

• Sub-Domain

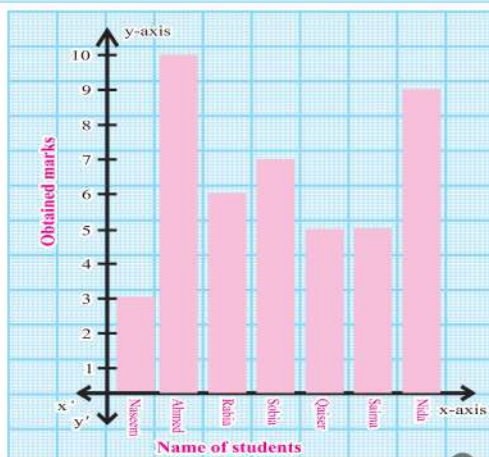
11

DATA MANAGEMENT

Students' Learning Outcomes:

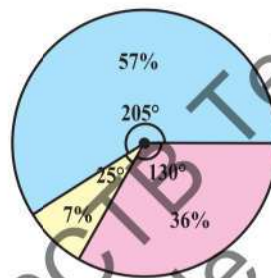
By the end of this sub-domain, students will be able to:

- Recognize different types of graphs.
- Differentiate between grouped and ungrouped data, continuous and discrete variables.
- Calculate mean, median, and mode.



If each bar in the bar graph represents the obtained marks of the students, then find

- Who has maximum marks?
- Who has minimum marks?



Key

- Before working age (1-7) years
- Working age (18-65) years
- After working (65+) years

If each sector in the pie graph represents the working stages of population of Pakistan.

- How much part of population represents working age?
- How much part of population represents after working age?

INTRODUCTION

In our previous classes, we have learnt about data.

“Data is a collection of any information and facts, such as numbers, words, measurements, observations or even description of things.”

Numerical data or information is very valuable because it helps us to understand the changes which take place in our daily lives. For example, numerical data or information about temperature helps us to know the condition of water. We can collect the data by several ways i.e. surveys, interviews, observation and questionnaire, etc.



Remember

A survey is a method of collecting information/data about a group.



11.1 GROUPED AND UNGROUPED DATA

Most surveys or observations produce a large amount of numerical data. It is so complicated to draw the meaningful conclusion from large range of numerical data. To make the meaningful results of data collection, the data is classified into two groups/categories.

Ungrouped data

The data which provides us information or data points individually is known as ungrouped data.

Example 1

The following table shows the scores of Pakistan cricket team in a one day international cricket match. The runs of each player are showing.



Name of players	Sarfraz	Umar	Hafeez	Shoaib	Yasir	Amir	Kamran	Shahid	Faheem	Waseem	Waqar
Runs	19	25	15	45	80	20	7	18	9	28	5

Grouped data

The data which is given in intervals or groups provide us the information about the data.

In the above example, the runs of the players can be organized into the following classes or groups or intervals.

0 – 19

20 – 39

40 – 59

60 – 79

80 – 99

The below frequency table shows the runs of the players in groups or classes.

Runs	Tally mark	Frequency
0 – 19	I	6
20 – 39		3
40 – 59	I	1
60 – 79		0
80 – 99	I	1



Activity -1

Ask the obtained marks of class-5 result of your all classfellows and show the result in grouped and ungrouped tables.

11.2 CONTINUOUS AND DISCRETE VARIABLES

Since, we are familiar about variable that a variable is a quantity whose value changes. Here, we will discuss two types of variables.

◆ Discrete variables

◆ Continuous variables



Remember

or data.

The frequency of an event is the number of times the event occurred in an information or study

(i) Discrete variables

“A variable whose values are obtained by counting is called discrete variable.” For example,

a

Blue marbles in the jar.

b

Number of tails when flipping a coin.

c

Number of children in a family.

(ii) Continuous variables

“A variable whose value is obtained by measurement is called continuous variable”. For example,

a

Height of a student of grade 6 is 5 feet.

b

Distance from a city to another city is 25.3 km.

c

Mass of student is 49.7 kg.

**Activity -2**

of continuous variables.

Observe in your surrounding and give at least three examples of discrete variables and three

**Exercise - 11.1**

- Define the following:
 - Data
 - Ungrouped data
 - Grouped data
- Observe the following tables and identify as grouped or ungrouped frequency table:

(i)

The table shows the ages of students.	
Name of students	Ages in years
Aliya	10
Shumaila	15
Samreen	8
Sameen	7
Aleezay	16
Laiba	11
Arsheena	10

(ii)

The table shows the obtained marks of grade 6 students.	
Marks	Frequency
1 – 20	5
21 – 40	10
41 – 60	12
61 – 80	21
81 – 100	25

- (iii) The below table shows the sales record of toys in a toys shop.

Prices of toys (Rs)	Tally mark	Frequency
1 – 100		40
101 – 200		20
201 – 300		45
301 – 400		30
401 – 500		35
501 – 600		38

- (iv) The table shows the distance covered by a bus in a week.

Days	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
Distance(km)	150	95	215	317	415	210	125

3. For each of the following, write discrete variable or continuous variable.

(i) Number of people in a school.

(ii) Mass of a mobile.

(iii) Number of pages in your Mathematics book.

(iv) The size of your shoe.

(v) The volume of your water bottle.

(vi) The number of players in football team.

4. A school keeps the record of blood groups of all students who donate blood every year. Is that record a discrete variable or continuous variable?



Practice -1

Count total number of trees and students in your school.

- Arrange your data in the form of grouped or ungrouped data.
- Which type of variable you need to use for your data?

11.3 GRAPHS

Have you seen the graphs around you, like in the newspapers, television, magazines and books etc? The purpose of graph is to show numerical facts in visual form, so that it can be understood quickly, easily and clearly.

In our previous classes, we have learnt about graphs.

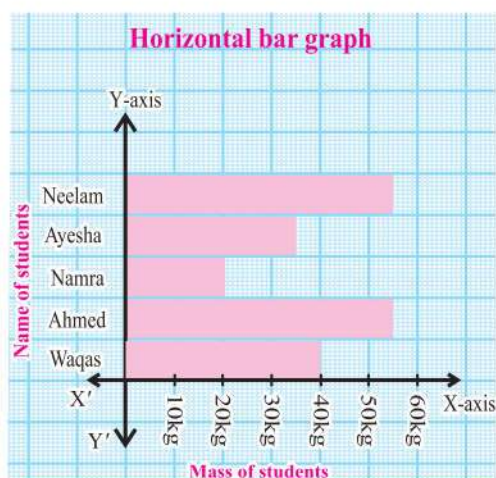
“A graph is a diagram representing a system of connections or interrelations among two or more things by a number of lines, pictures, distinctive dots and bars, etc”.

We use a graph to display a data in a simple and attractive way. There are different types of graphs such as pictograph, line graph, bar graph and multiple bar graph.

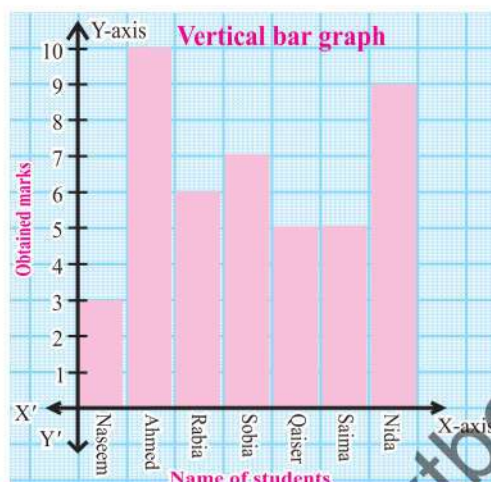
11.3.1 A Bar Graph

“A bar graph is a graphical display of data using bars of different heights.”

There are two types of bar graphs: ♦ Horizontal bar graph ♦ Vertical bar graph



This is bar graph showing the mass of grade 6 students.



This is bar graph showing the obtained marks of students in a class test.



Remember

A bar graph is also known as bar chart.



Note

Always choose a scale that allows all the data to be represented clearly in the available space.

Simple bar graphs

The procedure to draw the horizontal and vertical bar graphs is explained in the following examples.

Example

2

The students of different classes in a school were asked that how many of students had read the storybooks in summer vacation. The results are shown in the following table. Show this result in horizontal bar graph.

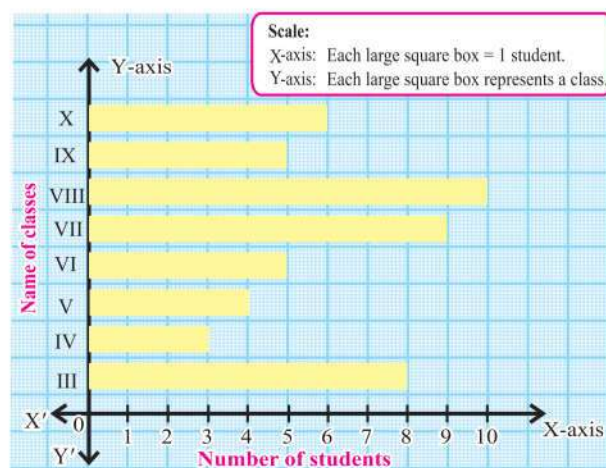
Name of class	III	IV	V	VI	VII	VIII	IX	X
Number of students	8	3	4	5	9	10	5	6

Solution

Horizontal bar graph for number of students in each class.

Steps of drawing a horizontal bar graph.

- Draw two lines XX' and YY' on a graph paper. Both lines are perpendicular to each other and cut at point 'O'. The horizontal line is called X-axis and the vertical line is called Y-axis.
- Give the title as Horizontal bar graph for number of students in each graph.
- Label X-axis as 'Number of students' and Y-axis as 'Name of classes'.
- Make a scale, for each axis (on X-axis each large square box = 1 student and on Y-axis large square box represents a class).
- Make bars of equal width at equal distance on Y-axis according to the given data.



Example 3 Given below are the seats won by different political parties in the polling outcome of a national assembly election 2018.

Political parties	A	B	C	D	E	F
Seats won	80	55	36	25	10	30

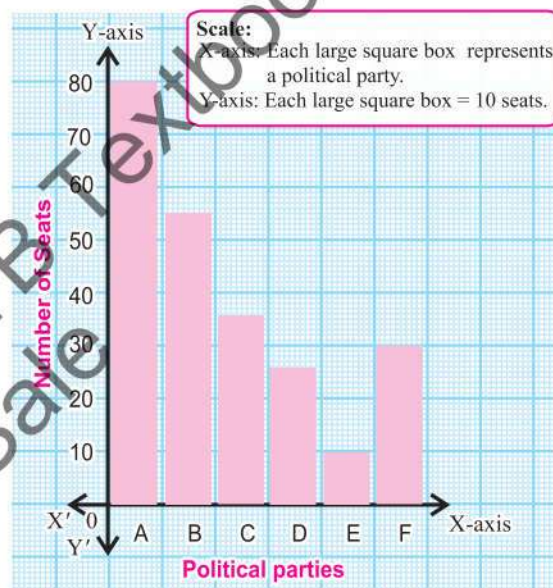
Draw a vertical bar graph to represent the polling results.

Solution

Vertical bar graph for polling results of 2018 election.

Steps of drawing a vertical bar graph.

- Draw two lines XX' and YY' on a graph paper. Both lines are perpendicular to each other and cut at point 'O'. The horizontal line is called X -axis and the vertical line is called Y -axis.
- Give the graph title (vertical bar graph for polling results of 2018 election).
- Label X -axis as "political parties" and Y -axis as "number of seats won".
- Make a scale, for each axis (on Y -axis each large square box = 10 seats. On X -axis each large square box represents a political party).
- Make bars of equal width at equal distance on X -axis according to the given information.



Remember

The graphs used in example 2 and example 3 are known as simple bar graphs.



Practice -1

The following table shows the number of copies of a book a publishing house distributed to the dealers in each year from 2015 to 2022.

Years	2015	2016	2017	2018	2019	2020	2021	2022
No. of copies	50,000	40,000	55,000	30,000	60,000	70,000	75,000	90,000

Use bar graph to illustrate the above information.

Multiple bar graphs

"A graph that shows the conformation/data through multiple bars on the graph is called multiple bar graph".

Each data value is represented by a column in the same graph. The procedure to draw the multiple bar graph is illustrated in the following example.

Example 4

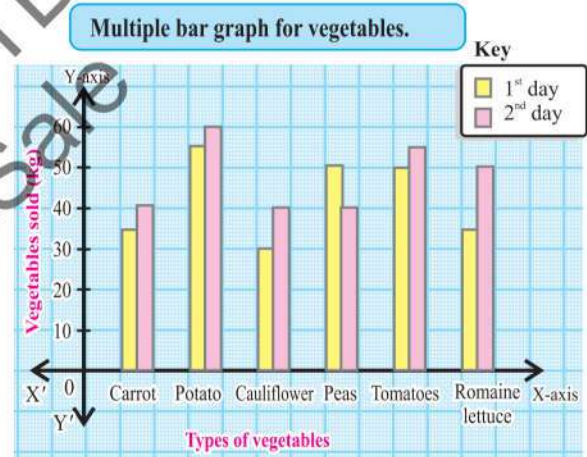
Nadeem is a vegetable shop owner. He recorded the number of kilograms of vegetables sold over two days period. The results are listed on the table below. Create a multiple bar graph to display the data. At the end, list three conclusions that can be made about the data.



Vegetables	1 st day sale	2 nd day sale
Carrot	35kg	40kg
Potato	55kg	60kg
Cauliflower	30kg	40kg
Peas	50kg	40kg
Tomatoes	50kg	55kg
Romaine lettuce	35kg	50kg

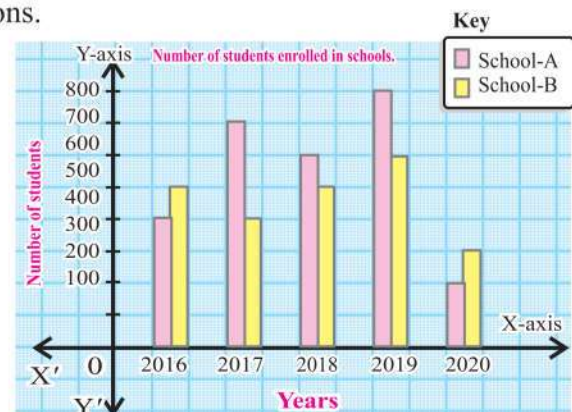
Solution**Steps of drawing a vertical bar graph.**

- Draw two lines XX' and YY' on the graph paper. Both lines are perpendicular to each other and cut at point 'O'. The horizontal line is called X-axis and the vertical line is called Y-axis.
- Give title as "multiple bar graph for vegetables."
- Label X-axis as "type of vegetables." and Y-axis as vegetables sold (kg).
- Make a key for bars of data.
- Make bars of equal width and at equal distance on x-axis.

**Example 5**

The graph below shows the number of students enrolled in two different schools of Lahore. Read the graph and answer the following questions.

- Make a table for enrolled students of both schools.
- How many less students enrolled in school-B in year 2020 than 2019?
- How many total number of students enrolled in school-A in 5 years?
- How many total students enrolled in 2017 in both schools?



Solution

(i)	Years	2016	2017	2018	2019	2020
	School-A	400	700	600	800	200
	School-B	500	400	500	600	300

(ii) $600 \text{ students} - 300 \text{ students} = 300 \text{ students}$.

(iii) $400 + 700 + 600 + 800 + 200 = 2700$

Hence, 2700 students enrolled in school-A in 5 years.

(iv) $700 \text{ students} + 400 \text{ students} = 1100 \text{ students}$.



Practice -2

Draw a multiple bar graph for the below set of data that shows the obtained marks of two students.

	Maths	English	Urdu	Science	Islamiyat	GK
Ali	35	25	28	37	45	48
Ahmed	40	30	45	35	25	40



Exercise - 11.2

1. The table shows the monthly sale of water coolers in a shop in 2018. Draw the horizontal bar graph.

Name of month	April	May	June	July	August	September
Number of water coolers	50	40	80	35	25	15

2. The following data shows the number of students of school who were late during a week. Construct a horizontal bar graph to show this information.

Days	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
Number of students	15	10	25	10	10	27

3. Draw a vertical bar graph for the money collected by a teacher for charity.

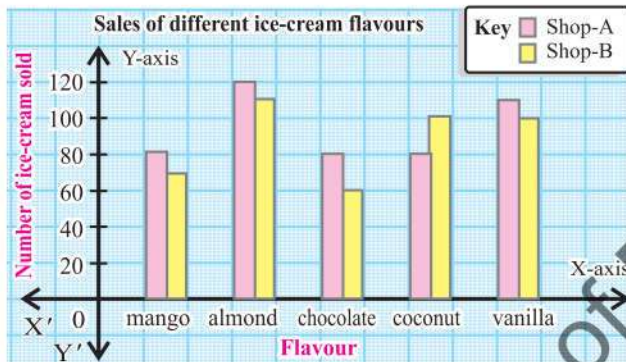
Name of students	Ahmed	Humaira	Sobia	Afshan	Aleezey	Rabia	Arslan	Saima
Amount (Rs)	20	90	50	70	30	20	40	10

4. Construct vertical bar graph using the daily sale of jeans and dress shirts sale at Ali's shop. The daily sale record is shown in the given table. Also, represent the result on horizontal bar graph.

Day	T-shirts	Dress shirts
Monday	40	20
Tuesday	60	25
Wednesday	35	20
Thursday	80	60
Friday	65	30
Saturday	28	37
Sunday	60	85



5. Draw the multiple bar graph for the data given Q.4.
6. The following bar graph shows the sales record of different flavoured ice-creams of two shope. Read the graph and give the answers of given questions.



- (i) How many mango ice-creams were sold by shop-A?
- (ii) Which flavour of ice-cream do people like most in shop-A?
- (iii) Find the total number of chocolate ice-creams sold by shop-A and shop-B.
- (iv) Which shop sold 80 coconut ice-creams?



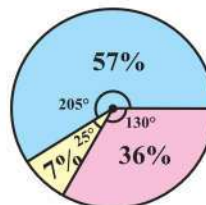
Project - 2

Visit your the nearest market and get the latest prices of the following vegetables, show the information on horizontal and vertical bar graphs.

(i)	Potato		Rs. _____	(v)	Tinda		Rs. _____
(ii)	Cauliflower		Rs. _____	(vi)	Tomato		Rs. _____
(iii)	Beet Root		Rs. _____	(vii)	Okra		Rs. _____
(iv)	Carrot		Rs. _____	(viii)	Bottle Gourd		Rs. _____

11.3.2 Pie Chart

A graph in which data is represented by “pie chart” or sector of circle is known as pie graph. When we have many quantities in data, we use a pie graph or pie chart to represent these quantities. It shows the contribution of each value to the total.



Key	
■	Before working age (1 – 7) years
■	Working age (18 – 65) years
■	After working (65+) years



Note

This information have been taken in the face.

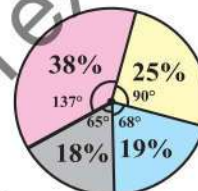
Reading a pie graph

We know that a circle has an angle of 360° . In a pie chart, the values/quantities are represented by sectors of a circle. It is useful to show or compare various quantities. Look the following example.

Example

6

Waqar did a survey in his school where he asked all the children to name their favourite colours. The results are shown in the pie chart. If there are 800 students in the school, then study the pie graph and answer the following questions:



Key	
■	Represented Red
■	Represented Orange
■	Represented black
■	Represented blue

There are 800 students in all

- Which is the most favourite colour among the students, and how many students like this colour?
- Which is the least favourite colour among students? Also, tell how many number of students like this colour?
- How many more students like blue colour than black colour?
- How many students like red colour?

Solution

Total number of students in the school = 800 students

- The most favourite colour among students is orange because its representing sector is the largest in the pie graph.

Orange colour sector represents = 38 %

Number of students like orange colour = 38% of 800

$$= \frac{38}{100} \times 800 = 304 \text{ students}$$

- The least favourite colour among students is black because the representing sector is the smallest in the pie graph.

Black colour represents = 18 %

Number of students like black colour = 18% of 800

$$= \frac{18}{100} \times 800 = 144 \text{ students}$$

- Blue sector represents = 19%

Number of students like blue colour = 19% of 800

$$= \frac{19}{100} \times 800 = 152 \text{ students}$$

Now, Blue sector 's students – black sector 's students

$$152 - 144 = 8 \text{ students}$$

Therefore, 8 more students like blue colour than black colour.

(iv) Red sector represents = 25%

Number of students like red sector = 25% of 800

$$= \frac{25}{100} \times 800$$

$$= 200 \text{ students}$$



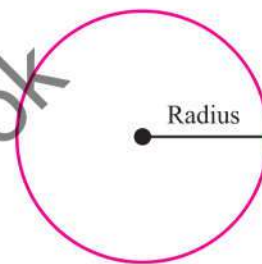
Activity -3

Take a blank sheet and draw a circle with compass and then use information given in example-3 then make a pie chart as given in particular example.

Drawing a pie chart

To draw a pie chart for given data, follow the following steps:

- (i) Draw a circle of any radius.
 - (ii) Find the central angle of each sector by using the formula.
- $$\text{Angle of sector} = \frac{\text{Value of the sector}}{\text{Total value}} \times 360^\circ$$
- (iii) Join the centre to the circle.
 - (iv) Start with the horizontal radius and draw radii making corresponding to the value of respective sectors.
 - (v) Repeat the process for all the data values or groups.
 - (vi) At the end shade every sector of the data values of different colours.



Example 7 Rabia wrote her daily time table on a paper and wanted to draw a pie chart of her time table. How can she draw the graph? If she spent her time as follow:

Activities	Sleeping	School	Watching T.V	Homework	Playing
Time (hours)	8	6	5	3	2

Solution

Total number of hours in a day = 24

$$\text{Angle of sleeping sector} = \frac{8 \text{ hours}}{24 \text{ hours}} \times 360^\circ$$

$$= 120^\circ$$

$$\text{Angle of school sector} = \frac{6 \text{ hours}}{24 \text{ hours}} \times 360^\circ$$

$$= 90^\circ$$

$$\text{Angle of homework sector} = \frac{2 \text{ hours}}{24 \text{ hours}} \times 360^\circ$$

$$= 30^\circ$$

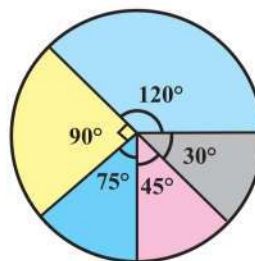
$$\text{Angle of T.V sector} = \frac{5 \text{ hours}}{24 \text{ hours}} \times 360^\circ$$

$$= 75^\circ$$

$$\text{Angle of playing sector} = \frac{3 \text{ hours}}{24 \text{ hours}} \times 360^\circ$$

$$= 45^\circ$$

Now, construct pie chart using angle of each sector.



Key	
	Sleeping time
	School time
	T.V watching time
	Playing time
	Home work time



Exercise - 11.3

1. Mr. Shahid gave his class surprise tests at the beginning and end of year. The given pie charts shows the grades. Study the pie graphs, do you think he must be happy with the results?



End of term result

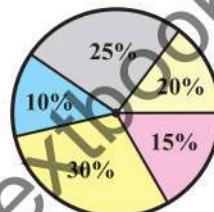


Beginning result

Key

- A - grades
- B - grades
- C - grades
- D - grades
- F - grades

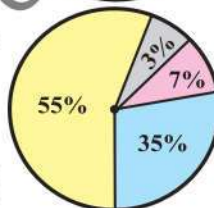
2. The pie graph shows the percentage of obtained marks. If Waqas obtained 300 marks out of 400 marks, then study the graph and find the marks obtained in each subject. Also, tell in which subjects Waqas obtained maximum and minimum marks?



Key

- English
- Science
- Mathematics
- Urdu
- Islamiyat

3. Aleezey is using social media network regularly. The following pie graph is showing the result of likes, comments, inbox messages and shares on her posts in the month of June 2022. Study the pie graph and answer the given questions.



Key

- Likes
- Comments
- Inbox Messages
- Shares

Aleezey received 6000 likes, comments and shares on her posts then.

- (i) Find the number of likes, comments, inbox messages and shares on Aleezey's posts.
- (ii) How many more likes are there than comments?
- (iii) How many less shares are there than likes?
- (iv) How many inbox messages are there?



Go Online

Visit the following link to play online game.

https://www.transum.org/software/sw/starter_of_the_day/students/pie_charts.asp

4. Draw the pie charts for the following data tables:

- (i) The following table shows the amount of money raised through fund raising by a group of students.

Income source	Income (Rs.)
Charity walk	3000
Donation from parents	2500
Donation from teachers	3200
Ticket selling	8400
Dressing-up-day	2500

- (ii) The following table shows the number of students in a school who achieved each grade.

Grades	Number of students
Distinction	100
Merit	250
Pass	400
Fail	100



Project -2

Draw a pie chart of your grade 5 final term obtained marks in each subject.

11.4 CALCULATION OF MEAN, MEDIAN AND MODE

The most common numerical measures in statistical data are mean, median and mode. These measures are also called averages in statistics. These are also known as measures of central tendency.

“Central tendency of data is the value that represents the whole data or the stages at which the largest number of items tends to concentrate and so it is called central tendency.”

(a) Mean

In order to find the mean of few number of quantities, first we add then divide them by the total number of quantities. It can be calculated by formula.

$$\text{Mean} = \frac{\text{Sum of quantities/values}}{\text{Number of quantities/values}}$$



Example 8 The height of 5 children is 1.7 metre, 1.9 metre, 1.4 metre, 1.6 metre and 1.3 metre. Find the mean height of the children.

Solution

$$\begin{aligned} \text{Mean height} &= \frac{\text{Sum of heights}}{\text{Number of children}} \\ &= \frac{1.7 + 1.9 + 1.4 + 1.6 + 1.3}{5} = \frac{7.9}{5} \\ &= 1.58 \text{ m} \end{aligned}$$

Therefore, the mean height of each children is 1.58 m.



Example 9 The mean of Rs. 75, Rs. 45, Rs. 60 and x is Rs. 67.5. Calculate the value of x .

Solution

$$\begin{aligned} \text{Since,} \\ \text{Mean value} &= \frac{\text{Sum of values}}{\text{Number of values}} \\ 67.5 &= \frac{75 + 45 + 60 + x}{4} \end{aligned}$$

$$67.5 = \frac{180 + x}{4}$$

$$67.5 \times 4 = 180 + x$$

$$270 = 180 + x \Rightarrow x = 270 - 180 = 90$$

Therefore, the required value of x is Rs. 90.

Practice -4

1. In Lahore the temperature of five days in winter was 18°C , 19°C , 20°C , 17°C and 37°C . Find the mean of temperature.
2. The mean of height of five numbers is 10. Find the value of x if the numbers are 79, 94, x , 105 and 200.

Activity -4

Find the mean of listed prices of your all books.

(b) Median

“The median in statistics is the middle most value in the given set of data.”

In order to find the median, we simply arrange the values/data in an ascending or descending order, then find the middle values.

The middle term can be calculated by using the following formula:

$$\text{Middle term} = \left(\frac{n+1}{2} \right)^{\text{th}} \text{ term, where } n \text{ is total number of observations}$$

Example 10 Find the median of the following set of data 45, 42, 53, 61, 44, 59 and 64.

Solution

$$\begin{aligned} \text{Middle term} &= \frac{n+1}{2} \\ &= \frac{7+1}{2} = 4 \end{aligned}$$

The median is the fourth term in the set of data. So, arrange the data in an ascending order.

Ascending order: 42, 44, 45, 53, 59, 61, 64
↓
4th term

Hence, the median is 53.

Example 11 Find the median of 92, 49, 87, 75, 101, 69, 105, 80, 115 and 108.

Solution

$$\text{Middle term} = \left(\frac{n+1}{2} \right)^{\text{th}} \text{ term} = \left(\frac{10+1}{2} \right)^{\text{th}} \text{ term} = 5.5^{\text{th}} \text{ term}$$

The median is the 5.5th term in the set of data. So, arrange the given set of data in an ascending order.

Ascending order: 49, 69, 75, 80, 87, 92, 101, 105, 108, 115

5.5th term

$$\text{Median} = \text{Mean of two middle values} = \frac{87+92}{2} = 89.5$$

Hence, the median is 89.5

**Remember**

If the given set of data is even then find the mean of two middle values to get median.



Practice -5 Find the median of the following set of data:

- (i) 7.5, 4.6, 2.9, 3.8, 7.5, 8.4, 5.9 (ii) 9.4, 8.7, 8.5, 7.4, 6.9, 8.9, 7.8, 9.9

**Activity -5**

Find the mode of all students marks of your class.

(c) Mode

“The value that appears most frequently in the set of data is called mode.”

A set of given data may have one mode, more than one mode, or no mode.

In order to find the mode in the given set of data, we simply, observe the data and write the most appeared value.

Example 12 Find the mode of 4, 4, 7, 8, 9, 13, 4, 13, 6, 0, 13, 15 and 13.

Solution

Arrange the data in an ascending order. 0, 4, 4, 4, 6, 7, 8, 9, 13, 13, 13, 13, 15

As, the most appeared value is 13. Therefore, mode = 13.

**Practice -6**

From the following sizes of kids trousers, find the model size of trousers.

28, 30, 25, 37, 29, 25, 33, 34, 25, 27 and 25.

**Exercise - 11.4**

1. Find the mean of the following data:
 - (i) 8, 9, 7, 14, 15, 10 and 20
 - (ii) 14kg, 13kg, 10kg, 15kg and 25kg
 - (iii) 13m, 14m, 37m, 24m, 20m, 16m and 19m
 - (iv) 2.4, 8.4, 7.8, 4.6, 7.7 and 9.7
 - (v) 134km, 415km, 168km, 250km, 424km, 342km and 241km
 - (vi) 42.6°C, 34°C, 37°C, 41°C, 38.6°C, 42.6°C, and 40.3°C
2. Nadeem read a storybook of 150 pages in 6 days. What is his per day reading of book pages?
3. The marks obtained by the students of grade 6 in the subject of Mathematics are given below. 85, 84, 80, 70, 71, 90, 55, 45, 78, 33, and 39. Find the mean of obtained marks.
4. The mean of 10 numbers is 29.01, seven number of them are 10, 15, 18, 34, 45, 26 and 32. The remaining three numbers are equal to k . What is the value of k ?
5. Find the mode and median for each set of data given in Q.1.
6. The scores of 12 students in a Mathematics class test are 18, 17, 18, 13, 9, 11, 13, 24, 22, 18, 11, 18. Find the model score of the class.
7. Find the median of the given set of data 47.5, 40, 25.5, 27.7, 49.8, 32.4, 9.6, 8.4, 40.

**Project - 4**

Visit your the nearest market. Get the prices of at least five electronic items then find the mean, median and mode of your collected data e.g.

(a)



(b)



(c)



(d)



(e)

**Go Online**

Visit the following link and complete the online quiz about mean, median and mode.

<http://www.mathgames.com/skill/7.90-interpret-charts-to-find-mean-median-mode-and-range>.

Review Exercise**11**

1. Each of the following question is followed by four suggested options. In each case select the correct option.
 - (i) The collection of any information or fact is called:
 - a. frequency
 - b. interval
 - c. information
 - d. data
 - (ii) The data can be classified into:
 - a. 2 types
 - b. 3 types
 - c. 4 types
 - d. 5 types

**Activity -6**

Collect the data of ages of 15 people and find the mean, median and mode of your collected data.

(iii) The tally marks show:

- a. data b. class c. frequency d. graph

(iv) The bar graph is also known as:

- a. Pie chart b. bar chart c. line graph d. pictograph

(v) Pie slices are used to draw:

- a. bar chart b. pie chart c. line graph d. pictograph

(vi) The total angle used in pie chart is:

- a. 90° b. 180° c. 270° d. 360°

(vii) The mean is also known as:

- a. speed b. distance c. average d. addition

2. Define the following:

- (i) data (ii) grouped data (iii) ungrouped data (iv) mean
(v) median (vi) mode (vii) bar graph

3. Which table is showing the grouped data and which is showing the ungrouped data?

(i) The sales record of a month for a bookshop is shown in the table.

Number of books	Days
201 – 250	4
251 – 300	8
301 – 350	15
351 – 400	5
401 – 450	10

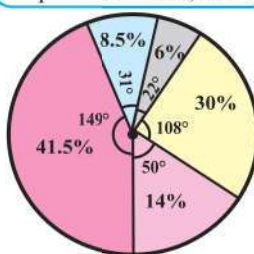
(ii) Number of visitors at an exhibition at expo-centre Lahore.

Days	Number of visitors
Saturday	150
Sunday	200
Monday	325
Tuesday	250
Wednesday	175
Thursday	100
Friday	195

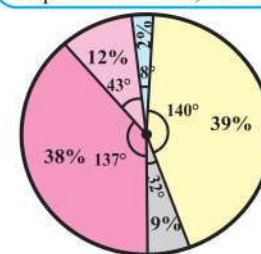
4. Draw a horizontal and vertical bar graphs of your obtained marks of each subject of your recent monthly test.

5. These pie charts show how people travel to work in Model Town and Awan Town. The population of the towns are given.

Model Town Lahore
Population = 100,000



Awan Town Lahore
Population = 125,000



Compare the both graphs and answer the following questions:

- (i) In which town do more people travel to work by cars and how many people are travelling by car?
(ii) In which town how many more people are travelling by bikes?
(iii) Find how many people are travelling by walk, train and bus in each town?

6. The number of students of 7 different classes is given below. Draw the vertical bar graph.

Classes	1	2	3	4	5	6	7
No. of students	15	25	18	29	30	25	35

7. The number of bed sheets manufactured by a factory during six consecutive weeks is given below. Represent this data using bar graph.

Weeks	1 st	2 nd	3 rd	4 th	5 th	6 th
No. of bed sheets	350	200	250	300	150	425

8. The following table shows the record of weekly pocket money of two students. Read the table and draw multiple bar graph.

	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
First student	35	60	70	90	80	100
Second student	60	80	50	100	55	50

Summary

- Data is a collection of any information and facts.
- The frequency of an event is the number of times the event occurred in an information/data.
- The data which provides us information or data points individually is called as ungrouped data.
- The data which is given in intervals or groups is called grouped data.
- The bar graph is graphical display of data using bars of different height.
- A graph in which data is represented by "pie chart" or sector of circle is known as pie graph.
- In the circle/pie chart the total angle of the circle is 360° .
- Angle of sector can be calculated by using the formula: $\text{Angle of sector} = \frac{\text{Value of the sector}}{\text{Total value}} \times 360^\circ$
- The average of given data is also known as mean.
- $\text{Mean} = \frac{\text{Sum of values}}{\text{Number of values}}$
- The middle term in any set of data is called median if it is arranged in an ascending or descending order.
- To calculate middle term, we use the formula: $\text{Median} = \left(\frac{n+1}{2} \right)^{\text{th}} \text{ term}$
- The most appeared value in any set of data is called mode.



Teaching Point

The questions in the exercises, practices and different activities are given as examples (symbols) for learning. You can use self generated questions (test items) conceptual type MCQ's, fill in the blanks, column matching, constructed response questions and (simple computations) based on cognitive domain (e.g. knowing = 40%, applying = 40% and reasoning = 20%) to assess the understanding of learners.

PROBABILITY

Students' Learning Outcomes:

By the end of this sub-domain, students will be able to:

- Explain experiments, outcomes, sample space, events, equally likely events and probability of a single event.
- Differentiate the outcomes that are equally likely and not equally likely to occur. (including real-world word problems).



What will happen when you roll a dice once? Predict your possible outcome/result.



INTRODUCTION

Have you ever heard about a toss before the start of cricket match between two cricket teams (e.g. Pakistan and Indian cricket teams) on television (TV)? A coin is used for toss to decide that which team will bat first.

Similarly, there are many aspects of uncertainty and chances in our routine life. e.g. Will Pakistan cricket team win the match? If throw a dice will I score double of 5? It is likely to rain. Should I go outside with umbrella?

To overcome these types of situations we use the term “probability” in Mathematics.

“Probability is the branch of Mathematics that measures how likely something will happen.”

We use it to find the probability of an outcome of an event. Here, we are going to learn a few basic concepts, probability formulas involved to calculate the probability of different types of situations.



12.1 EXPERIMENT

In statistics “an experiment is any procedure that can be repeated many times and has a well defined sets of possible outcomes”. There are two kinds of experiments.

Scientific experiment

This is an experiment for which outcome can be predicted with certainty beforehand such as oxygen and hydrogen or add two numbers such as 4 and 5, etc.

Random experiment

An experiment for which the outcome cannot be predicted except that it is known to be one of the set of possible outcomes, is called a random experiment. e.g. for any given triangle, without knowing the three angles, we can clearly say that the sum of measures of interior angles of a triangle is 180° . Similarly, tossing a coin and rolling a dice, are examples of random experiment.

Experiments			There are 4 green balls and 5 red balls in the bag. A ball is drawn at random.	
Possible outcomes				

Since, our interest lies in the random experiment, so in this text by experiment we mean random experiment.

12.2 SAMPLE SPACE AND AN EVENT

Note

An element of the sample space is called sample point.

Sample space

For a random experiment, the set of all possible outcomes of a random experiment is called a sample space and is denoted by S . The elements of S are called sample points of outcomes.

For example

- (i) Tossing a coin once, then

$$S = \{H, T\}$$

where H and T are the possible outcomes.



- (ii) Tossing a coin twice, then the possible outcomes in the sample space are $\{HH, HT, TH, TT\}$

- (iii) Consider an experiment of rolling a dice. The possible outcomes of this experiment are 1, 2, 3, 4, 5 and 6.



If we are interested in the number of dots on the upper face on the dice.

- (iv) Rolling two dice, then we have sample space

$$S = \left\{ \begin{array}{l} (1,1) (1,2) (1,3) (1,4) (1,5) (1,6) \\ (2,1) (2,2) (2,3) (2,4) (2,5) (2,6) \\ (3,1) (3,2) (3,3) (3,4) (3,5) (3,6) \\ (4,1) (4,2) (4,3) (4,4) (4,5) (4,6) \\ (5,1) (5,2) (5,3) (5,4) (5,5) (5,6) \\ (6,1) (6,2) (6,3) (6,4) (6,5) (6,6) \end{array} \right\}$$

Remember

Outcome: A possible result of a random experiment is called its outcome.

Practice -1

Write down the sample space for tossing a coin three times.

Events

Let S be the sample space of an experiment. Any subset E of S is called an event associated with the experiment. For example $E = \{HH, TT\}$ is an event associated with the experiment of tossing a coin twice. There are two types of events.

(i) Simple event

“An event which contains a single point of sample space is known as a simple event” e.g.

$S = \{H, T\}$ then $E = \{H\}$ is a simple event.

(ii) Compound event

“An event which contains more than one point of sample space is called compound event.” e.g.

If $S = \{HH, HT, TH, TT\}$ then $E_1 = \{HH, TT\}$ and

$E_2 = \{HT, TH\}$ represent compound events.

(iii) Equally Likely Events

Two events are said to be equally likely if they have equal chances of happening. For example when we toss a coin there are equal chances of getting a head or a tail. So, getting a head and tail by tossing a coin are equally likely events. When a dice is rolled then getting an odd number and getting an even number are equally likely events.

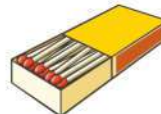


Note

If the random experiment results in P and $P \in S$. We say that the event A occurs or happens.

(iv) Not Equally Likely Events

An event is said to be not equally likely if one or more events are more likely to occur than others.



For example, if you were using a dice that always landed with six facing up, that would be not equally likely event. Similarly, a matchbox has 6 faces but all the faces are not equally likely. So, each face has different occurring value.

12.3

PROBABILITY OF SINGLE EVENT

We use a formula to calculate the probability of an event to occur. “How likely something is to occur / happen is called probability”.

Let S be the sample space of a random experiment, and E be an event. The probability that an event E will occur, is denoted by $P(E)$. So,

$$\text{Probability of an event occur} = \frac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}}$$

$$P(E) = \frac{n(E)}{n(S)}$$



Project - 1

Differentiate equally likely and not equally likely outcomes/events with examples from your surrounding.

**Note**

event.

Not equally likely event is also known as unfair event and equally likely event is also known as fair event.

**Practice -2**

Shahbaz and Amina has 4 new vehicles this year, 2 cars and 2 wagons. What is the probability that their next number will be a car?

Example 1

Find the Probability of getting an even number when a dice is rolled.

SolutionSample space = $S = \{1, 2, 3, 4, 5, 6\}$

$$n(S) = 6$$

$$n(E) = 3$$

$$\text{So, } P(E) = \frac{n(E)}{n(S)} = \frac{3}{6} = \frac{1}{2}$$

**Example 2**

(i) Find a probability that a tail will turn up if a coin is tossed.

(ii) Find a probability that an odd number has shown up if a fair dice is tossed.

Solution(i) Here $S = \{H, T\}$ Let A = set of tails

$$= \{T\}, n(A) = 1$$

Since, the outcomes are equally likely, then by using the formula:

$$P(A) = \frac{n(A)}{n(S)} = \frac{1}{2}$$

(ii) Here $S = \{1, 2, 3, 4, 5, 6\}$ Let B = set of odd numbers has shown up

$$= \{1, 3, 5\}$$

Since, the outcomes are equally likely then we have

$$P(A) = \frac{n(B)}{n(S)} = \frac{3}{6} = \frac{1}{2}$$

**Remember**

The probability of an event is always a number from 0 to 1 (inclusive). Look at the below probability scale.

**Do you Know?**

- If the probability of an event occurring is '0', we call it an impossible event.
- If the probability of an event occurring is '1', we call it certain event.

Example 3

Aleezey is playing cards with her friend when she draws a card from a pack of 56 cards numbered from 1 to 56. What is probability of drawing a number that is perfect of square?

SolutionHere $S = \{1, 2, 3, \dots, 56\}$ B = Set of perfect square numbers.

$$= \{1, 4, 9, 16, 25, 36, 49\}$$

Since, the outcomes are equally likely then we have $P(B) = \frac{n(B)}{n(S)} = \frac{7}{56} = \frac{1}{8}$





Exercise - 12.1

1. Each of the letters in the words Jammu Kashmir are on separate cards, face down on the table. If you pick a card at random, what is the probability that its letters will be J and K?
2. What is the probability of rolling a dice at:
 - (i) rolling a '5'?
 - (ii) rolling a number greater than '6'?
 - (iii) rolling a number less than or equal to '3'?
3. Two unbiased coins are tossed what is the probability of obtaining:
 - (i) all tails
 - (ii) one head
 - (iii) one tail
 - (iv) all heads
4. There are 5 green colours, 3 red colours and 4 blue colours in a box. What is the probability that:
 - (i) all colours are green
 - (ii) all colours are blue
5. Consider a random experiment of tossing three coins each coin toss only once.
 - (i) Find the sample space S if we want to determine the exact sequence of tails and heads obtained.
 - (ii) Find the probability of two tails only.
6. What is the probability of number at least 5 when two dice are rolling once. Also find the probability of number at least 1.
7. There are 200 Covid-19 patients in a private hospital in Lahore. Of these 20 are selected to undergo a special treatment that increases the percentage cured rate from 60% to 85%. What is the probability that a patient received a special treatment if the patient is known to be cured?
8. Two girl students and two boy students enter in a classroom and take 4 seats random in a row. What is the probability that the boy students take the two end seats?



Review Exercise

12

1. Each of the following questions is followed by four suggested options. In each case select the correct option.
 - (i) The branch of Mathematics that measures how likely it is that something will happen, is called:
 - a. event
 - b. probability
 - c. rounding off
 - d. estimation
 - (ii) There are _____ kinds of experiments:
 - a. 1
 - b. 2
 - c. 3
 - d. 4
 - (iii) The set of all possible outcomes of a random experiment is called:
 - a. sample space
 - b. event
 - c. head
 - d. tail
 - (iv) A possible result of random experiment is called:
 - a. event
 - b. estimation
 - c. outcome
 - d. scientific experiment
 - (v) The sample space for rolling a dice is:
 - a. $S = \{1, 2, 3, 4, 5, 6\}$
 - b. $S = \{0, 1, 2, 3\}$
 - c. $S = \{0, 1, 2, 3, 4\}$
 - d. $S = \{1, 2, 3, 4, 5\}$

(vi) An event that contains a single point is called:

- a. simple event b. compound event c. likely event d. exclusive event

(vii) The probability of single event can be calculated by using the formula:

- a. $P(E) = \frac{n(F)}{S}$ b. $P = \frac{E}{S}$ c. $P(E) = \frac{n(E)}{n(S)}$ d. $P(E) = \frac{E}{n(S)}$

(viii) The probability of heads for tossing a coin is:

- a. 1 b. 2 c. $\frac{1}{2}$ d. $\frac{1}{3}$

(ix) The probability of letter 'E' in the word MATHEMATICS is:

- a. $\frac{2}{11}$ b. $\frac{1}{11}$ c. $\frac{3}{11}$ d. $\frac{2}{2}$

2. Define the following in a short way:

- | | | |
|-------------------------|-----------------------------|----------------------------|
| (i) Probability | (ii) Experiment | (iii) Sample space |
| (iv) Outcome | (v) Simple event | (vi) Compound event |
| (vii) Random experiment | (viii) Equally likely event | (ix) Scientific experiment |

3. The two coins are tossed. Find the probability that two tails are obtained.

4. The two dice are rolled, find the probability that the sum is an even prime.

5. A spinner is divided into 8 equal sectors numbered 1 through 8. If the spinner is spun, is it certain, likely, unlikely, equally likely or impossible that it lands on a number less than 5? Also, find its probability of numbers less than 5.

Summary

- An experiment for which the outcome cannot be predicted except that it is known to be one of a set of possible outcomes is known as random experiment.
- The set of all possible outcomes of a random experiment is known as sample space. It is usually denoted by S .
- Let S be a sample space of an experiment. Any subset E of S is known as an event associated with the experiment.
- Two events are said to be equally likely if they have equal chances of happening.
- An event containing one sample points is called a simple event.
- An event containing at least two sample points is called a compound event.
- Let S be a sample space and E be an event of an experiment then the probability of the event can be calculated by using the formula.

$$\text{Probability of an event occur} = \frac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}}$$

$$P(E) = \frac{n(E)}{n(S)}$$



Go Online

To play the different online games and practice the mathematical concepts visit the following links.

♦ <https://www.educatingyouth.net/videos>

♦ <https://www.calculator.net/probability-calculator.html>

ANSWERS



Exercise - 1.1

- (i) 1, 2, 3, 5, 6, 10, 15, 30
(iv) 1, 2, 3, 6, 13, 26, 39, 78
(vi) 1, 2, 59, 118
(viii) 1, 2, 4, 5, 10, 22, 44, 55, 110, 220
- (i) 7, 14, 21, 28, 35
(iii) 19, 38, 57, 76, 95
(v) 59, 118, 177, 236, 295
4. No even prime numbers
6. 51, 55, 57, 63, 65, 69, 75, 77, 81, 85, 87, 91, 93, 95, 97, 99
- (ii) 1, 41
(v) 1, 3, 5, 7, 15, 21, 35, 105
(vii) 1, 3, 5, 11, 15, 33, 165
- (ii) 13, 26, 39, 52, 65
(iv) 43, 86, 129, 172, 215
3. 31, 37, 41, 43, 47, 53, 61
5. 2
7. 1



Exercise - 1.2

- (i) 2, 10
(ii) 3, 35
(iii) 2, 90
2. (i) $5^4 \times 7$
(ii) $2^2 \times 5^4 \times 7^3$
(iii) $3^4 \times 5^3$
(iv) $11^4 \times 13^1 \times 19^3$
- (i) $2 \times 2 \times 2 \times 2 \times 2$
(iii) $2 \times 5 \times 11$
(v) 5×23
(vi) $2 \times 2 \times 3 \times 3 \times 3 \times 5$
(viii) $3 \times 3 \times 5 \times 7 \times 7$
(x) $3 \times 3 \times 3 \times 3 \times 3 \times 5 \times 5$
- (i) 2^6
(ii) 5^3
(iii) $2^2 \times 5^2$
(iv) $2^2 \times 5^3$
(v) $2^2 \times 47$
(vi) $2^3 \times 3^4$
(vii) $2^3 \times 3^3 \times 7$
(viii) $2^3 \times 3^4 \times 5$
(ix) $2^3 \times 3^3 \times 5 \times 7$
- Activity based.
6. 182
7. 70
- (i) $2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 5 \times 5$
(ii) $3 \times 3 \times 3 \times 3 \times 7 \times 7 \times 11 \times 11 \times 11$
(iii) $5 \times 5 \times 5 \times 5 \times 5 \times 7 \times 7 \times 11 \times 11 \times 11 \times 11$
(iv) $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3 \times 13 \times 13 \times 13 \times 17 \times 17$



Exercise - 1.3

- (i) 3
(ii) 5
(iii) 2
(iv) 5
(v) 14
(vi) 18
- (i) 10
(ii) 2
(iii) 4
(iv) 32
(v) 6
(vi) 5
- 1
4. No, A number greater than 8 cannot be factor of 8.
5. 2 books of each subject
6. 10 students
7. 5cm



Exercise - 1.4

- (i) 72
(ii) 196
(iii) 180
(iv) 1575
(v) 975
(vi) 5250
- (i) 126
(ii) 225
(iii) 84
(iv) 196
(v) 480480
- 75 cm
4. 42 minutes or 2520 sec
5. 3364
- 09 : 00 pm or 21 : 00
7. 240



Exercise - 1.5

- (i) 250
(ii) 224
(iii) 2312
(iv) 2695
(v) 25300
(vi) 60800
- 397120
3. 630
4. 288



Exercise - 1.6

- (ii), (v), (vi), (viii) and (x)
- (i) 169
(ii) 64
(iii) 529
(iv) 289
(v) 961
(vi) 1849
(vii) 9
(viii) 3249
(ix) 5041
(x) 8649

Review Exercise

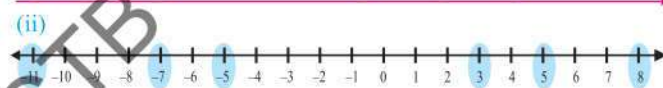
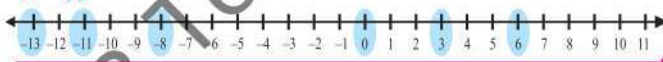
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- (i) a
(viii) c
(ix) a
(x) b
- 8, 643
3. 12
4. 12:30 p.m.
5. 363
6. 1:00 p.m.



Exercise - 2.1

- 5, 6, 7, 8, 9, 10
2. 1
3. 0, 1, 2, 3, 4, 5, 7, 8, 9
- 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12
- (i) Yes.
(ii) 0 is whole number but it is not natural number.
(iii) All natural numbers are whole numbers.
- (i) $S = 4$, $P = 7$, $A = 12$
(ii) $A = 54$, $T = 57$, $X = 61$, $R = 65$
- (i) -48
(ii) +48
- (i) -4
(ii) -2
(iii) +1
(iv) A
(v) C
(vi) F
- (i)



- (i) -14
(ii) +24
(iii) -65
(iv) +39
(v) 0
(vi) +420
- +12, -17, +26, -57, 0
12. 7
13. 8
14. 97°F



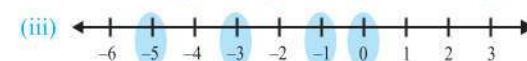
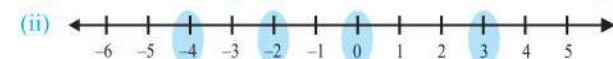
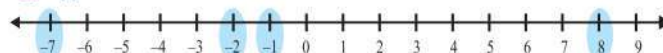
Exercise - 2.2

- (i) -7, -4, -2, 0, 1, 3, 4
(ii) -6, -3, -1, 0, 6, 7, 17
- (i) 16, 7, 0, -1, -2, -4
(ii) 24, 13, 8, 5, 0, -10
- (i) 45
(ii) 116
(iii) 209
(iv) 0
(v) 2813
(vi) 501
- (i) 0, 4, 9, 12, 13, 16, 17
(ii) 12, 14, 18, 44, 48, 84
- (i) 34, 21, 17, 16, 12, 9
(ii) 215, 215, 149, 141, 109, 85, 0

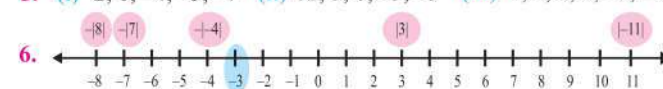
Review Exercise

2

- (i) d
(vii) a
(viii) a
(ix) c
(x) a
(xi) b
- (i)



- (i) -4, -3, 0, 2, 4, 7
(ii) -13, -11, -7, 0, 1, 4
(iii) -6, -4, 0, 4, 5, 15
- (i) 2, 0, -4, -5, -7
(ii) 12, 5, 0, -3, -5
(iii) 8, 5, 3, 0, -2, -6



7. 1st floor 8. 15°C 9. Activity based
10. 15, 0, 18, 15, 13, 215, 131, 54 11. 22 km



Exercise - 3.1

1. (i) +16 (ii) -10 (iii) -14 (iv) +9 (v) +11
(vi) +39 (vii) +7 (viii) +1 (ix) +15
2. (i) +6 (ii) -50 (iii) +6 (iv) -75 (v) -5
(vi) -97 (vii) +2 (viii) -5 (ix) 29
4. -156 6. 62 7. 24 km



Exercise - 3.2

1. (i) +20 (ii) -24 (iii) 0 (iv) -22 (v) -25
(vi) +30 (vii) 0 (viii) -14
2. (i) -874 (ii) +360 (iii) 285 (iv) -943
(v) 0 (vi) +89
3. (i) -17 (ii) -9 (iii) -6 (iv) +6 (v) -108
(vi) undefined (vii) 0 (viii) -228
4. (i) -8 (ii) -10 (iii) 7 (iv) 720 (v) 0 (vi) 1
9. 222 10. Rs. 165 11. 30°C



Exercise - 3.3

1. (i) -25 (ii) 95 (iii) 186 (iv) 2 (v) 41860 (vi) 230
2. (i) 18.8 (ii) 6.8 (iii) 8.8 (iv) 26.76 (v) -72.96
(vi) 46.8 (vii) 6.3
3. (i) $1\frac{43}{99}$ (ii) $15\frac{13}{135}$ (iii) $2\frac{1}{18}$ (iv) $1\frac{11}{18}$ (v) $1\frac{5}{11}$
(vi) $-1\frac{1}{3}$ (vii) $1\frac{19}{35}$ (viii) $43\frac{5}{12}$
4. Rs. 108,000 5. Rs. 87,500 6. 7.1 m
7. Rs. 16,250 8. 2.5 m 9. $7\frac{7}{8}$ metres

Review Exercise

3

1. (i) b (ii) a (iii) b (iv) a (v) b (vi) b (vii) d
(viii) b (ix) c (x) b (xi) d (xii) d (xiii) c (xiv) d
3. (i) 74 (ii) -9 (iii) -0.65 (iv) 4.71 4. -80 5. -18
6. 50 7. 585 8. -200 9. 935 pages
10. 6 hours 11. 27 kg 12. Rs. 180



Exercise - 4.1

1. (i) 80 : 50 (ii) 35 : 55 (iii) 46 : 70 (iv) 85 : 50
(v) 35 : 85 (vi) 55 : 85 : 50 (vii) 46 : 62 : 70
2. (i) 3 : 5 (ii) 7 : 12 (iii) 34 : 37 (iv) 53 : 59
(v) 101 : 103 (vi) $x : y$ (vii) $x : x+1$ (viii) $a - b : p + q$
3. (i) $\frac{5}{6}$ (ii) $\frac{11}{13}$ (iii) $\frac{1}{2}$ (iv) $\frac{17}{18}$ (v) $\frac{15}{17}$
(vi) $\frac{17}{19}$ (vii) $\frac{4}{5}$ (viii) $\frac{2}{3}$
4. $\frac{2}{3}$, 2:3 5. Expenses = 33000 Savings = 22000 6. Moez: Rs. 26,400
Zurain: Rs. 33,000
Abdul Hadi: Rs. 39,600



Exercise - 4.2

1. (i) Rs. 857 (ii) Rs. 8570 (iii) Rs. 21,425
2. 3 km/h 3. 400 km 4. 6 m/h 5. Rs. 1725
6. (i) Rs. 17.5 (ii) Rs. 1407.5



Exercise - 4.3

1. (i) 33% (ii) 30% (iii) 50%
2. (i) $\frac{1}{4}$ (ii) $\frac{1}{5}$ (iii) $\frac{21}{25}$ (iv) $\frac{17}{50}$ (v) $\frac{93}{100}$
3. (i) 8 (ii) 9 litre (iii) 56 (iv) 720 (v) 92.5
(vi) 270 (vii) 7.84 (viii) 80.1 4. 50%
5. 280 students 6. 525 students 7. (i) 37.5% (ii) 62.5
8. Rs. 43,750 9. Rs. 600 10. Rs. 5.43m
11. Rs. 6000 student 12. march 13. Aleem 14. 31.45
15. 9 16. Rs. 51,750 17. Rs. 7320.32 18. 17.65%



Exercise - 4.4

1. (i) 19:45 (ii) 21:31 (iii) 20:35
2. (i) 03:48 p.m. (ii) 11:15 p.m. (iii) 07:43 p.m.
3. 10:30 p.m. 4. 04:45 5. 30 minutes



Exercise - 4.5

1. (i) $3\frac{8}{9}$ m/sec (ii) $11\frac{1}{4}$ m/sec (iii) $69\frac{4}{9}$ m/sec
2. (i) 90 km/h (ii) 468 km/h (iii) $1225\frac{4}{9}$ km/h
3. 107 km/h 4. 160 km 5. yes 6. 84.29 km/h

Review Exercise

4

1. (i) b (ii) c (iii) a (iv) b (v) d (vi) b (vii) b (viii) c
(ix) b (x) b (xi) d (xii) b (xiii) c
3. (i) 9:10 (ii) 1:3 (iii) 13:21 (iv) 7:8:9 (v) 2:3:4 (vi) 4 to 5 to 6
4. Rabia: Rs. 3948, Saima: Rs. 6580, Alina: Rs. 9212
5. 333.3 words 6. 35% 7. 240 votes 8. Rs. 1950
9. 160 10. Bananas = 345 Apples = 390 11. 60 boxes 12. yes
13. Rs. 90,000 14. 12 hours



Exercise - 5.1

1. (i) It is a set because all geometrical instruments are distinct
(ii) It is a set because all vehicles are distinct
(iii) It is not a set because tea cups are repeating
(iv) It is a set because all flowers are distinct
2. (i) False (ii) False (iii) True (iv) True (v) True
(vi) True (vii) False (viii) False (ix) True
3. (i) $\in$ (ii) $\notin$ (iii) $\in$ (iv) $\notin$ (v) $\notin$
(vi) $\in$ (vii) $\notin$ (viii) $\in$ (ix) $\in$
4. (i) A = {spring, summer, autumn, winter}
(ii) B = {Wednesday} (iii) Activity based (iv) Activity based
(v) E = {p, u, n, j, a, b} (vi) F = {Rajab, Shaban, Ramadan, Shawwal,
Dhu al Qidha, Zhu al Hajj}
(vii) G = {Imran Khan, Nawaz Sharif, Shahid Khaqan Abbasi,
Yousaf Raza Ghalani}

(viii) $H = \{2, 3, 5, 7\}$ (ix) $I = \{a, e, i, o\}$

5. (i) A = Set of first five whole number
 (ii) B = Set of four games
 (iii) C = Set of English alphabets
 (iv) D = Set of all provincial capitals of Pakistan
 (v) E = Set of last six months of solar colander
 (vi) F = Set of whole numbers between 20 and 31
 (vii) G = Set of all provinces of Pakistan
 (viii) H = Set of even numbers upto 14
 (ix) I = Set of three fruits



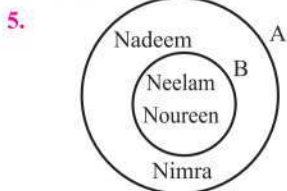
Exercise - 5.2

1. (i) empty set (ii) empty set (iii) not an empty set
 (iv) empty set
 2. (i) finite (ii) finite (iii) finite (iv) infinite (v) finite
 (vi) finite (vii) finite (viii) finite (ix) infinite
 3. (i) $n(A) = 3$, not singleton (ii) $n(B) = 1$, singleton
 (iii) $n(C) = 5$, not singleton (iv) $n(D) = 1$, singleton
 (v) $n(E) = 5$, not singleton (vi) $n(F) = 1$, singleton
 4. (i) $\supset$ (ii) $\longleftarrow, \subseteq$ (iii) $\subset$ (iv) $\longleftrightarrow$ (v) $\supset$
 (vi) $\subset$ (vii) $\supset$ (viii) $\supset$
 5. (i) equal (ii) equal (iii) equal (iv) equal
 (v) equivalent (vi) equivalent
 6. **Proper subsets:** $\{ \}$, $\{\text{Red}\}$, $\{\text{Yellow}\}$, $\{\text{Blue}\}$, $\{\text{Red, Yellow}\}$,
 $\{\text{Red, Blue}\}$, $\{\text{yellow, Blue}\}$
Improper Subsets: $\{\text{Red, Yellow, Blue}\}$
 8. $n(A) = 6$ 9. Activity based



Exercise - 5.3

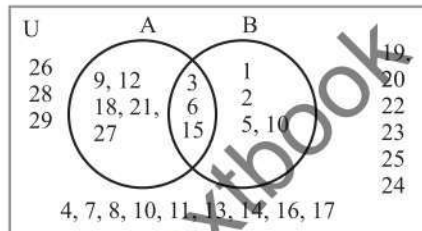
1. $U = \{\text{goat, cat, hen, dog}\}$
 $A = \{\text{hen, cat, dog}\}$
 2.
 3.
 4. (i) $A = \{1, 2, 3, 4, 5, 6\}$, $B = \{1, 3, 5\}$
 (ii) $U = \{2, 4, 5, 6, 7, 8, 9, 10, 13\}$, $A = \{7, 13\}$, B
 (iii) $X = \{1, 3, 5, 7, 9\}$, $Y = \{ \}$



Review Exercise

5

1. (i) a (ii) b (iii) a (iv) c (v) b
 (vi) d (vii) c (viii) b (ix) a (x) x
 3. (i) Activity based (ii) B = Set of negative whole numbers
 (iii) $\{\text{school diary}\}$
 4. $\{ \}$, $\{1\}$, $\{2\}$, $\{1, 2\}$, $\{8\}$, $\{1, 8\}$, $\{2, 8\}$, $\{1, 2, 8\}$
 5. $U = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$,
 $A = \{1, 2, 3, 5, 7\}$, $B = \{2, 4, 6, 8, 9\}$



Exercise - 6.1

1. (i) 21, 25 (ii) 307, 317 (iii) 42, 49 (iv) -78, -76
 2. (i) 25, 36, 49 (ii) 83, 81, 79 (iii) 24, 28, 32 (iv) 13, 21, 34



Exercise - 6.2

1. (i) $n + 6 = 20$ (ii) $n - 27 = 5$ (iii) $4n < 7$ (iv) $2n > 25$
 2. (i) False (ii) True (iii) True (iv) False (v) False
 3. (i) 18 (ii) 6 (iii) 400 (iv) $\frac{3}{10}$ (v) $\frac{15}{22}$
 4. (i) $C = 9, 0; V = n$ (ii) $C = 7, 2, 9, V = p$
 (iii) $C = \frac{1}{3}, V = xy$ (iv) $C = 7, 4, 9, V = i$
 (v) $C = 8, V = m, n$ (vi) $C = 9, -37, V = x, y$



Exercise - 6.3

1. (i) $3a + 8$ (ii) $5p - 5$ (iii) $5a \div 8b$
 (iv) $4ny$ or $n(4y)$ (v) Rs. 170n
 2. (i) $2n$ (ii) n (iii) $n - 3$
 3. (i) $n; 3$ (ii) $2n; 6m$ (iii) ℓmn (iv) $4xy, 6y, -5$ (v) $ab, 2$
 4. (i) 4, 3 (ii) 23 (iii) $\frac{2}{5}$ (iv) 1, 85 (v) 15, 81

	Constant	Variable	Coefficient
i	1	a	8
ii	-4	a, b	3, 2
iii	0	p, q	1
iv	C	xy	a, b
v	C	a, b	3, 4

6. $3a, 5a, 9a, 76a$ 7. (i) $x + y$ (ii) $6a$
 $21m, 4m$ (iii) $4x - y$ (iv) $p + pq + q + r$
 $8p, 3p$ (v) $12a^2 + 3b^2$ (vi) $\frac{1}{y} + \frac{2}{x} + \frac{3}{2}$
 $2s$ (vii) $2xy + yz + 4zx$
 8ℓ (viii) $9x + 6x^2$
 8. $(25g + 27b), (23g + 30b), 48g + 57b$ 9. $75b, 53f, 114c$

**Exercise - 6.4**

- (i) $5x$ (ii) $11p+13q$ (iii) $15a$ (iv) $30xy$
(v) $98pq+2qr+23rs$ (vi) $15xy+6pq+54$ (vii) $13x+10y+5z$
- (i) $11p - q$ (ii) $\frac{6}{7}a + \frac{1}{3}b$ (iii) $15xy+11yz$
(iv) $38x+11y-8z$ (v) $13p+14q-r+10$ (vi) $5q+13r$
- (i) $12a-23b$ (ii) $13xy-7z$ (iii) $9x-5y+4z$
(iv) $-\frac{2}{3}l - \frac{2}{5}m + 14$ (v) $-49l-11m+8n$
- (i) $-x+5y-12z$ (ii) $2(3x-3yz)$ (iii) $6m+2n-2$
(iv) $13p-4q+7r$
- $p+2q+3r$ 6. $8x+8y+8z+8$ 7. $3x+7y+5z$

**Exercise - 6.5**

- (i) $8x+3y$ (ii) a (iii) $3a+42\frac{1}{3}b$ (iv) $4x+7a$
(v) $a+b$ (vi) $4x-11y+2z$
- $8b+(12b-3b)+(20b-2b)$, Total bags of sweets = 35 bags.
- (i) -5 (ii) -76 (iii) $\frac{85}{14}$ (iv) 62
(v) $\frac{7}{12}$ (vi) $\frac{9}{8}$ 4. -336 5. 29

Review Exercise

6

- (i) c (ii) b (iii) b (iv) a (v) a (vi) a
(vii) b (viii) d (ix) a (x) c
- (i) $23x-6y-4z$ (ii) $22xy+2zy+zx$
- (i) $2x+y+1$ (ii) $-2x-4y-2z+3$
- | | Coefficient | Variable | Constant |
|---|--------------------|----------|----------|
| a | a, b | p, q | c |
| b | $2.5, \frac{1}{3}$ | x, y | 14 |
| c | 13, q | p, q | 5 |
- (i) $17p+2q$ (ii) $4xy+7yz+4zx-30$ 7. $12x-4y+8z$
- (i) $7p+q-10r+45s$ (ii) $-14x-8y+12$ (iii) $5x-y-8$
- 24 10. 36, 49

**Exercise - 7.1**

- (i) $4(9)+4=40$ (ii) $2(4)-2=6$ (iii) $2n-3=20$
(iv) $\frac{1}{2}x+6=22$ (v) $3(6)\div 3=6$ (vi) $x+7=14$
- {i, ii, iv, vii, xi} are expressions
{iii, v, vi, viii, ix and x} are equations

**Exercise - 7.2**

- i, iii, iv are linear equation
- (i) $n+8=18$ (ii) $y-7=14$
(iii) $(x+1)+(x+2)=29$ (iv) $4n=2n+23$
- (i) 12 (ii) 7 (iii) 5 (iv) 35 (v) 15 (vi) $15\frac{2}{3}$
- 4 5. 5 6. 20 7. 9 8. 90

- 37 10. 30 years 11. 80 and 120 12. 43, 560 kg

Review Exercise

7

- (i) b (ii) b (iii) a (iv) a (v) b (vi) a
(vii) c (viii) a (ix) a (x) b
- (i) -15 (ii) 4 (iii) 10 (iv) $4\frac{1}{2}$ (v) -3.5 (vi) $-\frac{41}{560}$
- (i) -10 (ii) 0.98 (iii) $1\frac{7}{20}$ (iv) 1 (v) $\frac{3}{4}$ (vi) 9
- 13 5. 47 years 6. 60 and 40

**Exercise - 8.1**

- (i) 14.8 cm (ii) 13 cm (iii) 16 cm (iv) 14 cm
- (i) $A=18\text{ cm}^2$, $P=16\text{ cm}$ (ii) $A=16\text{ cm}^2$, $P=16\text{ cm}$
(iii) $A=14\text{ cm}^2$, $P=18\text{ cm}$ (iv) $A=5\text{ cm}^2$, $P=12\text{ cm}$
- $A=45000\text{ mm}$, $P=860\text{ mm}$ 4. 12.25 m 5. 30 cm, 32 cm
- 32 cm 7. 36 km^2 8. Rs. 14400 9. Rs. 11000
- (i) 96 cm^2 (ii) 41 cm^2 , 1 cm^2 (iii) 24 cm^2
- Rs. 163500 12. 264 m^2

**Exercise - 8.2**

- (i) $\overline{BC}$ (ii) $\overline{RS}$ and $\overline{PT}$ (iii) $\overline{AF}$ and $\overline{CE}$
- (i) 24 cm^2 (ii) 50 cm^2 (iii) 18 cm^2 (iv) 25 cm^2
(v) 12 cm^2 (vi) 25 cm^2
- (i) 3.2 cm^2 (ii) 57 cm^2 (iii) 49 cm^2
- 14 cm 5. 11 cm 6. 7 cm 7. 55 cm 8. 49500 flowers

**Exercise - 8.3**

- (i) 60 cm^3 , 96 cm^2 (ii) 729 cm^3 , 486 cm^2 (iii) 5145 mm^3 , 8470 mm^2
- (i) 352 cm^2 , 448 cm^3 (ii) 9.5 cm^2 , 580 cm^3
(iii) 3 cm , 948 cm^3 (iv) 0.5 cm , 56 cm^3
(v) 9 cm , 990 cm^2 (vi) 17 m , 4913 m^3
(vii) 11 m , 726 cm^2
- 25000 m^3 4. 3375 5. 2400 cm^2 6. 90 m^3
- 598.5 m^3 8. 1248 cm^2 9. Rs. 3800

Review Exercise

8

- (i) c (ii) c (iii) c (iv) b (v) a (vi) d (vii) b
(viii) b (ix) c (x) b (xi) c (xii) c (xiii) b
(xiv) b (xv) a (xvi) a (xvii) c (xviii) a (xix) a
- (i) 16 cm^2 (ii) 17 cm^2 (iii) 12 cm^2
- (i) 34 cm , 30 cm^2 (ii) 33 cm
- Momina 6. Rs. 121,500 7. 25 cm^2
- Rs. 60,000 9. 180 cm^3 10. 70 sheets
- 510 square feet

**Exercise - 9.1**

- (i) Cuboid (ii) Cone (iii) Sphere (iv) Cube
(v) Cylinder (vi) Hemisphere
- | 1-D | 2-D | 3-D |
|------|-----------------------------|------------------------|
| Line | Square, rectangle, triangle | Cube, cuboid, cylinder |

3. (i) Surfaces = 3
Vertices = 0
Edges = 2
(iv) Surfaces = 1
Vertices = 1
Edges = 1
- (ii) Surfaces = 6
Vertices = 8
Edges = 12
(v) Surfaces = 1
Vertices = 0
Edges = 1
- (iii) Surfaces = 6
Vertices = 8
Edges = 12



Exercise - 9.2

1. (i) Parallel lines (ii) Intersecting lines
(iii) Intersecting lines (iv) Parallel lines
(v) Intersecting lines (vi) Intersecting lines
2. (i) $a = 30^\circ$, $b = 150^\circ$, $c = 150^\circ$ (ii) $r = 120^\circ$, $p = q = 60^\circ$
3. (i) d and f , b and h , a and g , c and e , m and i , ℓ and o , n and j , k and p
(ii) d and e , a and h , m and k , n and ℓ
(iii) f and b , c and g , i and o , j and p
4. (i) $x = 80^\circ$ (ii) $x = 120^\circ$ (iii) $x = 23$
(iv) $x = 22.5$ (v) $x = 80^\circ$



Exercise - 9.3

Activity based

Review Exercise 9

1. (i) c (ii) b (iii) b (iv) d (v) c
(vi) c (vii) b (viii) d (ix) b (x) d
4. (i) $x = 34^\circ$ (ii) 247° 5. Activity based



Exercise - 10.1

Activity based



Exercise - 10.2

Activity based



Exercise - 10.3

1. (i) 54° (ii) 92° (iii) 48° 2. No 3. 83° , 63° and 34°
4. (i) $x = 132^\circ$, $y = 35^\circ$ (ii) $x = 57^\circ$, $y = 43^\circ$
(iii) $x = 15^\circ$, $y = 156^\circ$
5. (i) 129° (ii) 130° (iii) 87° (iv) 68°
6. Each interior angle = 60° , Each exterior angle = 120° 7. 60°

Review Exercise 10

1. (i) d (ii) a (iii) b (iv) c (v) c (vi) c
(vii) c (viii) c (ix) c (x) c (xi) c
7. (i) 90° (ii) $x = 60^\circ$, $y = 40^\circ$



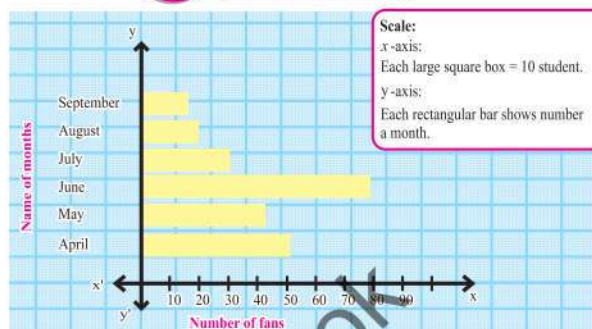
Exercise - 11.1

2. (i) Ungrouped (ii) Grouped (iii) Grouped
(iv) Ungrouped
3. (i) Discrete variable (ii) Continuous variable
(iii) Discrete variable (iv) Continuous variable
(v) Continuous variable (iii) Discrete variable
4. Continuous

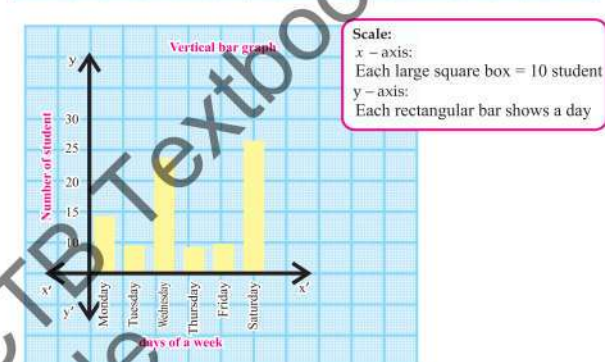


Exercise - 11.2

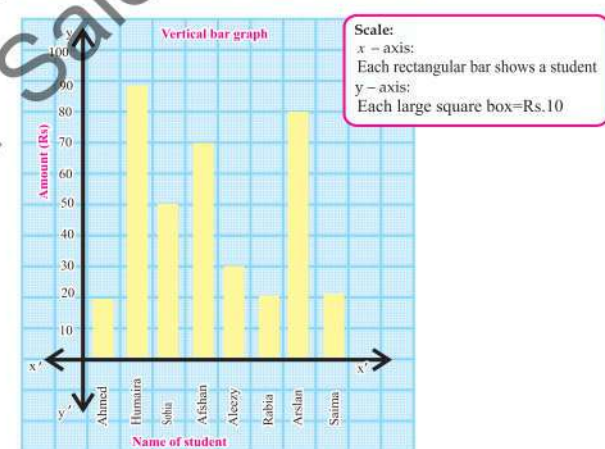
1.



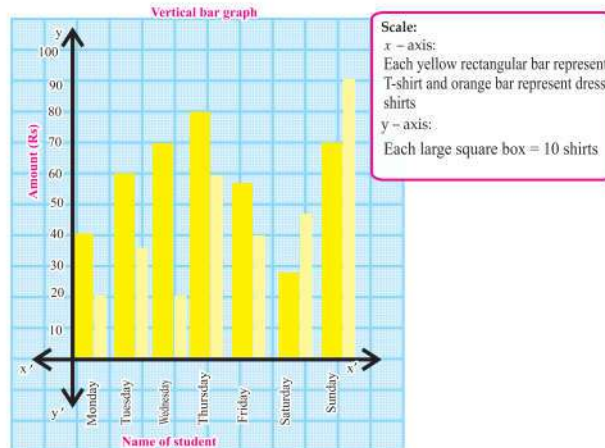
2.



3.



4.



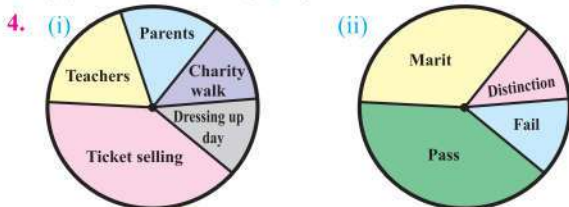
6. (i) 80 (ii) almond (iii) 140 (iv) A



Exercise - 11.3

- He must be happy because the end of term result is better
- English: 75 Science: 60 Mathematics: 90
Urdu: 30 Islamiat: 45
- (i) likes: 3,300 , comments: 2100 ,
Inbox message: 420 , shares: 180

- (ii) 1200 (iii) 3,120 (iv) 420



Exercise - 11.4

- (i) 11.86 (ii) 15.4 kg (iii) 20.4 m (iv) 6.8
(v) 282 km (vi) 39.4°C
- 25 pages
- 66.4 marks
- $k = 36.7$

	(i)	(ii)	(iii)	(iv)	(v)	(vi)
Mode	no mode	no mode	no mode	no mode	no mode	42.6
Median	10	14	19	7.75	250	40.3

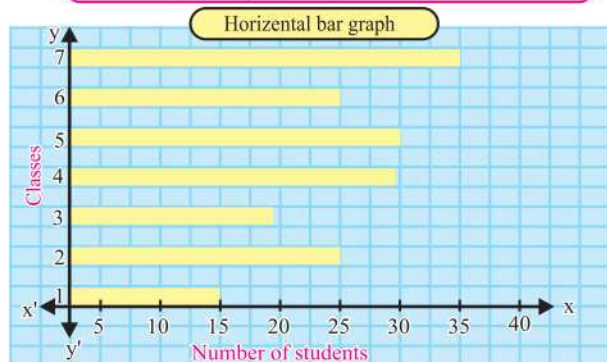
6. 18 7. 30.05

Review Exercise 11

- (i) d (ii) a (iii) c (iv) b (v) b
(vi) b (vii) d (viii) c (ix) d
- (i) grouped data (ii) Ungrouped data 4. Activity based
- (i) Model town , 41, 500 people (ii) 18, 750 more people

Name of town	Walk	Train	Bus
Model town	6,000	14, 000	8,500
Awan town	2,500	11, 250	15, 000

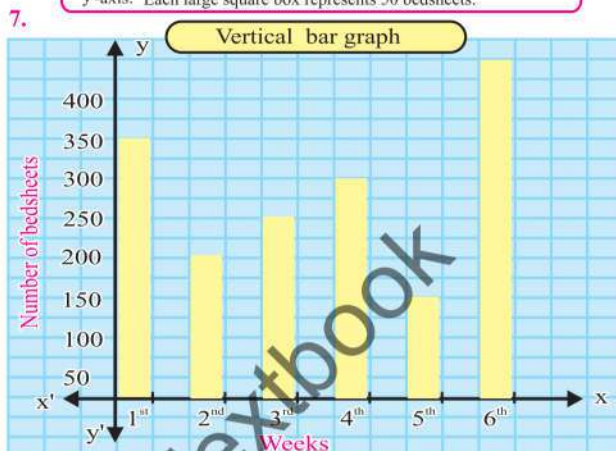
6. Scale: x-axis: Each large square box represent 5 students.
y-axis: Each green bar represent a class.



Scale:

x-axis: Each orange bar represents the weeks.

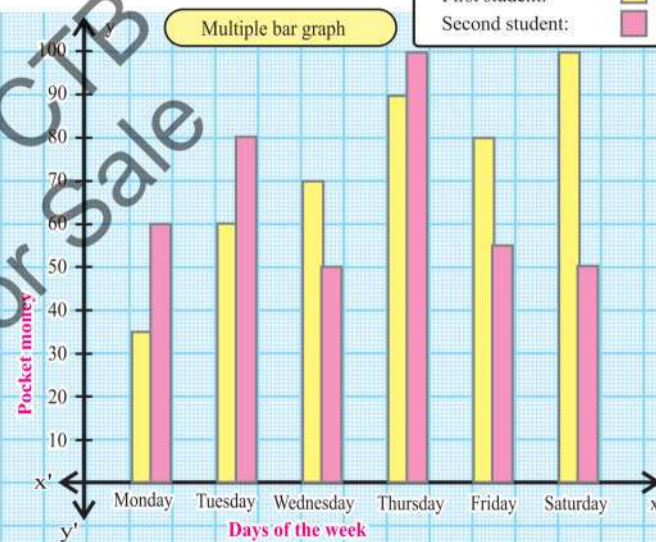
y-axis: Each large square box represents 50 bedsheets.



8.

Key

First student: ■
Second student: ■



Exercise - 12.1

- (i) $\frac{1}{5}$ 2. (i) $\frac{1}{6}$ (ii) 0 (iii) $\frac{1}{12}$
- (i) $\frac{1}{4}$ (ii) $\frac{1}{2}$ (iii) $\frac{1}{2}$ (iv) $\frac{1}{4}$ 4. (i) $\frac{1}{12}$ (ii) $\frac{1}{12}$
- (i) {HHH, HHT, HTH, THH, HTT, THT, TTH, TTT} (ii) $\frac{3}{8}$
- $\frac{11}{36}$ and $\frac{11}{36}$ 7. $\frac{459}{10000}$ or 0459 8. $\frac{1}{2}$ 9. $\frac{35}{143}$

Review Exercise 12

- (i) b (ii) b (iii) a (iv) c (v) a
(vi) a (vii) c (viii) c (ix) c (x) b
- $\frac{1}{4}$ 4. $\frac{1}{36}$ 5. equally likely $\frac{1}{2}$

Answers

187

NOT FOR SALE-PESRP

GLOSSARY

Absolute Value: The positive distance from zero to any number on the number line is called absolute value.

Algebraic Equation: An algebraic equation is an open mathematical sentence which contains an equal sign i.e

Area: The size of surface or amount of surface covered by any closed shape is called area.

Altitude: The altitude is measure of the shortest distance between the base and its top.

Bar Graph: A bar graph is a graphical display of data using of different heights.

BODMAS: BODMAS, stands for brackets, order of operation, multiplication division, addition and subtraction.

Cube: A 3-dimensional solid body bounded by six square surfaces is known as cube.

Cuboid: A 3 – dimensional solid body bounded by six rectangular surfaces is known as cuboid.

Cylinder: A 3-dimensional solid body with two circular surfaces of equal radius and one curved surface between them is known as cylinder.

Cone: A 3-dimensional solid body with one circular surface and a slanting surface is known as cone.

Data: Data is a collection of any information or facts.

Empty Set: A set which does not contain any element is called empty set.

Even Numbers: Even number is a number which is multiple of 2.

Factor: A factor is a number which divide the dividend completely leaving no remainder.

Grouped Data: The data which is given in intervals / groups, it provides us the information about groups it is known as grouped data.

Graph: A graph is a diagram representing a system of interrelation among two or more things by a number of lines, pictures distinctive dots and bars etc.

Set: A set is a collection of well defined and distinct objects.

Finite Set: A set having a finite number of elements is known as finite set.

Infinite Set: A set having unlimited number of elements is known as infinite set.

Singleton Set: A set which contains only one element is called singleton set.

Subset: A subset is a set which contain in another set.

Superset: A superset is a set which contain another set.

Subset: A proper subset of a set A is subset of A which is not equal to set A.

Improper Subset: Improper subset is a subset which contains all the elements of original set.

Natural Numbers: Natural numbers are counting numbers. 1, 2, 3, ..., are all natural numbers.

Continued Ratio: The comparison among three or more quantities is called continued ratio.

Rate: A rate is a special kind of ratio in which the two terms are in different units.

Universal set: A universal set is a collection of all elements or members of all related sets known as subsets of universal set.

Venn diagram: A diagram that represent the possible logical relationship between finite collection of sets pictorially.

Pattern: A pattern is a some phenomenon that repeats regularly based on a set rule or condition.

Parallel lines: Two or more lines which extends in the same directions and remain the same distance apart.

Intersecting lines: Two lines that are intersecting each other at a single point is called intersecting lines.

Point of intersection: The point where two lines are intersecting each other.

Perpendicular lines: Two or more lines that meet or intersect each other at right angle.

Transversal: A line that passes through two or more of the parallel lines.

Vertically opposite angles: When two lines intersect at a point then opposite angles formed because of intersection.

Symmetry: A line that divide an object into two identical pieces.

Reflective symmetry: If a shape or pattern is reflected in a mirror line or line of symmetry then this type of symmetry is called reflective symmetry.

Rotational symmetry: If a shape is rotated about a point to another position and still look the same then the shape has rotational symmetry.

Order of symmetry: Number of times a shape looks the same in one full turn is called order of symmetry.

Centre of rotation: A find point around which the rotation occurs is called the centre of rotation.

Acute angle: An angle which is less than 90° .

Right angle: An angle which is equal to 90° .

Obtuse angle: An angle which is greater than 90° but less than 180° .

Straight angle: An angle which is equal to 180° .

Reflex angle: An angle which is greater than 180° but less than 360° .

Complete angle: An angle which is equal to 360° .

Complementary angles: If the sum of two angles is equal to 90° , then angles are complementary angles.

Supplementary angle: If the sum of two angles is 180° , then the angles are supplementary angles.

Equilateral triangle: A triangle with all three sides are equal.

Isosceles triangle: A triangle with 2 equal sides.

Scalene triangle: A triangle with all three sides are different.

Acute triangle: A triangle with all three interior angles are acute angles.

Obtuse triangle: A triangle with one interior angle is obtuse angle.

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Mathematical Symbols / Notations

Symbol	Name	Symbol	Name
$=$	equal to	$\times$	multiplication
$\neq$	not equal to	$\div$	division
$>$	greater than	$ $	absolute value
$<$	less than	$\Rightarrow$	material implication
$\leq$	less than or equal to	$\forall$	for all
$\geq$	greater than or equal to	$\{ \}$	set brackets
$+$	addition	$\emptyset$	empty set
$-$	subtraction	$\in$	belongs to
$\subset$	subset	$\notin$	does not belong to
$\subsetneq$	proper subset	$()$	parentheses
$\supset$	superset	$\parallel$	parallel
$\perp$	perpendicular	$\approx$	approximately equal to
$\%$	percentage	π	pi
$\angle$	angle	$\cup$	universal set
$:$	ratio		

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