Fsc Part 1 Chapter 4

Work And Energy

Work

When a constant force is applied on an object and move it through certain distance, then we said that work is done by the force on the object.

OR

Work is also defined as "The scalar or dot product of force and displacement". i-e;

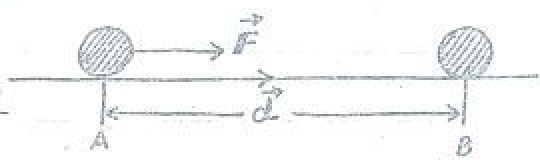
$$W = \vec{F} \cdot \vec{d} = F d \cos\theta$$

Explanation

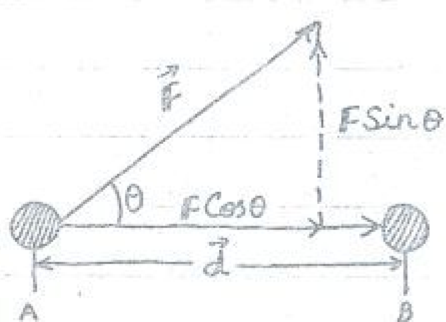
Consider an object is being pulled by a constant force $\vec{F}$ and moves the object from position A to B through a displacement $\vec{d}$ in the direction of force, then work done by the force is equal to the product of magnitudes of force $\vec{F}$ and displacement $\vec{d}$, so

Work = Magnitude Of Force × Distance

$$\boxed{W = F.d}$$



If force $\vec{F}$ is applied at a certain angle θ with the displacement $\vec{d}$, then work done by the force is the product of magnitude of displacement $\vec{d}$ and the



component of force in the direction of displacement i-e; $F\cos\theta$, so work done is given as

$$W = (F\cos\theta)(d) = Fd\cos\theta$$

Where

$$\vec{a}\cdot\vec{b} = ab\cos\theta$$

$$\Rightarrow \quad \vec{F}\cdot\vec{d} = Fd\cos\theta$$

So, work done is

$$W = Fd\cos\theta$$

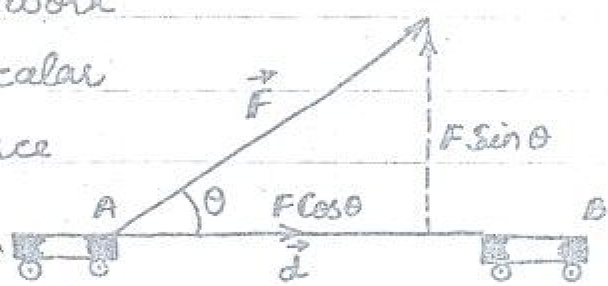
or $$\boxed{W = \vec{F}\cdot\vec{d}}$$

Hence, work is scalar product of $\vec{F}$ and $\vec{d}$. So, work is a scalar quantity.

WORK DONE BY A CONSTANT FORCE

Consider a body is being pulled by a constant force $\vec{F}$ at a certain angle θ to the direction of motion of the body. The force moves the body from position A to B through a certain displacement $\vec{d}$ as shown in figure.

As we know that work done by the force is scalar or dot product of force $\vec{F}$ and displacement $\vec{d}$, so



$$W = \vec{F}\cdot\vec{d} = Fd\cos\theta$$

or $$W = (F\cos\theta)\,d$$

Where the quantity $F\cos\theta$ is the component of the force in the direction of the displacement $\vec{d}$, so work done by a constant force can be written as

$$W = (F\cos\theta)\, d$$

or WORK DONE = (COMPONENT OF $\vec{F}$ IN THE DIRECTION OF $\vec{d}$) (MAGNITUDE OF DISPLACEMENT $\vec{d}$)

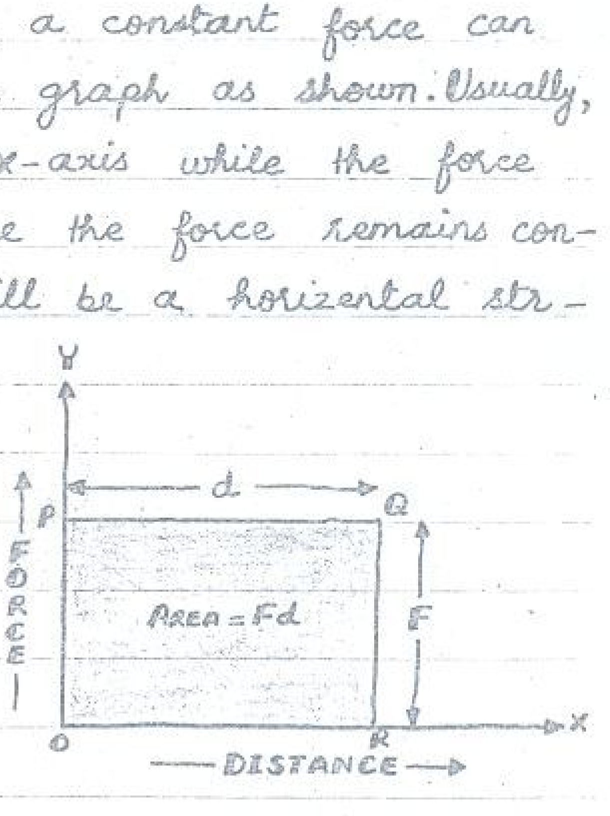
Hence, the work done by a constant force is defined as

DEFINITION ⇒ The product of the magnitudes of the displacement and the component of the force in the direction of the displacement is called work done by a constant force.

EXPLANATION FROM GRAPH OR FORCE-DISPLACEMENT CURVE

The work done by a constant force can be plotted on a simple graph as shown. Usually, distance is taking along x-axis while the force along Y-axis. In this case the force remains constant, then the graph will be a horizental straight line.

If the constant force $\vec{F}$ (newton) and the displacement $\vec{d}$ (meter) are in the same direction, then the work done is Fd (joule). It is clear from the



graph that the shaded area is also Fd.

Hence, the area under a force – displacement curve can be taken to represent the work done by the force. If force $\vec{F}$ is not in the direction of the displacement $\vec{d}$, then the graph is plotted between $F\cos\theta$ and d to represent the work done.

Positive Work or Maximum Work

If force $\vec{F}$ and the displacement $\vec{d}$ are in the same direction, then work is said to be positive or maximum work.

Here $\theta = 0^\circ$

As $W = \vec{F}\cdot\vec{d} = Fd\cos\theta$

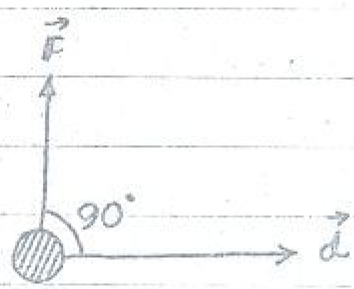
$W = Fd\cos 0^\circ = Fd(1)$ $\quad (\because \cos 0^\circ = 1)$

$$\boxed{W = Fd}$$

The work done is also positive if $\theta < 90^\circ$.

Null or Zero Work or Minimum Work

If force $\vec{F}$ and the displacement are perpendicular, then work is said to be null or zero work or minimum work.



Here $\theta = 90^\circ$

As $W = \vec{F}\cdot\vec{d} = Fd\cos\theta$

$W = Fd\cos 90^\circ$

$W = Fd(0)$ $\quad (\because \cos 90^\circ = 0)$

$$\boxed{W = 0}$$

Negative Work

If force $\vec{F}$ and the displacement $\vec{d}$ are in opposite directions, then work is said to be negative work.

Here $\theta = 180^\circ$

As $W = \vec{F} \cdot \vec{d} = Fd\cos\theta$

$W = Fd\cos 180^\circ$

$W = Fd(-1)$ $(\because \cos 180^\circ = -1)$

$$\boxed{W = -Fd}$$

The work done is also negative if $\theta > 90^\circ$.

Units Of Work

In SI-units, the unit of work is joule (J).

As

$$W = Fd$$

$$\boxed{1J = 1N \times 1m}$$

Joule

Definition ⟹ The work is said to be one joule if a force of one newton acts on a body and moves it through a distance of one meter in the direction of the force.

Dimension Of Work

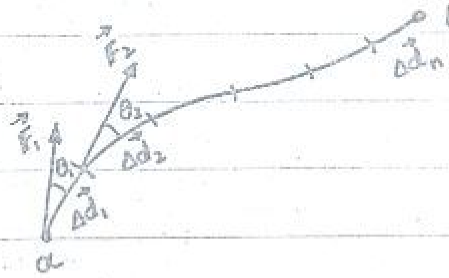
The dimension of work is $[ML^2T^{-2}]$.

WORK DONE BY A VARIABLE FORCE

In many cases, the force does not remains constant during the process of work done.

For example, when a body or a rocket moves away from the earth, then work is done against the force of gravity and this force is inversely proportional to the square of the distance from the centre of the earth. Hence, in such cases force does not remains constant. Similarly, the force exerted by a spring varies with the displacement of the spring.

Consider a particle is moving in x-y plane from point 'a' to point 'b' along a path as shown in figure. As, force does not remain constant during the process



of work done from point a to b, so to find work along this path, divided the path into 'n' short intervals of displacements $\Delta\vec{d}_1$, $\Delta\vec{d}_2$, ------, $\Delta\vec{d}_n$ and the forces acting during these intervals are $\vec{F}_1$, $\vec{F}_2$, -------, $\vec{F}_n$ respectively. It is supposed that the force during each short interval is approximately constant.

Now, the work done for the first interval of displacement $\Delta \vec{d}_1$ can be written as

$$W_1 = \vec{F}_1 \cdot \Delta \vec{d}_1 = F_1 \cos\theta_1 \cdot \Delta d_1$$

Similarly, work done for other intervals are

$$W_2 = \vec{F}_2 \cdot \Delta \vec{d}_2 = F_2 \cos\theta_2 \cdot \Delta d_2$$

$$W_3 = \vec{F}_3 \cdot \Delta d_3 = F_3 \cos\theta_3 \cdot \Delta d_3$$

$$\vdots \qquad\qquad \vdots$$

$$W_n = \vec{F}_n \cdot \Delta \vec{d}_n = F_n \cos\theta_n \cdot \Delta d_n$$

So, the total work done can be calculated by adding all these terms as

$$W_{TOTAL} = W_1 + W_2 + \text{-----------} + W_n$$

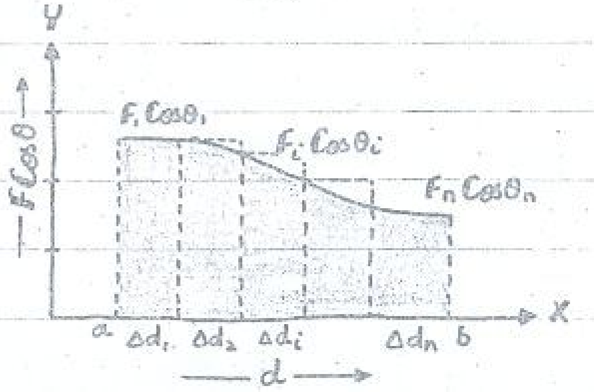
$$W_{TOTAL} = F_1\cos\theta_1 \cdot \Delta d_1 + F_2\cos\theta_2 \cdot \Delta d_2 + \text{-----} + F_n\cos\theta_n \cdot \Delta d_n$$

$$\boxed{W_{TOTAL} = \sum_{i=1}^{n} F_i \cos\theta_i \, \Delta d_i} \longrightarrow (1)$$

FORCE - DESPLACEMENT CURVE

The work done by a variable force can be represented graphically by plotting $F\cos\theta$ along Y-axis and d along X-axis as shown in figure.

The displacement d has been divided into n short intervals and the value of $F\cos\theta$ represented by straight horizontal line



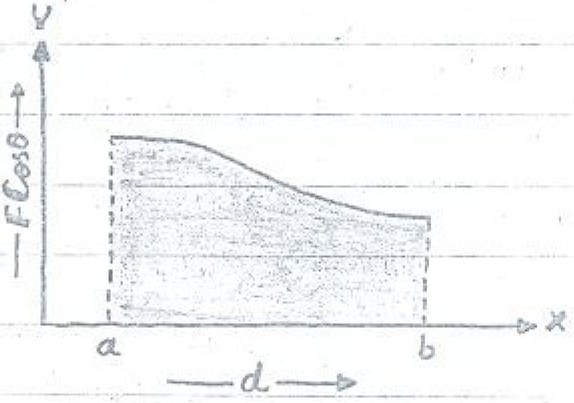
at the begining of each interval.

From graph, the area of ith shaded rectangle is $F_i \cos\theta_i \cdot \Delta d_i$ which is the work done during the ith interval. Hence, the work done in equation ① equal to the sum of areas of all the rectangles. If we sub-divided the distance into large number of small intervals, so that each interval Δd becomes very small, then the work done as in equation ① becomes more accurate. Consider Δd approaches to zero, then work done will be exact given as

$$\boxed{W_{TOTAL} = \lim_{\Delta d \to 0} \sum_{i=1}^{n} F_i \cos\theta_i \cdot \Delta d_i} \longrightarrow ②$$

In the limit $\Delta d \to 0$, the area of the rectangles approaches the area between the $F\cos\theta$ and d from a to b as shown in figure.

Hence, the work done by a variable force in moving a particle between two points a and b is equal to the area under the $F\cos\theta$ and d-curve between points a and b as shown in figure.



GRAVITATIONAL FIELD

The space or region around the earth within which its gravitational force acts on a body is called is gravitational field.

WORK DONE IN GRAVITATIONAL FIELD

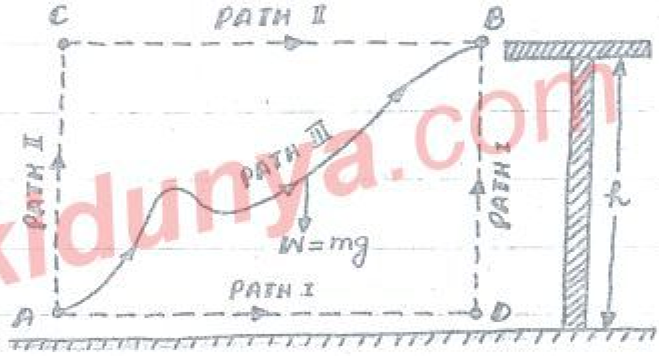
Consider an object of mass m is moved with constant velocity from point A to B along various paths in a gravitational field as shown in figure.

Let us consider three paths from point A to B given as

PATH I ⟶ ADB

PATH II ⟶ ACB

PATH III ⟶ AB



First we find work done in moving a body from A to B through PATH I as (Work from A to D is)

$$W_{AD} = \vec{F} \cdot \vec{d} = Fd\cos\theta$$

In this case, the gravitational force is equal to the weight of the object, so

$$F = W = mg$$

Also direction of mg is perpendicular to the displacement, so $\theta = 90°$

$$\therefore W_{AD} = Fd\cos\theta = mgd\cos 90°$$

$$W_{AD} = mgd(0) \qquad (\because \cos 90° = 0)$$

$$W_{AD} = 0$$

Now work done from D to B is

$$W_{DB} = \vec{F} \cdot \vec{d} = Fd\cos\theta$$

As $F = W = mg$

Here direction of weight mg is opposite to the displacement h, so $\theta = 180°$ and $d = h$

$$\therefore W_{DB} = Fd\cos\theta = mgh\cos 180°$$

$$W_{DB} = mgh(-1) = -mgh \quad (\because \cos 180° = -1)$$

Hence total work done from A to B through PATH I is

$$W_{ADB} = W_{AD} + W_{DB} = 0 + (-mgh) = 0 - mgh$$

$$\boxed{W_{ADB} = -mgh} \longrightarrow (1)$$

Now we consider the path ACB (PATH II)

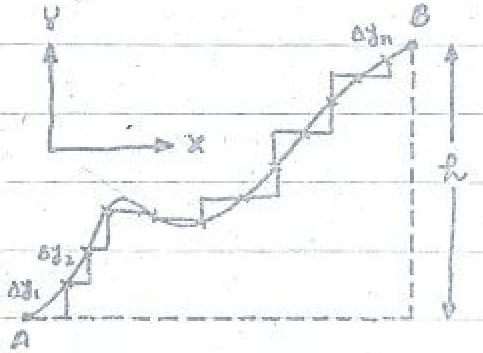
The work done along AC is $(-mgh)$ as weight mg is opposite to the displacement and work done along CB is zero because weight mg is perpendicular to the displacement.

So total work done along path ACB is

$$W_{ACB} = W_{AC} + W_{CB} = -mgh + 0$$

$$\boxed{W_{ACB} = -mgh} \longrightarrow (2)$$

Now consider the path III which is a curved path. To find work done divide the path into a series of horizontal and vertical steps as shown.



There is no work done along the horizontal steps because the weight mg is perpendicular to the displacement for these steps. Here work is done for the vertical displacements $\Delta y_1, \Delta y_2, \text{-------}, \Delta y_n$ by the force of gravity. As, the weight mg is opposite to the displacement of each interval, so work done along path AB is

$$W_{AB} = -mg\Delta y_1 - mg\Delta y_2 - \text{------} - mg\Delta y_n$$

$$W_{AB} = -mg(\Delta y_1 + \Delta y_2 + \text{---------} + \Delta y_n)$$

Where

$$\Delta y_1 + \Delta y_2 + \text{-------} + \Delta y_n = h$$

So

$$\boxed{W_{AB} = -mgh} \longrightarrow (3)$$

It is clear that the work done along each path is $-mgh$ as from eq. (1), (2) and (3).

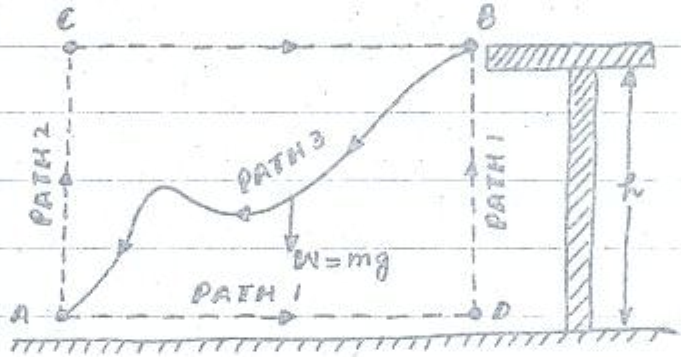
Hence, work done in a gravitational field is independent of the path followed.

WORK DONE IN A GRAVITATIONAL FIELD ALONG A CLOSED PATH

Consider an object of mass m is moved with constant velocity in a gravitational field along a closed path ADBA or ACBA as shown in figure.

Now we find work done along a closed path ADBA i-e; from A to D, D to B and then B to A.

First we find the work done in moving the object from A to D as



$$W_{AD} = \vec{F} \cdot \vec{d} = Fd\cos\theta$$

In this case, the gravitational force is equal to the weight of the object, so

$$F = W = mg$$

Also direction of weight mg is perpendicular to the displacement, so $\theta = 90°$

$$\therefore W_{AD} = Fd\cos\theta = mgd\cos 90°$$

$$W_{AD} = mgd(0) \qquad (\because \cos 90° = 0)$$

$$W_{AD} = 0 \longrightarrow ①$$

Now work done from D to B is

$$W_{DB} = \vec{F} \cdot \vec{d} = Fd\cos\theta$$

As $\qquad F = W = mg$

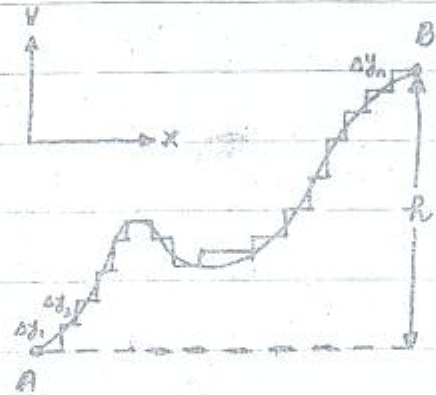
Here direction of weight mg is opposite to the displacement h, so

$$\theta = 180° \quad \text{and} \quad d = h$$

$$\therefore W_{DB} = Fd\cos\theta = mgh\cos 180° \qquad (\because \cos 180° = -1)$$

$$W_{DB} = mgh(-1) = -mgh \longrightarrow ②$$

Now the work done of curved path from B to A can be calculated by dividing the path into a series of horizontal and vertical steps



as shown. There is no work done along the horizontal steps becuse weight mg is perpendicular to the displacement for these steps. Here work is done for the vertical displacements $\Delta y_1, \Delta y_2, ---------, \Delta y_n$ by the force of gravity. As, weight mg is in same direction of the displacement of each step, so work done along path BA is

$$W_{BA} = mg(\Delta y_1 + \Delta y_2 + --------- + \Delta y_n)$$

Where

$$\Delta y_1 + \Delta y_2 + --------- + \Delta y_n = h$$

So

$$W_{BA} = mgh \longrightarrow (3)$$

Hence total work done of the closed path ADBA is

$$W_{TOTAL} = W_{AD} + W_{DB} + W_{BA}$$
$$= 0 + (-mgh) + mgh$$
$$= 0 - mgh + mgh$$
$$\boxed{W_{TOTAL} = 0}$$

Hence, the work done along a closed path in a gravitational field is zero.

Conservative Field

The field in which the work done along a closed path is zero, called conservative field.

OR

The field in which the work done along a closed path is independent of the path followed, called conservative field.

For example, gravitational field, electric field, magnetic field.

Power

The measure of the rate of work done is called power.

Average Power

The total work done divided by total time is called average power.

If work ΔW is done in time Δt, then the average power P_{av} or $\langle P \rangle$ is given as

$$\boxed{P_{av} = \langle P \rangle = \frac{\Delta W}{\Delta t}}$$

Instantaneous Power

The measure of the rate of work done at a particular instant is called instantaneous power.

As power is given as

$$P = \frac{\Delta W}{\Delta t}$$

If work ΔW is done in very short interval

Units Of Power

The SI-units of power is Watt (W).

As

$$P = \frac{\Delta W}{\Delta t}$$

$$\boxed{1\,\text{Watt} = \frac{1\,J}{1\,S}}$$

Watt → Power is said to be one watt if one joule of work is done in one second.

Kilo-Watt Hour (KWh)

It is commercial unit of electrical energy.

Defenition → One kilo-watt hour is the work done in one hour by an agency whose power is one kilo-watt.

Also

$$1\,KWh = 36 \times 10^5\,J = 3.6 \times 10^6\,J = 3.6\,MJ$$

Proof: As we know that

$$1\,KW = 1000\,W = 10^3\,J/S$$

$$1\,h = 3600\,S = 36 \times 10^2\,S$$

So

$$1\,KWh = 10^3\,J/S \times 36 \times 10^2\,S$$

$$\boxed{1\,KWh = 36 \times 10^5\,J}$$

or

$$\boxed{1\,KWh = 3.6 \times 10^6\,J}$$

or

$$\boxed{1\,KWh = 3.6\,MJ} \qquad (\because 1M = 10^6)$$

ENERGY

The capacity of a body to do work is called energy.

There are two basic forms of energy

(a) KINETIC ENERGY (K.E.)

(b) POTENTIAL ENERGY (P.E.)

KINETIC ENERGY

The energy possessed by a body due to its motion is called kinetic energy.

Its formula is given as

$$K.E. = \frac{1}{2} mv^2$$

or $$K.E. = \frac{1}{2} m(\vec{v}.\vec{v}) \qquad (\because v^2 = \vec{v}.\vec{v})$$

Where m is the mass of the body and v is the velocity of the body.

POTENTIAL ENERGY

The energy possessed by a body due to its position is called potential energy.

GRAVITATIONAL P.E.

The energy required to raise the body from one point to another against the gravitational field is called gravitational P.E.

Its formula is given as

$$\text{Gravitational P.E.} = mgh$$

Where m is the mass of the body and h is its height.

Elastic P.E

The energy stored in a compressed spring is the potential energy possessed by the spring due to its compressed or stretched state is called elastic P.E.

Its formula is given as

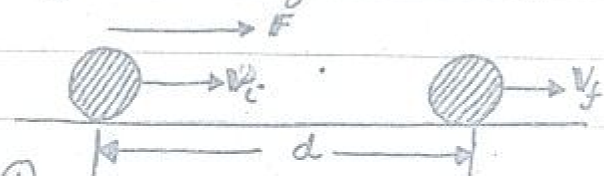
$$\boxed{\text{Elastic P.E.} = \frac{1}{2} K x^2}$$

Where K is the spring constant and x be the displacement from the mean position.

Work - Energy Principle or Relation or Theorem or Prove That Work Is Change In K.E.

Consider a body of mass m is moving with velocity V_i. Let a force F acts on the body through a distance d and increases its velocity from V_i to V_f, then from third equation of motion

$$2ad = V_f^2 - V_i^2$$



$$\therefore d = \frac{1}{2a}(V_f^2 - V_i^2) \longrightarrow ①$$

From Newton's second law of motion

$$F = ma \longrightarrow ②$$

Multiplying equation ① and ②, we have

$$Fd = \frac{ma}{2a}(V_f^2 - V_i^2)$$

$$Fd = \frac{1}{2} m (V_f^2 - V_i^2)$$

$$\boxed{Fd = \frac{1}{2} m V_f^2 - \frac{1}{2} m V_i^2} \longrightarrow (3)$$

This relation is known as work-energy principle or relation or theorem.

Where

Fd = Work done on the body.

$\frac{1}{2} m V_i^2$ = Initial K.E.

$\frac{1}{2} m V_f^2$ = Final K.E.

So, equation (3) can be written as

Work Done = Final K.E. − Initial K.E

$$\boxed{\text{Work Done} = \text{Change in K.E.}}$$

Hence work done on a body is equal to change in its kinetic energy.

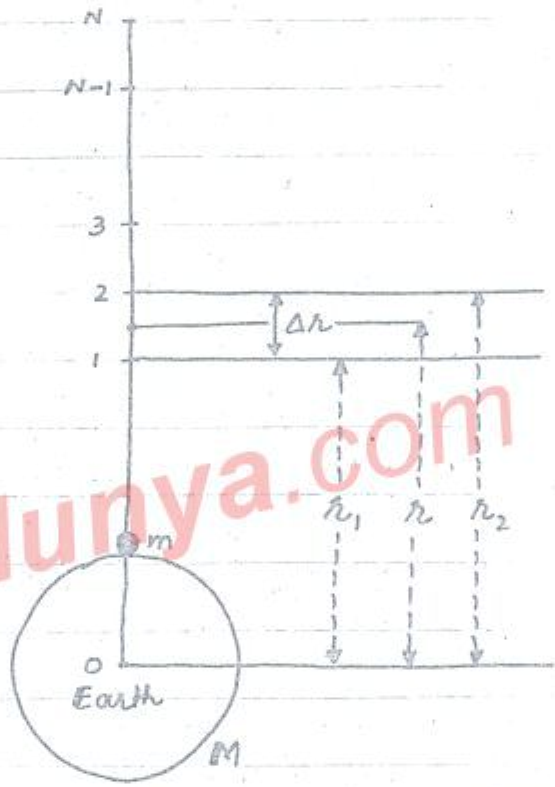
Similarly, the work done in lifting the body from the surface of earth or in compressing the spring is equal to change or increase in its gravitational P.E. and elastic P.E. respectively.

ABSOLUTE POTENTIAL ENERGY

The work done in lifting the body from surface of earth to a far off distance at infinity against gravitational force is stored in the body as its absolute P.E.

DERIVATION

Consider a body of mass m is lifted from surface of earth of mass M and radius R as shown. As we know that the relation to find work done by the gravitational force or $P.E = mgh$ is true only near the surface of the earth where the gravitational force is nearly constant. So when body is lifted through a large distance from point 1 to N in the gravitational field, then



the gravitational force does not remain constant as it is inversely proportional to the square of the distance i-e; $F \propto 1/r^2$.

To overcome this difficulty, we divide the distance from 1 to N into large number of small steps each of length Δr, so that the gravitational force for each step is practically remains constant. Hence, the total work done can be calculated by adding the work done during all the steps from 1 to N.

First we find the work done during first step from point 1 to 2:

Let

r_1 = Distance of point 1 from centre of earth.

r_2 = Distance of point 2 from centre of earth.

So distance from point 1 to 2 is

$$\Delta r = r_2 - r_1 \longrightarrow (1)$$

Now the gravitational force between body and earth from the centre of first step and centre of earth is

$$F = \frac{GMm}{r^2} \longrightarrow (2)$$

Where

G = Universal Gravitational Constant

r = Distance between centre of first step and centre of earth given as

$$r = \frac{r_1 + r_2}{2}$$

Calculation Of r^2

As

$$r = \frac{r_1 + r_2}{2}$$

From equation (1)

$$\Delta r = r_2 - r_1$$

$$\Rightarrow r_2 = \Delta r + r_1$$

$$\therefore r = \frac{r_1 + \Delta r + r_1}{2} = \frac{2r_1 + \Delta r}{2} = \frac{2r_1}{2} + \frac{\Delta r}{2}$$

$$\Rightarrow \quad r = r_1 + \frac{\Delta r}{2}$$

Taking square on both sides

$$r^2 = \left(r_1 + \frac{\Delta r}{2}\right)^2$$

$$\Rightarrow \quad r^2 = r_1^2 + \frac{\Delta r^2}{4} + 2\, r_1 \frac{\Delta r}{2}$$

$$\Rightarrow \quad r^2 = r_1^2 + \frac{\Delta r^2}{4} + r_1 \cdot \Delta r$$

As Δr is very small, so $\frac{\Delta r^2}{4} <<< r_1^2$, hence this term can be neglected as compared to r_1

$$\therefore \quad r^2 = r_1^2 + r_1 \Delta r$$

$$\Rightarrow \quad r^2 = r_1^2 + r_1(r_2 - r_1) \quad (\because \Delta r = r_2 - r_1)$$

$$\Rightarrow \quad r^2 = r_1^2 + r_1 r_2 - r_1^2$$

$$\Rightarrow \quad \boxed{r^2 = r_1 r_2}$$

Hence, work done from point 1 to 2 is

$$W_{1\to2} = \vec{F} \cdot \vec{\Delta r} = F \Delta r \cos\theta$$

As body moves opposite to the gravitational force, so $\theta = 180^\circ$

$$\therefore \quad W_{1\to2} = F \Delta r \cos 180^\circ = F \Delta r (-1)$$

$$W_{1\to2} = -F \Delta r$$

Put values of F and Δr, we get

$$W_{1\to2} = -\frac{GMm}{r^2}(r_2 - r_1) = -GMm\left(\frac{r_2 - r_1}{r^2}\right)$$

$$\Rightarrow \quad W_{1\to2} = -GMm\left(\frac{r_2 - r_1}{r_1 r_2}\right) \quad (\because r^2 = r_1 r_2)$$

$$\Rightarrow \quad W_{1\to2} = -GMm\left(\frac{r_2}{r_1 r_2} - \frac{r_1}{r_1 r_2}\right)$$

$$\Rightarrow \quad W_{1\to2} = -GMm\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

Similarly, the work done during other steps is given as

$$W_{2\to 3} = -GMm\left(\frac{1}{r_2} - \frac{1}{r_3}\right)$$

$$W_{3\to 4} = -GMm\left(\frac{1}{r_3} - \frac{1}{r_4}\right)$$

$$\vdots \qquad\qquad \vdots$$

$$W_{N-1\to N} = -GMm\left(\frac{1}{r_{N-1}} - \frac{1}{r_N}\right)$$

Now the total work done in displacing the body from point 1 to N is

$$W_{TOTAL} = W_{1\to 2} + W_{2\to 3} + \cdots\cdots + W_{N-1\to N}$$

$$\Rightarrow W_{TOTAL} = -GMm\left(\frac{1}{r_1} - \frac{1}{r_2}\right) - GMm\left(\frac{1}{r_2} - \frac{1}{r_3}\right) - \cdots\cdots - GMm\left(\frac{1}{r_{N-1}} - \frac{1}{r_N}\right)$$

$$\Rightarrow W_{TOTAL} = -GMm\left(\frac{1}{r_1} - \frac{1}{r_2} + \frac{1}{r_2} - \frac{1}{r_3} + \cdots\cdots + \frac{1}{r_{N-1}} - \frac{1}{r_N}\right)$$

$$\Rightarrow W_{TOTAL} = -GMm\left(\frac{1}{r_1} - \frac{1}{r_N}\right)$$

If point N is at an infinite distance from the earth, then

$$r_N = \infty \quad \text{and} \quad \frac{1}{r_N} = \frac{1}{\infty} = 0$$

So

$$W_{TOTAL} = -GMm\left(\frac{1}{r_1} - 0\right)$$

$$\Rightarrow W_{TOTAL} = -GMm\left(\frac{1}{r_1}\right)$$

$$\Rightarrow W_{TOTAL} = -\frac{GMm}{r_1}$$

The general expression for the gravitational P.E. U of a body at a distance r from the earth is given as

$$U = W_{TOTAL} = -\frac{GMm}{r}$$

This is also known as the absolute value of gravitational P.E. of a body at a distance r from the centre of the earth

Now, the absolute P.E. on the surface of the earth where r = R (radius of earth) is

$$\boxed{\text{Absolute P.E.} = U_g = -\frac{GMm}{R}}$$

Negative sign shows that the earth's gravitational field is attractive for the body of mass m.

Escape Velocity

It is observed that if a body projected upward, it comes back to the ground from certain height due to force of gravity. If we increased the initial velocity then the body rises to greater height before coming back. If we go on increasing the initial velocity of the body, a stage comes when body go out of the gravitational field and it will not return to the ground.

DEFINITION → The initial velocity with which a body goes out of the earth's gravitational field is known as escape velocity.

It is denoted by V_{esc}.

DERIVATION

Consider a body of mass m is moving with initial velocity V_{esc} projected upward to escape from the earth's gravitational field. The escape velocity V_{esc} corresponds to the initial K.E. gained by the body which lifted it to an infinite distance from the surface of earth is given as

$$\text{Initial K.E.} = \frac{1}{2} m V_{esc}^2$$

As we know that the work done in lifting the body from surface of earth to an infinite distance is called absolute P.E. given as

$$\text{Absolute P.E.} = \frac{GMm}{R}$$

Where

M = Mass of earth.

R = Radius of earth.

If initial K.E. of the body is equal to the absolute P.E. then it will escape out of the gravitational field, so

$$\text{Initial K.E} = \text{Absolute P.E}$$

$$\Rightarrow \frac{1}{2} m V_{esc}^2 = \frac{GMm}{R}$$

$$\Rightarrow \quad V_{esc}^2 = \frac{2GM}{R}$$

Taking square root on both sides

$$\boxed{V_{esc} = \sqrt{\frac{2GM}{R}}} \longrightarrow (1)$$

If body lies on the surface of the earth, then gravitational force between body and earth becomes equal to the weight of the body, so

Gravitational Force = Weight of Body

$$\Rightarrow \quad \frac{GMm}{R^2} = mg$$

$$\Rightarrow \quad \frac{GM}{R \times R} = g$$

$$\Rightarrow \quad \frac{GM}{R} = gR$$

Put this value in equation (1), so

$$\boxed{V_{esc} = \sqrt{2gR}} \longrightarrow (2)$$

Where

Acc. due to Gravity $= g = 9.8 \ m/s^2$

Radius of Earth $= R = 6.4 \times 10^6 \ m$

So

$$V_{esc} = \sqrt{2 \times 9.8 \times 6.4 \times 10^6}$$

or $\quad V_{esc} = 11.2 \times 10^3 \ m/s$

or $\quad V_{esc} = 11.2 \ Km/s$

Hence, the value of escape velocity V_{esc} comes out to be approximately $11.2 \ Km/s$.

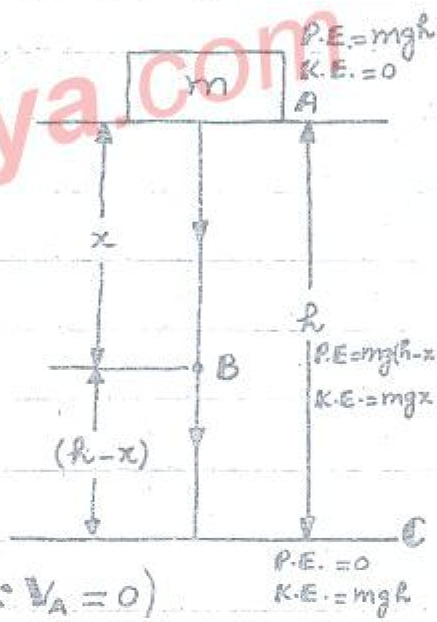
Interconversion Of P.E. And K.E. or Work - Energy Equation

As we know that when a body is lifted, then its K.E. is converted into P.E. and when body falls from certain height, then its P.E. is converted into K.E.

Now we consider the case when body is falling from certain height.

Case (i) When Friction Is Not Present

Consider a body of mass m is at rest at point A as shown in figure. Let body is at certain height h from the surface of earth at point C.



Now at position A, the body has

$$\text{P.E. at } A = mgh$$

$$\text{K.E. at } A = \frac{1}{2} m V_A^2 = 0 \qquad (\because V_A = 0)$$

We assume that there is no air friction and it falls down. Let us calculate P.E. and K.E. at position B. The distance covered by the body from position A to B is x, so its height from C is $(h-x)$. Hence

$$\text{P.E. at } B = mg(h-x)$$

and $\text{K.E. at } B = \frac{1}{2} m V_B^2$.

The velocity V_B at point B can be calculated as

Initial Velocity $= V_i = V_A = 0$

Final Velocity $= V_f = V_B$

Acceleration $= a = g$

Distance $= S = x$

Using third equation of motion

$$V_f^2 - V_i^2 = 2aS$$

or $$V_B^2 - (0)^2 = 2gx$$

or $$V_B^2 = 2gx$$

So

$$\text{K.E. at } B = \frac{1}{2} m V_B^2 = \frac{1}{2} m (2gx)$$

$$\text{K.E. at } B = mgx$$

Now the total energy at position B is

$$\text{TOTAL ENERGY AT } B = \text{P.E. at } B + \text{K.E. at } B$$

$$= mg(h - x) + mgx$$

$$= mgh - mgx + mgx$$

$$\text{TOTAL ENERGY AT } B = mgh$$

Now, at position C just before hitting the earth, P.E. and K.E. given as

$$\text{P.E. at } C = mgh = 0 \qquad (\because h = 0)$$

and $$\text{K.E. at } C = \frac{1}{2} m V_C^2$$

The velocity V_C can be calculated as

Initial Velocity $= V_i = V_A = 0$

Final Velocity $= V_f = V_C$

Acceleration $= a = g$

Distance $= S = h$

Again using third equation of motion

$$V_f^2 - V_i^2 = 2aS$$

or $$V_c^2 - (0)^2 = 2gh$$

or $$V_c^2 = 2gh$$

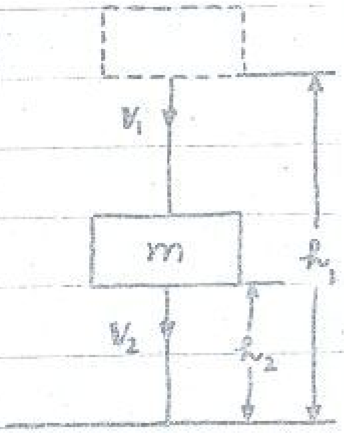
So

$$K.E. \text{ at } C = \frac{1}{2} m V_c^2 = \frac{1}{2} m (2gh)$$

$$K.E. \text{ at } C = mgh$$

Hence, K.E. at point C is equal to the original value of P.E. of the body at point A. Actually when a body falls, its velocity increases which results in the increase in its K.E. While P.E. of the falling body decreases as its height decreases. Thus we write as

$$\boxed{\text{Loss in P.E.} = \text{Gain in K.E.}}$$



or $$mg(h_1 - h_2) = \frac{1}{2} m (V_2^2 - V_1^2)$$

Where V_1 and V_2 are the velocities of the body at heights h_1 and h_2 respectively as shown in figure. This equation holds only if frictional force is not present.

Case (ii) WHEN FRICTION IS PRESENT

If force of friction f is present during the downward motion of the body, then a part of P.E. is used in doing work against friction fh and the remaining P.E. is converted into K.E.

Gain in K.E. = $mgh - fh$

or mgh = Gain in K.E. + fh

Where

mgh = Loss in P.E.

fh = Work done against friction

So

Loss in P.E. = Gain in K.E. + Work done against friction

CONSERVATION OF ENERGY

The law of conservation of energy stated as "Energy cannot be destroyed, it can be transformed from one kind into another but the total energy remains constant."

The kinetic and potential energies are both different forms of the same quantity. Total energy of a body is the sum of K.E. and P.E. As we know that for a falling body, the P.E. may be converted into K.E. and vice versa but the total energy remains constant.

Mathematically,

Total Energy = P.E. + K.E. = Constant

This is one of the basic law of physics.

Fsc Part 1 Chapter 4

In our daily life, we observe many energy transformations from one form to another such as electrical and chemical energies are easily transferred into heat. All the energies ultimately transferred into heat and wasted to environment.

For example, the P.E. of a falling body changes into K.E. But on hitting the ground, K.E. is converted into heat and sound and absorbed by the ground.

Fsc Part 1 Chapter 4

SHORT QUESTIONS & ANSWERS

4.1 A person holds a bag of groceries while standing still, talking to a friend. A car is stationary with its engine running. From the standpoint of work, how are these two situations similar?

Ans. In both cases, the displacement is zero i.e., $\vec{d} = 0$ and hence, no work is done because $W = \vec{F} \times \vec{d}$, $W = \vec{F} \times 0 = 0$.

4.2 Calculate the work done in kilo joules in lifting a mass of 10 kg (at a steady velocity) through a vertical height of 10 m.

Ans. As work = Force × displacement

Here Force = Weight = mg & Displacement = Height

Therefore

Work = Weight × height

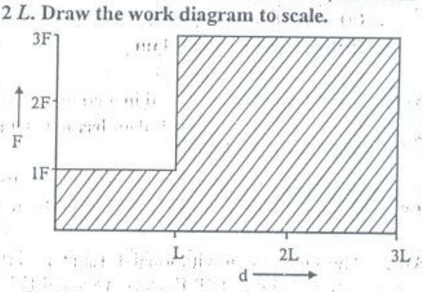
$W = mgh$

$W = 10\ kg \times 9.8\ ms^{-2} \times 10\ m = 980\ J = 0.98\ kJ$

Hence approximately 1k J work is done.

4.3 A force *F* acts through a distance *L*. The force is then increased to 3 *F*, and then acts through a further distance of 2 *L*. Draw the work diagram to scale.

Ans. Work diagram is shown in the figure which is the force – distance a graph. The area under force distance graph (Shaded Area) is equal to the amount of work done. It happens to be 7 units.



4.4 In which case is more work done? When a 50 kg bag of books is lifted through 50 cm, or when a 50 kg crate is pushed through 2m across the floor with a force of 50 N?

Ans. In 1st case when 50 kg bag is lifted through 50 cm or 0.5 m the work done is:

$W_1 = mgh$

$= 50\ kg \times 9.8\ ms^{-2} \times 0.5\ m = 245\ J$

In 2nd case when 50 kg crate is pushed through 2 m by a force of 50 N, the work done is

$W_2 = F \times d = 50\ N \times 2m = 100\ J \quad (\because 1\ Nm = 1J)$

Hence more work is done in the 1st case.

4.5 An object has 1 J of potential energy. Explain what does it mean?

Ans. An object has P.E when it is in a force field such as gravitational field or it is in a constrained state such as a compressed spring or extended spring. In the 1st case where an object is raised up by doing 1 J of work on it, the work appears as its gravitational P.E. In 2nd case when a spring is compressed or extended by doing 1J work on it, the work is conserved in the spring as its elastic P.E.

4.6 A ball of mass m is held at a height h_1 above a table. The tabletop is at a height h_2 above the floor. One student says that the ball has potential energy mgh_1 but another says that it is $mg(h_1 + h_2)$. Who is correct?

Ans. Both statements are correct. It is a matter of relative position for specifying the P.E of the ball. The P.E relative to the tabletop is mgh_1 and the P.E relative to the ground is $mg(h_1+h_2)$.

4.7 When a rocket re-enters the atmosphere, its nose cone becomes very hot. Where does this heat energy come from?

Ans. When a rocket reenters the atmosphere with high velocity, its nose becomes very hot due to air friction. The work done against the air friction is changed into heat and the nose of the rocket becomes very hot.

4.8 What sort of energy is stored in the following:

(a) Compressed spring

(b) Water in a high dam

(c) A moving car

Ans. (a) Elastic P.E. is stored in a compressed spring.

(b) The water in a high dam has gravitational P.E. due to its height with respect to the basin.

(c) A moving car has kinetic energy due to its motion.

4.9 A girl drops a cup from a certain height, which breaks into pieces. What energy changes are involved?

Ans. The cup has gravitational P.E. at a certain height. When it is dropped, from a certain height, its P.E. decreases and K.E. increases. Just before striking the floor, the whole of P.E. is changed into K.E. On striking the ground, the K.E. is changed into sound, heat and work to break the cup into pieces.

4.10 A boy uses a catapult to throw a stone, which accidentally smashes a green house window. List the possible energy changes.

Ans. The elastic P.E. of the catapult is transferred to stone as its kinetic energy. When the stone strikes the green house window, the K.E. of the stone is changed into sound, heat and work done in breaking the window glass.