Unit-9

CHORDS OF A CIRCLE

In this unit, students will learn how to

Prove the following theorems alongwith corollaries and apply them to solve appropriate problems.

- *One and only one circle can pass through three non collinear points.*
- *A straight line, drawn from the centre of a circle to bisect a chord (which is not a diameter) is perpendicular to the chord.*
- *Perpendicular from the centre of a circle on a chord bisects it.*
- *If two chords of a circle are congruent then they will be equidistant from the centre.*
- *Two chords of a circle which are equidistant from the centre are congruent.*

Basic concepts of the circle

A **circle** is the locus of a moving point P in a plane which is always equidistant from some fixed point O. The fixed point O not lying on the circle is called the centre, the constant distance OP is its radius whereas the boundary traced by moving point P is called circumference of the circle.

Note that the **radial segment** of a circle is a line segment, determined by the centre and a point on the circle. There is only one centre point whereas all the radii of a circle are equal in length.

In the adjoining figure (i) of the circle, the length of radial segment = $m\overline{OP} = m\overline{OQ} = m\overline{OT}$

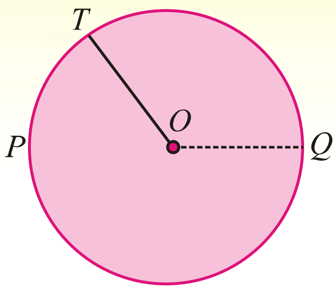


Fig. (i)

$2\pi r$ is the **circumference** of a circle with radius r whereas an irrational number π being the ratio of the circumference and the diameter of a given circle.

An **arc** ACB of a circle is any portion of its circumference.

A **chord** AKB of a circle is a line segment joining any two points A and B on the circumference of a circle. Whereas diameter POQ is the chord passing through the centre of a circle. Evidently diameter bisects a circle.

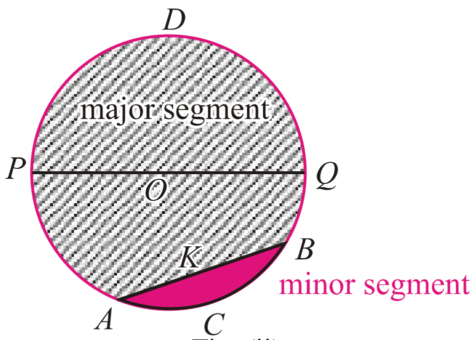


Fig. (ii)

A **segment** is the portion of a circle bounded by an arc and a corresponding chord. Evidently any chord divides a circle into two segments.

In figure (ii) the bigger area shown by slanting line segments is the major segment whereas the smaller area shown by shading is the minor segment.

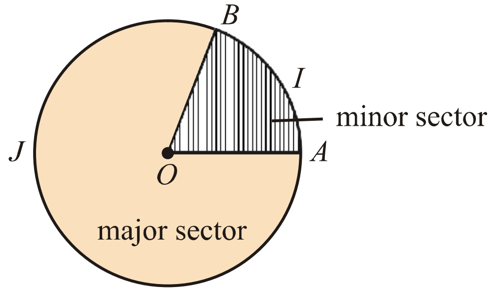


Fig. (iii)

A **sector** of a circle is the plane figure bounded by two radii and the arc intercepted between them. Any pair of radii divides a circle into two sectors.

In the figure (iii) $OAIB$ is the minor sector, whereas $OAJB$ is the major sector of the circle.

$\angle AOB$ is the central angle of a circle whose vertex is at the centre O and its arms meet at the end points of the arc AB.

THEOREM 1

9.1(*i*) One and only one circle can pass through three non-collinear points.

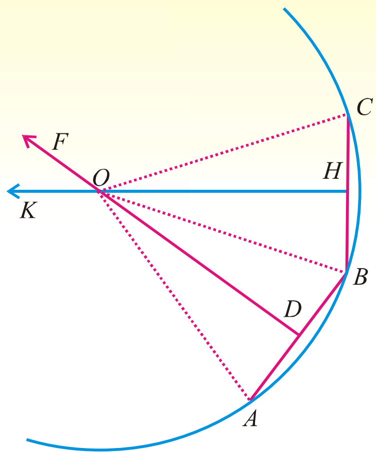
Given: A, B and C are three non collinear points in a plane.

To prove: One and only one circle can pass through three non-collinear points A, B and C.

Construction: Join A with B and B with C.

Draw $\overline{DF} \perp$ bisector to $\overline{AB}$ and $\overline{HK} \perp$ bisector to $\overline{BC}$.

So, $\overline{DF}$ and $\overline{HK}$ are not parallel and they intersect each other at point O. Also join A, B and C with point O.



Proof:

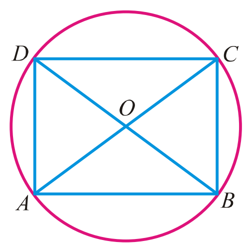
Statements	Reasons
Every point on $\overline{DF}$ is equidistant from A and B.	$\overline{DF}$ $\perp$ bisector to $\overline{AB}$ (construction)
In particular $m\overline{OA} = m\overline{OB}$ (i)	
Similarly every point on $\overline{HK}$ is equidistant from B and C.	$\overline{HK}$ is $\perp$ bisector to $\overline{BC}$ (construction)
In particular $m\overline{OB} = m\overline{OC}$ (ii)	
Now O is the only point common to $\overline{DF}$ and $\overline{HK}$ which is equidistant from A, B and C.	
i.e., $m\overline{OA} = m\overline{OB} = m\overline{OC}$	Using (i) and (ii)
However there is no such other point expect O.	
Hence a circle with centre O and radius OA will pass through A, B and C. Ultimately there is only one circle which passes through three given points A, B and C.	

Example: Show that only one circle can be drawn to pass through the vertices of any rectangle.

Given: $ABCD$ is a rectangle.

To Prove: Only one circle can be drawn through the vertices of the rectangle $ABCD$.

Construction: Diagonals $\overline{AC}$ and $\overline{BD}$ of the rectangle meet each other at point O.

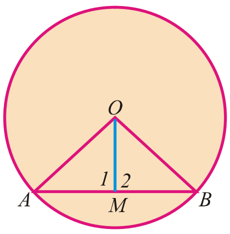


Proof:

Statements		Reasons
$ABCD$ is a rectangle.		Given
$\therefore \quad m\overline{AC} = m\overline{BD}$	(i)	Diagonals of a rectangle are equal.
$\because \ \overline{AC}$ and $\overline{BD}$ meet each other at O		Construction
$\therefore \ m\overline{OA} = m\overline{OC}$ and $m\overline{OB} = m\overline{OD}$	(ii)	Diagonals of rectangle bisect each other
$\Rightarrow \ m\overline{OA} = m\overline{OB} = m\overline{OC} = m\overline{OD}$	(iii)	Using (i) and (ii)
i.e., point O is equidistant from all vertices of the rectangle $ABCD$. Hence $\overline{OA}$, $\overline{OB}$, $\overline{OC}$ and $\overline{OD}$ are the radii of the circle which is passing through the vertices of the rectangle having centre O.		

THEOREM 2

9.1(*ii*) A straight line, drawn from the centre of a circle to bisect a chord (which is not a diameter) is perpendicular to the chord.



Given: M is the mid point of any chord $\overline{AB}$ of a circle with centre at O.

Where chord $\overline{AB}$ is not the diameter of the circle.

To prove: $\overline{OM} \perp$ the chord $\overline{AB}$.

Construction: Join A and B with centre O.

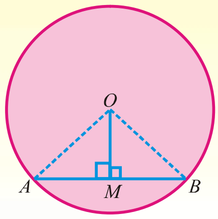
Write $\angle 1$ and $\angle 2$ as shown in the figure.

Proof:

Statements		Reasons
In $\Delta\, OAM \leftrightarrow \Delta OBM$		
$m\overline{OA} = m\overline{OB}$		Radii of the same circle
$m\overline{AM} = m\overline{BM}$		Given
$m\overline{OM} = m\overline{OM}$		Common
$\therefore \quad \Delta OAM \cong \Delta OBM$		S.S.S $\cong$ S.S.S
$\Rightarrow \quad m\angle 1 = m\angle 2$	(i)	Corresponding angles of congruent triangles
i.e., $m\angle 1 + m\angle 2 = m\angle AMB = 180^{o}$	(ii)	Adjacent supplementary angles
$\therefore \ m\angle 1 = m\angle 2 = 90^{o}$		From (i) and (ii)
i.e., $\overline{OM} \perp \overline{AB}$		

THEOREM 3

9.1(*iii*) Perpendicular from the centre of a circle on a chord bisects it.



Given: $\overline{AB}$ is the chord of a circle with centre at O so that $\overline{OM} \perp$ chord $\overline{AB}$.

To prove: M is the mid point of chord $\overline{AB}$

i.e., $m\overline{AM} = m\overline{BM}$

Construction: Join A and B with centre O.

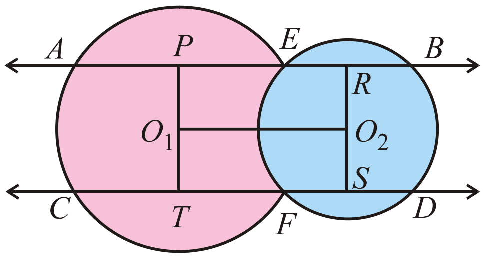
Proof:

Statements	Reasons
In $\angle rt\ \Delta^s\ OAM \leftrightarrow OBM$	
$m\angle OMA = m\angle OMB = 90^o$	Given
hyp. $m\overline{OA}$ = hyp. $m\overline{OB}$.	Radii of the same circle
$m\overline{OM} = m\overline{OM}$	Common
$\therefore\ \Delta OAM \cong \Delta OBM$	In $\angle rt\ \Delta^s$ H.S $\cong$ H.S
Hence, $m\overline{AM} = m\overline{BM}$	Corresponding sides of congruent triangles
$\Rightarrow\ \overline{OM}$ bisects the chord $\overline{AB}$.	

Corollary 1: $\perp$ bisector of the chord of a circle passes through the centre of a circle.

Corollary 2: The diameter of a circle passes through the mid points of two parallel chords of a circle.

Example: Parallel lines passing through the points of intersection of two circles and intercepted by them are equal.



Given: Two circles have centres O_1 and O_2. They intersect each other at points E and F.

Line segment $\overline{AB}$ || Line segment $\overline{CD}$

To Prove: $m\overline{AB} = m\overline{CD}$

Construction: Draw $\overline{PT}$ and $\overline{RS} \perp$ both $\overline{AB}$ and $\overline{CD}$ and join the centres O_1 and O_2.

Proof:

Statements	Reasons
$PRST$ is a rectangle	Construction
$\therefore\ m\ \overline{PR} = m\ \overline{TS}$ (i)	
Now $m\ \overline{PR} = m\ \overline{PE} + m\ \overline{ER}$	
$= \frac{1}{2} m\ \overline{AE} + \frac{1}{2} m\ \overline{EB}$	By Theorem 3

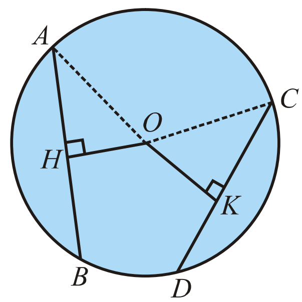
$= \frac{1}{2}(m\,\overline{AE} + m\,\overline{EB})$	
$m\,\overline{PR} = \frac{1}{2}(m\,\overline{AB})$ (ii)	$m\overline{AE} + m\overline{EB} = m\overline{AB}$
Similarly $m\,\overline{TS} = \frac{1}{2}m\overline{CD}$ (iii)	
$\Rightarrow \frac{1}{2}m\overline{AB} = \frac{1}{2}m\overline{CD}$	Using (i), (ii) and (iii)
i.e., $m\overline{AB} = m\overline{CD}$	

EXERCISE 9.1

1. Prove that, the diameters of a circle bisect each other.
2. Two chords of a circle do not pass through the centre. Prove that they cannot bisect each other.
3. If length of the chord $\overline{AB}$ = 8cm. Its distance from the centre is 3 cm, then find the diameter of such circle.
4. Calculate the length of a chord which stands at a distance 5cm from the centre of a circle whose radius is 9cm.

THEOREM 4

9.1(*iv*) If two chords of a circle are congruent then they will be equidistant from the centre.



Given: $\overline{AB}$ and $\overline{CD}$ are two equal chords of a circle with centre at O.

So that $\overline{OH} \perp \overline{AB}$ and $\overline{OK} \perp \overline{CD}$.

To prove: $m\overline{OH} = m\overline{OK}$

Construction: Join O with A and O with C.

So that we have $\angle rt\Delta^s$ OAH and OCK.

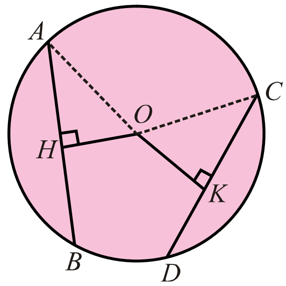
Proof:

Statements	Reasons
$\overline{OH}$ bisects chord $\overline{AB}$	$\overline{OH} \perp \overline{AB}$ By Theorem 3
i.e., $m\overline{AH} = \frac{1}{2}m\overline{AB}$ (i)	
Similarly $\overline{OK}$ bisects chord $\overline{CD}$	$\overline{OK} \perp \overline{CD}$ By Theorem 3
i.e., $m\overline{CK} = \frac{1}{2}m\overline{CD}$ (ii)	

But $m\overline{AB} = m\overline{CD}$	(iii)	Given
Hence $m\overline{AH} = m\overline{CK}$	(iv)	Using (i), (ii) & (iii)
Now in $\angle rt\ \Delta^s\ OAH \leftrightarrow OCK$		Given $\overline{OH} \perp \overline{AB}$ and $\overline{OK} \perp \overline{CD}$
hyp $\overline{OA}$ = hyp $\overline{OC}$		Radii of the same circle
$m\overline{AH} = m\overline{CK}$		Already proved in (iv)
$\therefore$ $\Delta OAH \cong \Delta OCK$		H. S postulate
$\Rightarrow$ $m\overline{OH} = m\overline{OK}$		

THEOREM 5

9.1(*v*) Two chords of a circle which are equidistant from the centre, are congruent.



Given: $\overline{AB}$ and $\overline{CD}$ are two chords of a circle with centre at O.

$\overline{OH} \perp \overline{AB}$ and $\overline{OK} \perp \overline{CD}$, so that $m\overline{OH} = m\overline{OK}$

To prove: $m\overline{AB} = m\overline{CD}$

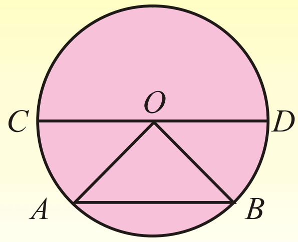
Construction: Join A and C with O. So that we can form $\angle rt\Delta^s\ OAH$ and OCK.

Proof:

Statements		Reasons
In $\angle rt\ \Delta^s\ OAH \leftrightarrow OCK$.		
$\because$ hyp $\overline{OA}$ = hyp $\overline{OC}$		Radii of the same circle.
$m\overline{OH} = m\overline{OK}$		Given
$\therefore$ $\Delta\ OAH \cong \Delta\ OCK$		H.S Postulate
So $m\overline{AH} = m\overline{CK}$	(i)	Corresponding sides of congruent triangles
But $m\overline{AH} = \frac{1}{2} m\overline{AB}$	(ii)	$\overline{OH} \perp$ chord $\overline{AB}$ (Given)
Similarly $m\overline{CK} = \frac{1}{2} m\overline{CD}$	(iii)	$\overline{OK} \perp$ chord $\overline{CD}$ (Given)
Since $m\overline{AH} = m\overline{CK}$		Already proved in (i)
$\therefore$ $\frac{1}{2} m\ \overline{AB} = \frac{1}{2} m\ \overline{CD}$		Using (ii) & (iii)
or $m\ \overline{AB} = m\ \overline{CD}$		

Example: Prove that the largest chord in a circle is the diameter.

Given: $\overline{AB}$ is a chord and $\overline{CD}$ is the diameter of a circle with centre point O.



To prove: If $\overline{AB}$ and $\overline{CD}$ are distinct, then $m\overline{CD} > m\overline{AB}$.

Construction: Join O with A and O with B then form a $\Delta\ OAB$.

Proof: Sum of two sides of a triangle is greater than its third side.

$\therefore$ In $\Delta OAB \Rightarrow m\overline{OA} + m\overline{OB} > m\overline{AB}$ (i)

But $\overline{OA}$ and $\overline{OB}$ are the radii of the same circle with centre O.

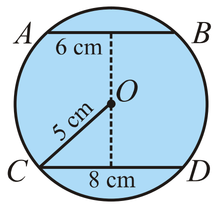
So that $m\overline{OA} + m\overline{OB} = m\overline{CD}$ (ii)

$\Rightarrow$ Diameter $\overline{CD} >$ chord $\overline{AB}$ using (i) & (ii).

Hence, diameter CD is greater than any other chord drawn in the circle.

EXERCISE 9.2

1. Two equal chords of a circle intersect, show that the segments of the one are equal corresponding to the segments of the other.
2. AB is the chord of a circle and the diameter CD is perpendicular bisector of AB. Prove that $m\overline{AC} = m\overline{BC}$.
3. As shown in the figure, find the distance between two parallel chords AB and CD.



MISCELLANEOUS EXERCISE 9

Multiple Choice Questions

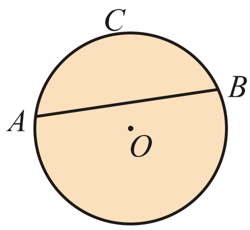
1. Four possible answers are given for the following questions.

Tick (✓) the correct answer.

(*i*) In the circular figure, ADB is called

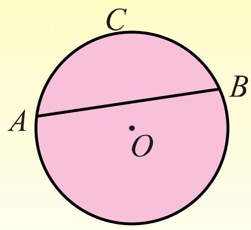
(*a*) an arc (*b*) a secant

(*c*) a chord (*d*) a diameter



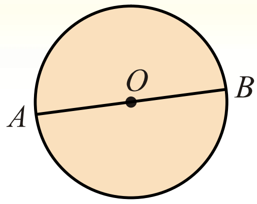
(*ii*) In the circular figure, $A\overset{\frown}{C}B$ is called

(*a*) an arc (*b*) a secant
(*c*) a chord (*d*) a diameter



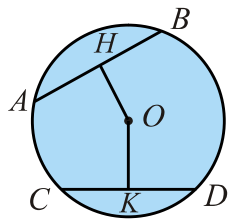
(*iii*) In the circular figure, *AOB* is called

(*a*) an arc (*b*) a secant
(*c*) a chord (*d*) a diameter



(*iv*) In a circular figure, two chords $\overline{AB}$ and $\overline{CD}$ are equidistant from the centre. They will be

(*a*) parallel (*b*) non congruent
(*c*) congruent (*d*) perpendicular



(*v*) Radii of a circle are

(*a*) all equal (*b*) double of the diameter
(*c*) all unequal (*d*) half of any chord

(*vi*) A chord passing through the centre of a circle is called

(*a*) radius (*b*) diameter
(*c*) circumference (*d*) secant

(*vii*) Right bisector of the chord of a circle always passes through the

(*a*) radius (*b*) circumference
(*c*) centre (*d*) diameter

(*viii*) The circular region bounded by two radii and the corresponding arc is called

(*a*) circumference of a circle (*b*) sector of a circle
(*c*) diameter of a circle (*d*) segment of a circle

(*ix*) The distance of any point of the circle to its centre is called

(*a*) radius (*b*) diameter (*c*) a chord (*d*) an arc

(*x*) Line segment joining any point of the circle to the centre is called

(*a*) circumference (*b*) diameter
(*c*) radial segment (*d*) perimeter

(*xi*) Locus of a point in a plane equidistant from a fixed point is called

(*a*) radius (*b*) circle (*c*) circumference (*d*) diameter

(*xii*) The symbol for a triangle is denoted by

(*a*) $\angle$ (*b*) Δ (*c*) $\perp$ (*d*) $\odot$

(*xiii*) A complete circle is divided into

(*a*) 90 degrees (*b*) 180 degrees (*c*) 270 degrees (*d*) 360 degrees

(*xiv*) Through how many non collinear points, can a circle pass?

(*a*) one (*b*) two (*c*) three (*d*) none

Q.2. Differentiate between the following terms and illustrate them by diagrams.

(*i*) A circle and a circumference.
(*ii*) A chord and the diameter of a circle.
(*iii*) A chord and an arc of a circle.
(*iv*) Minor arc and major arc of a circle.
(*v*) Interior and exterior of a circle.
(*vi*) A sector and a segment of a circle.

SUMMARY

- $2\pi r$ is the **circumference** of a circle with radius r.
- πr^2 is the **area** of a circle with radius r.
- Three or more points lying on the same line are called **collinear points** otherwise they are **non-collinear points**.
- The circle passing through the vertices of a triangle is called its **circumcircle** whereas ⊥ bisectors of sides of the triangle provide the centre.
- One and only one circle can pass through three non-collinear points.
- A straight line, drawn from the centre of a circle to bisect a chord (which is not a diameter) is perpendicular to the chord.
- Perpendicular from the centre of a circle on a chord bisects it.
- If two chords of a circle are congruent, then they will be equidistant from the centre.
- Two chords of a circle which are equidistant from the centre are congruent.