher on Eid Day by giving her a self-made greeting card. She chooses a purple coloured square sheet of paper. A side of that paper measures 19.5 cm. Find the area of paper she chooses for the card.
4. The area of a square plot is 900 m^2 . Find the length of the side of the plot.
5. 400 students sit in rows in such a way that the number of rows is equal to the number of students in a row. How many students are there in each row?

SUMMARY

- When a number is multiplied by itself, the resultant number is known as its square.
- Perfect squares are the numbers that are squares of natural numbers.
- The power " $\frac{1}{2}$ " of perfect square of any positive number is called square root of that positive number.
- Square root of any positive number can be find from the following two methods:
 - Prime Factorization Method
 - Long Division Method

REVIEW EXERCISE 1 (b)

1. Choose the correct option.
 - (i) The ratio of 2 years to 6 months.
 (a) 1 : 4 (b) 2 : 6 (c) 4 : 1 (d) 1 : 3
 - (ii) The comparison of two quantities with same kinds is known as:
 (a) rate (b) ratio (c) average rate (d) proportion
 - (iii) The value of m in the given proportion: $6 : m :: 5 : 10$
 (a) 12 (b) 30 (c) 25 (d) 15
 - (iv) The equality of two ratios is known as:
 (a) ratio (b) increase ratio (c) rate (d) proportion

- (v) In proportion, product of means and product of extremes are:
 (a) less (b) equal (c) greater (d) not equal
- (vi) If the cost price is Rs. 450 and selling price is Rs. 550 then Profit will be:
 (a) 50 (b) 100 (c) 150 (d) 200
- (vii) If the marked price is Rs. 780 and selling price is Rs. 750 then discount will be:
 (a) 10 (b) 20 (c) 30 (d) 40
- (viii) The tax imposed on the income of an individual is called _____.
 (a) GST (b) VAT (c) income tax (d) property tax
- (ix) Rate of Zakat is _____.
 (a) 1% (b) 1.5% (c) 2.5% (d) 3%
- (x) Rate of ushr on a land irrigated by natural sources is _____.
 (a) 5% (b) 10% (c) 15% (d) 20%
- (xi) Which of the following cannot be at the ones place of a perfect square number?
 (a) 1 (b) 4 (c) 8 (d) 9
- (xii) The number of zeros in square of 40 is:
 (a) 1 (b) 2 (c) 3 (d) 4
- (xiii) How many natural numbers lies between 2^2 and 3^2 ?
 (a) 4 (b) 5 (c) 7 (d) 9
- (xiv) The ones place in the square of 16 is:
 (a) 4 (b) 6 (c) 7 (d) 8
- (xv) A square shaped garden has an area of 100m^2 . Find the length of one side of the garden.
 (a) 9 (b) 10 (c) 11 (d) 12

2. Write the following in ratio form and reduce it into lowest form:

- (i) $\frac{75}{125}$ (ii) $\frac{200}{500}$ (iii) 185 : 85.
 (iv) 5 week to 60 days (v) 4 km to 800 m (vi) 3 litres to 1800 ml

3. (i) Increase 15 in the ratio of 8 : 5 (ii) Increase 120 in the ratio of 13 : 10

4. (i) Decrease 510 in the ratio of 7 : 10 (ii) Decrease 999 in the ratio of 2 : 9

5. If the price of a shirt is increased from Rs. 3200 to Rs. 3600, what will be the increased in ratio?

6. If old price of an object was Rs. 890 and new price is Rs. 560. What will be decreased in ratio?

7. A shopkeeper purchased 20 packets of almonds for Rs. 16000. How much will he have to pay if he buys 25 packets?

8. A train covers 140 km in 2 hours. What will be the speed of the train per hour. Also find in how much distance will it cover in 4 hours?

9. Find the value of variable in the following proportions.

- (i) $5 : 4 :: x : 20$ (ii) $a : 6 :: 55 : 10$ (iii) $e : 10 :: 25 : 50$ (iv) $q : 3 :: 24 : 30$

10. Two watches cost Rs. 16,000. How much money will be required to buy 14 such watches?
11. Mohsin took 5 days to finish a book, reading 100 pages daily. How many pages must he read in a day to finish it in 10 days?
12. If 42 men can reap a field in 16 days, in how many days can 20 men reap the same field?
13. 14 men can dig a well in 8 days. How many men can dig it in 4 days?
14. A fort had enough food for 60 soldiers for 40 days. How long would the food last if 30 more soldiers join after 16 days?
15. Ahmad travelled 300 kilometres in 5 hours. Find the unit rate in kilometres / hour.
16. An international phone call costs Rs.10 for 4 minutes. Find the unit rate in rupees / minute.
17. Hassan reads 18 pages in 9 minutes. Find the unit rate in pages / minute.
18. A car consumes 10 litres of fuel for a distance of 260 km. Find the unit rate in km/litre.
19. A car travels 600 km in 10 hours on Monday and 200 km in 3 hours on Tuesday. What will be the average speed of the car?
20. The cost price of an article is Rs. 3000 and the selling price is 5000. Find the profit percentage.
21. Ahsan bought a car for Rs. 1500000 and sold it for Rs. 1200000. Find loss percentage.
22. If the discount on an item is Rs. 500 and the selling price is Rs. 4000. Find the marked price.
23. Adeel earns Rs. 100000 monthly. Find the amount of income tax on his income.
24. Zawar pays Rs. 10000 as property tax at the rate of 2%. Find the total worth of the property.
25. The price of a washing machine is Rs. 80000. If the rate of GST is 17%, then find the total price of washing machine including GST.
26. Find the amount of zakat on 200000 rupees.
27. Shahwar has two crops of worth Rs. 500000 and Rs. 800000 respectively irrigated by artificial resources. Find the amount of ushr.
28. The price of packet of a rusk is Rs. 270. If the amount of value added tax is Rs. 80, then find the rate of value added tax.
29. Find the squares of the following natural numbers:

(i) 53	(ii) 69	(iii) 288	(iv) 500
--------	---------	-----------	----------
30. Check whether the following numbers are perfect squares or not:

(i) 441	(ii) 572	(iii) 425	(iv) 3025
---------	----------	-----------	-----------
31. Find the square root of following numbers by prime factorization method.

(i) 169	(ii) 289	(iii) $\frac{49}{625}$	(iv) 3025
---------	----------	------------------------	-----------
32. Find the square root of following numbers by division method:

(i) 361	(ii) 729	(iii) $\frac{100}{121}$	(iv) 6.76
---------	----------	-------------------------	-----------
33. Hooria has a square shaped mat with an area of 289 square cm. She wants to decorate the mat by putting fringe around the edges. How many cm of fringe, she needs to buy?
34. What is the length of a side of a square having area 441 square metres?