CIRCULAR MOTION:

Motion of a body on a circular path is called circular motion.

EXAMPLES:

(i) Motion of planets around the sun.
(ii) A stone whirled around by a string.
(iii) A car turning around a corner.
(iv) Satellites in orbits around the earth.

During circular motion, velocity of body changes continuously. This change in velocity per unit time gives acceleration which is produced by different forces.

In motion of planet around the sun, this acceleration is produced by gravitational force. The coulomb force of attraction is responsible for motion of electron around the nucleus.

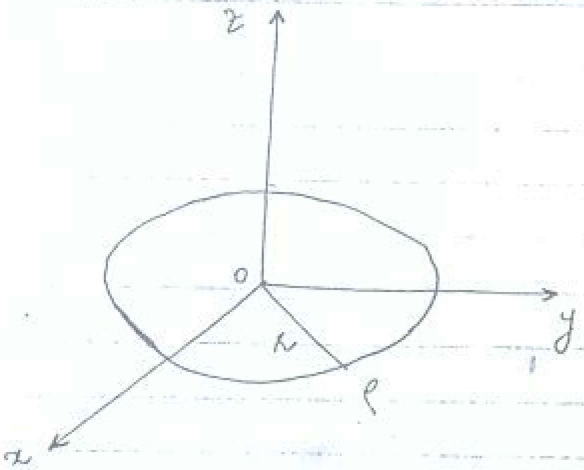
ANGULAR DISPLACEMENT:

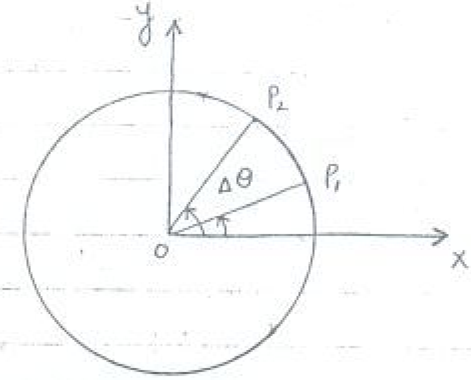
Angle ($\Delta\theta$) swept by a body in time interval Δt during the angular motion, at centre of its circular path is called "Angular Displacement"

EXPLANATION:

Consider the motion of a particle 'P' attached at the end of massless rigid rod of length 'r' whose other end is pivoted at the centre 'O' of the circular path. Suppose the particle 'P' moves in a circle of radius 'r' whose centre lies at the origin of xy-plane.

P 5.2





The axis of rotation is along z-axis with pivot 'O' as the origin of co-ordinates. Let,

$\vec{OP_1}$ = Position vector of particle at time 't' making angle θ with x-axis.

$\vec{OP_2}$ = Position vector of particle at time $(t + \Delta t)$ making angle $(\theta + \Delta\theta)$ with x-axis.

From figure,

$P_1 \hat{O} P_2 = \Delta\theta$ = Angular disp. of particle 'P' during time interval "Δt"

SIGN OF CONVENTION:

(i) The angular displacement $\Delta\theta$ is taken as positive, when the rotation of particle 'P' is "Anti-clockwise"

(ii) The angular displacement $\Delta\theta$ is taken as negative, when the rotation of particle 'P' is taken as "Clockwise".

small value of $\Delta\theta$, the angular disp. is a vector quantity.

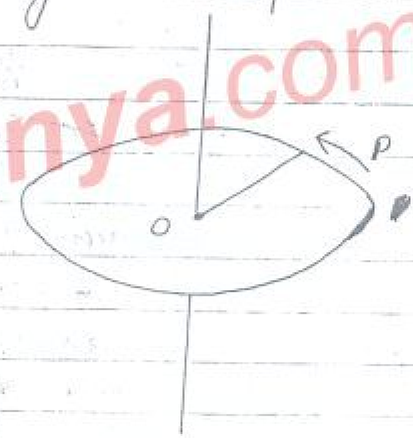
The direction associated with $\Delta\theta$ is along the axis of rotation and is given by "Right Hand Rule"

RIGHT HAND RULE:

According to this rule,

"Grasp the axis of rotation in right hand such that curl of fingers is in the direction of rotation. Then thumb points in the direction of angular displacement."

According to right hand rule the direction of angular disp. is out of the plane of circle, when the particle rotated in anti-clockwise direction.



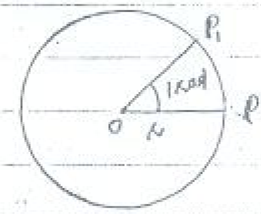
UNITS:

The units of angular displacement are
(i) Degree
(ii) Revolution
(iii) Radian

RADIAN:

It is SI-unit of angular disp. It is defined as, "Angle made by an arc whose length is equal to the radius of circle is called radian

From Figure,
when arc $PP_1 = r$
and $P\hat{O}P_1 = 1$ radian

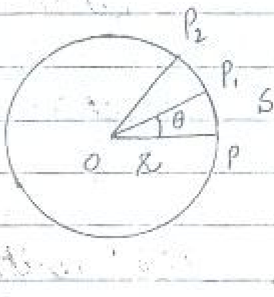


PROVE THAT $S = r\theta$:

Here,
S = Arc Length
r = Radius of circular path.
θ = Angle made by an arc in radians at the centre of circular path.

PROOF:
Consider the motion of the particle 'P' on a circular path of radius 'r'.

In moving from P to P_1, the arc $PP_1 = S$ makes angle "θ" measured in radians at the centre of the circle.



Also,
Length of arc $PP_2 = r$
So that, $P\hat{O}P_2 = 1$ radian

Diagrame shows that angle subtended at the centre of the circle is proportional to length of arc

So,

$$\frac{\text{arc } PP_1}{\text{arc } PP_2} = \frac{\theta \text{ radian}}{1 \text{ radian}}$$

$$\Rightarrow \quad \frac{S}{r} = \frac{\theta}{1}$$

$$\Rightarrow \quad \boxed{S = r\theta}$$

which is required relation

SHOW THAT 1 radian = 57.3°:

PROOF:

As we know that

$$S = r\theta$$

$$\Rightarrow \quad \theta = \frac{S}{r} \text{ radian}$$

Consider a particle moves in a circle of radius "r" and covers distance $S = 2\pi r$ in its one revolution.

$$\therefore \quad \theta = \frac{2\pi r}{r} \text{ radian}$$

Here,

1 revolution = 2π radians

Also,

1 revolution = 360 Degrees

$$\therefore \quad 2\pi \text{ radians} = 360 \text{ Degrees}$$

$$1 \text{ radian} = \frac{360^\circ}{2\pi}$$

$$\boxed{1 \text{ radian} = 57.3^\circ}$$

which is required relation.

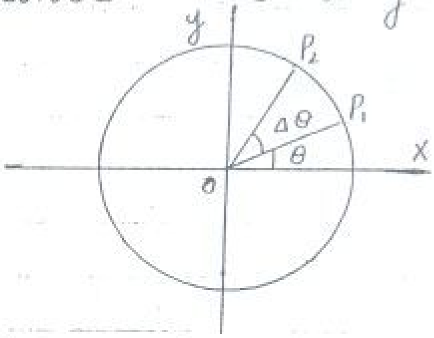
ANGULAR VELOCITY:

The rate of change of angular displacement is called angular velocity.

It is denoted by "ω" (omega).

Average Angular Velocity:

Consider a body moving on a circumference of a circle of radius 'r'. If $\Delta\theta$ is the angular disp. during the time interval Δt, Then,



$$\text{Average Angular velocity} = \frac{\text{Total angular disp.}}{\text{Total Time}}$$

$$\omega_{ave} = \frac{\Delta\theta}{\Delta t}$$

or

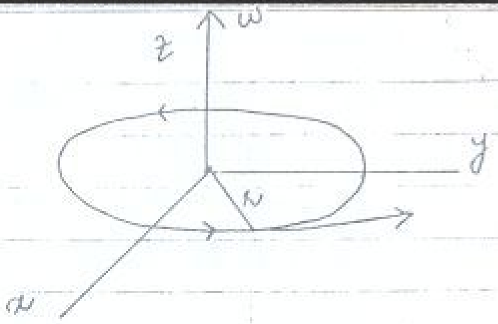
$$\langle\omega\rangle = \frac{\Delta\theta}{\Delta t}$$

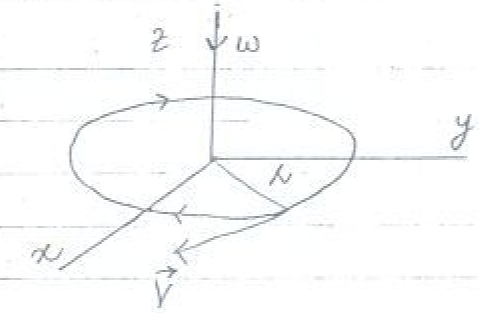
Representation Of Angular Velocity:

Angular velocity is a vector quantity. It is represented by a line drawn parallel to the axis of rotation. The length of line gives magnitude of angular velocity. The direction of angular velocity is given by "Right Hand Rule"

Right Hand Rule:

According to this rule, "Grasp the axis of rotation in the right hand with fingers curled around the direction of rotation, then the thumb gives the direction of rotation"





Instantaneous Angular Velocity:

If the angular velocity is not uniform during the motion of the body along a circular path, Then we define instantaneous angular velocity. So, it is defined as "The angular displacement $\Delta\theta$ during a small interval of time Δt approaching zero."

So,

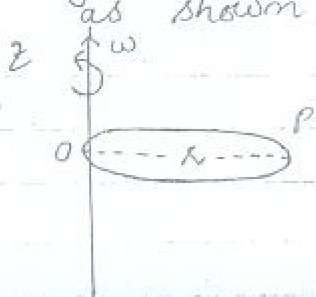
$$\vec{\omega} = \lim_{\Delta t \to 0} \frac{\overrightarrow{\Delta\theta}}{\Delta t}$$

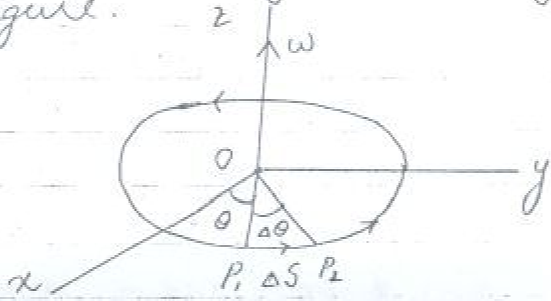
Units:

(i) rad / Sec or rad S^{-1}
(ii) degree / Sec or degree S^{-1}
(iii) revolution / min or revolution min^{-1}

Relation Between Linear And Angular Velocity $V = r\omega$:

Consider a rigid body rotating about z-axis with angular velocity "ω" as shown in figure.





Let a point "P_1" in the rigid body rotates in a circle of radius r, whose centre lies at the origin of xy-plane. Suppose the point 'P_1' moves through a distance $P_1P_2 = \Delta S$ in a time interval Δt.

The arc $P_1P_2 = \Delta S$ subtends an angle $\Delta\theta$ at the centre of the circle.

So,

$$\Delta S = r\,\Delta\theta$$

Dividing by 'Δt' B.S, we get,

$$\frac{\Delta S}{\Delta t} = r\,\frac{\Delta\theta}{\Delta t}$$

Taking Limit $\Delta t \to 0$ B.S.

$$\lim_{\Delta t\to 0}\frac{\Delta S}{\Delta t} = r\lim_{\Delta t\to 0}\frac{\Delta\theta}{\Delta t} \longrightarrow (1)$$

As,

$$\lim_{\Delta t\to 0}\frac{\Delta S}{\Delta t} = V$$

and

$$\lim_{\Delta t\to 0}\frac{\Delta\theta}{\Delta t} = \omega$$

Put in (1) we get,

$$\boxed{V = r\omega}$$

Which is required relation.

Here 'V' is the magnitude of instantaneous velocity and its direction is always along the tangent to the circle, that is why the linear velocity of point 'P' is also known as tangentile velocity

Angular Acceleration:

The rate of change of angular velocity is called angular acceleration. It is denoted by "α".

If ω_i and ω_f are the values of instantaneous angular velocity of a rotating body at instants "t_i" and "t_f", the average angular acceleration during the interval $t_f - t_i$ is given by;

$$\vec{\alpha}_{ave} = \frac{\vec{\omega}_f - \vec{\omega}_i}{t_f - t_i}$$

$$\vec{\alpha}_{ave} = \frac{\Delta \vec{\omega}}{\Delta t}$$

It is a vector quantity and has the same direction as that of "ω". It is measured in rad s^{-2}.

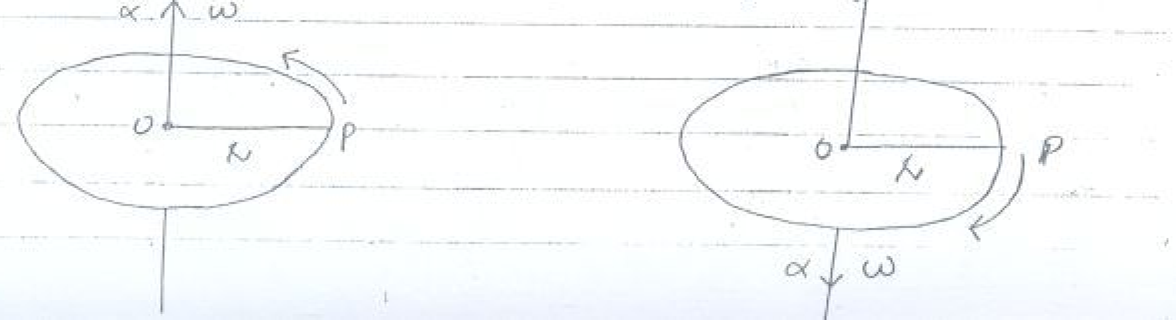
Instantaneous Angular Acceleration:

It is defined as, "The angular acceleration of a body at any instant during its circular motion." So,

$$\vec{\alpha} = \lim_{\Delta t \to 0} \frac{\Delta \vec{\omega}}{\Delta t}$$

Representation Of Angular Acceleration:

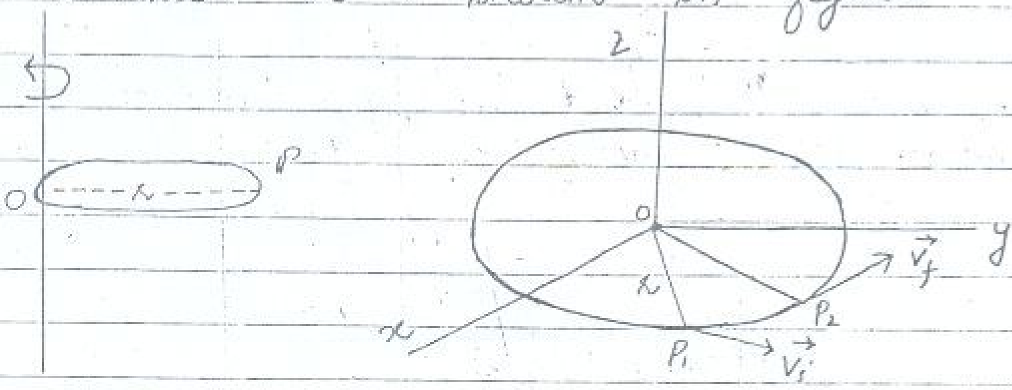
The angular acceleration is a vector quantity whose direction is along the axis of rotation, given by "right hand rule"



PROVE THAT $a = r\alpha$.

PROOF:

Consider a rigid body rotating about z-axis as shown in figure.



Let a point P in the rigid body rotates in circle of radius "r" whose centre lies at the origin of xy-plane.

Suppose the point P moves from point P_1 to P_2 in time interval Δt.

$\vec{V_i}$ and $\vec{V_f}$ are instantaneous linear velocities of point P at instants "t_i" and "t_f" respectively.

Now,

$$V_i = r\omega_i \longrightarrow (i)$$

$$V_f = r\omega_f \longrightarrow (ii)$$

∴ We can write,

$$V_f - V_i = r(\omega_f - \omega_i)$$

Divide B.S by $t_f - t_i$

$$\frac{V_f - V_i}{t_f - t_i} = r\left(\frac{\omega_f - \omega_i}{t_f - t_i}\right)$$

or $$\frac{\Delta V}{\Delta t} = r\frac{\Delta \omega}{\Delta t}$$

Taking Limit $\Delta t \rightarrow 0$ B.S.

$$\lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = r \lim_{\Delta t \to 0} \frac{\Delta \omega}{\Delta t} \quad \text{(iii)}$$

As,

$$\lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = a$$

and,

$$\lim_{\Delta t \to 0} \frac{\Delta \omega}{\Delta t} = \alpha$$

Put these values in eq. (3)

$$\therefore \boxed{a = r\alpha}$$

where,

a = linear or tangential acceleration of point P of rigid body

In vector form above eq. can be written as,

$$\boxed{\vec{a} = \vec{\alpha} \times \vec{r}}$$

Comparison Of Linear And Angular Equations:

The equations of angular motion are exactly analogous to those in linear motion.

Linear Motion	Angular Motion
(i) $v_f = v_i + at$	(i) $\omega_f = \omega_i + \alpha t$
(ii) $2aS = v_f^2 - v_i^2$	(ii) $2\alpha\theta = \omega_f^2 - \omega_i^2$
(iii) $S = v_i t + \frac{1}{2} at^2$	(iii) $\theta = \omega_i t + \frac{1}{2}\alpha t^2$
(iv) $V_{av} = \frac{v_i + v_f}{2}$	(iv) $\langle\omega\rangle = \frac{\omega_i + \omega_f}{2}$
(v) $F = ma$	(v) $\tau = I\alpha$

CENTRIPETAL FORCE:

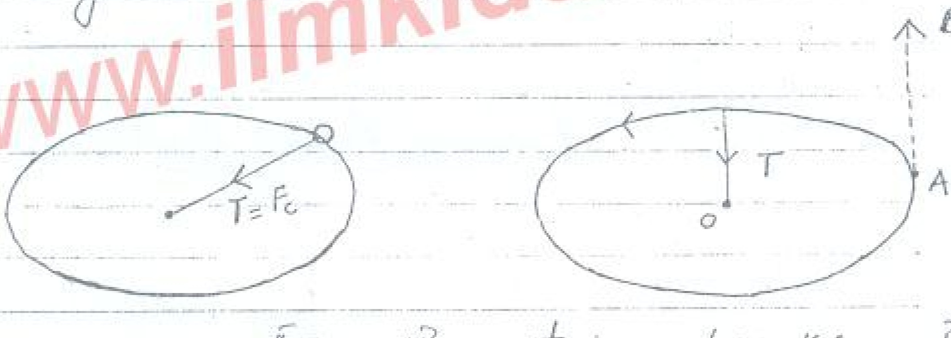
"The force required to keep a body revolving on a circular path and is always directed towards the centre of circle is called "Centripetal Force"

OR

"The force required to bend the normally straight path of the particle into a circular path is called the Centripetal force."

EXAMPLE:

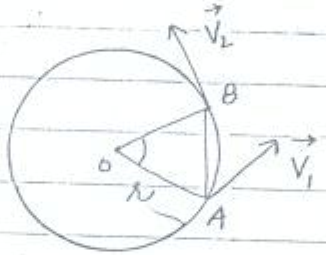
"When a ball is whirled in a horizontal circle at the end of a string. Then tension in a string behaves as centripetal force in order to keep the ball moving in circle.

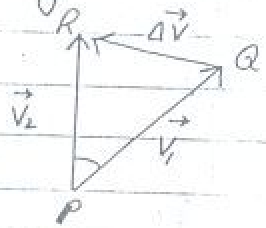


If the string breaks, when the ball is at point 'A', the ball will follow straight line path AB. This shows that ball will not continue along the circular path in the absence of tension force which acts as centripetal force.

DETERMINATION:

Consider a particle of mass 'm' moving on a circular path of radius 'r' with uniform linear speed 'V' and constant angular velocity 'ω'.





As the particle moves from A to B with uniform speed 'V' but the velocity of the particle changes because of change in direction. Hence, the acceleration of the particle is,

$$a = \frac{\Delta V}{\Delta t} \longrightarrow (i)$$

Where 'Δt' is time taken by the particle to travel from A to B, are $\vec{V_1}$ and $\vec{V_2}$. Now time taken to travel distance is,

$$\Delta t = \frac{S}{V} \longrightarrow (ii)$$

By putting value of 'Δt' in (i)

$$\therefore \quad a = \frac{\Delta V}{S/V}$$

$$\Rightarrow \quad a = V\,\frac{\Delta V}{S} \longrightarrow (iii)$$

Let us draw a triangle PQR as shown in fig.

$$\overrightarrow{QR} = \Delta\vec{V} = \text{Change in vel.} = \vec{V_2} - \vec{V_1}$$

Now,

$m\angle AOB = m\angle QPR$ ∵ [Angle b/w lines]

As,

$$V_1 = V_2 \quad \text{and} \quad \overline{OA} = \overline{OB}$$

ΔOAB and ΔPQR are isosceles triangles and they are also similar. Hence we can write

$$\frac{\overline{AB}}{\overline{QR}} = \frac{\overline{OA}}{\overline{PQ}}$$

$$\Rightarrow \quad \frac{\overline{AB}}{\Delta V} = \frac{r}{V} \longrightarrow (iv)$$

When $\Delta t \to 0$, the point 'B' is very close to point 'A' on circle. Then,

$$\text{arc } \overline{AB} \simeq \text{Line } AB$$

So we can substitute $\overline{AB} = S$ in Eq. (iv)

$$\frac{S}{\Delta V} = \frac{r}{V}$$

$$\text{or} \quad \Delta V = \frac{SV}{r}$$

By putting the value of 'ΔV' in Eq. (iii)

$$a = \frac{V\,(SV/r)}{S}$$

$$\boxed{a = \frac{V^2}{r}} \quad \text{——— (v)}$$

This is the instantaneous acceleration produced by centripetal force and is called "centripetal acceleration".

It is denoted by a_c. This acceleration is directed along the radius towards the centre of the circle.

Multiply eq. (v) B.S by 'm' we get,

$$ma = \frac{mv^2}{r}$$

According to 2nd law of motion,

$$F = ma$$

So,

$$\boxed{F_c = \frac{mv^2}{r}} \quad \text{——— (vi)}$$

Which is required expression for centripetal force. As,

$$v = r\omega$$

$$\therefore \quad F_c = \frac{m(r\omega)^2}{r}$$

$$\boxed{F_c = mr\omega^2}$$

This is expression of centripetal force in terms of angular velocity.

EXAMPLES:

(i) Earth revolves around the sun because of the centripetal force provided by gravitational force between sun and earth.

(ii) When a car takes turn around a corner then centripetal force is provided by friction between road and tyres of car.

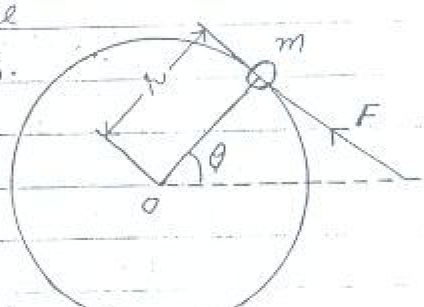
MOMENT OF INERTIA:

The product of mass and square of the moment arm is called "Moment Of Inertia"

EXPLANATION:

Consider a mass 'm' attached to a massless rod pivoted at

'O'. Let us assume that pivot point 'O' is frictionless and system is in horizontal plane. A force 'F' acts on the mass perpendicular to the rod and produce acceleration in the mass.



According to Newton's 2nd Law,

$$F = ma \longrightarrow (i)$$

The linear or tangential acceleration of mass 'm' is given by,

$$a = r\alpha$$

So, Eq. (i) becomes as,

$$F = mr\alpha \longrightarrow (ii)$$

Multiplying B.S by 'r'

$$Fr = mr^2\alpha$$

$$\tau = mr^2\alpha \quad \because (\tau = rF)$$

The quantity (mr^2) is known as moment of inertia represented by 'I'

$$I = mr^2$$

The moment of inertia plays the same role in angular motion as the inertial mass in linear motion.

DEPENDANCE:

(i) Mass of a particle 'm'.
(ii) Distance of mass from axis of rotation.

UNITS:

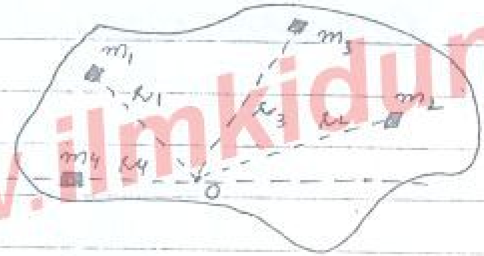
SI-units of moment of inertia is $kg.m^2$.

DIMENSION

$[ML^2]$ or $[ML^2T^0]$

MOMENT OF INERTIA OF RIGID BODY:

Consider a rigid body which is made up of n small pieces of masses i-e; $m_1, m_2 ------- m_n$ at distances $r_1, r_2 ----- r_n$ from axis of rotation 'O'.



Let the body is rotating with angular acceleration α. So the magnitude of torque acting on mass m_1 is,

$$\tau_1 = m_1 r_1^2 \alpha_1$$

Similarly, the torque on m_2 is,

$$\tau_2 = m_2 r_2^2 \alpha_2$$

And so on.

Since the body is rigid, so all the masses are rotating with the same angular acceleration α.

$\therefore$ Total Torque is given by,

$$\tau_{TOTAL} = (m_1 r_1^2 \alpha + m_2 r_2^2 \alpha + \cdots\cdots + m_n r_n^2 \alpha)$$

$$\tau_{TOTAL} = (m_1 r_1^2 + m_2 r_2^2 + \text{------} + m_n r_n^2)\,\alpha$$

$$\left(\sum_{i=1}^{n} m_i r_i^2\right)\alpha$$

$$\tau = I\alpha$$

where,

I = Moment of inertia of rigid body

$$= \left(\sum_{i=1}^{n} m_i r_i^2\right)$$

ANGULAR MOMENTUM:

The magnitude of angular or rotational motion in a body is called its "Angular Momentum".

It is also defined as,

A particle is said to be possess an angular momentum about a refrence axis if it moves such that its angular position changes relative to the refrence axis.

Mathematically,

The cross product of position vector $\vec{r}$ and liner momentum $\vec{P}$ is called angular momentum.

$$\vec{L} = \vec{r} \times \vec{P}$$

where $\vec{L}$ is angular momentum. It is a vector quantity.

EXPLANATION:

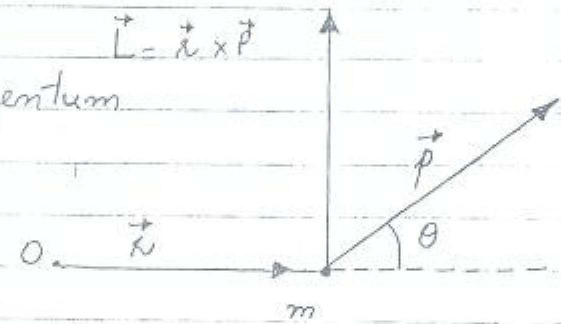
Consider a particle whose position is given by position vector $\vec{r}$

w.r.t an origin, 'O'.
Its linear momentum is,

$\vec{P} = m\vec{V}$

And, angular momentum is,

$\vec{L} = \vec{r} \times \vec{P}$



Its magnitudes is,

$|\vec{L}| = rp\sin\theta$

Where, 'θ' is angle b/w $\vec{r}$ and $\vec{P}$.

The angular momentum '$\vec{L}$' is perpendicular to the plane containing $\vec{r}$ and $\vec{P}$ and its direction is determined by right hand rule as shown in fig.

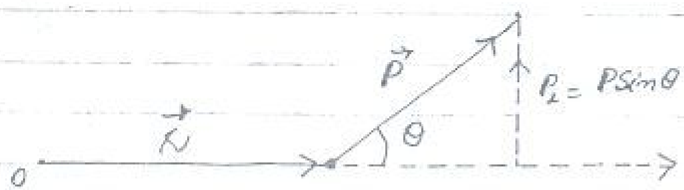
As we know that,

$\vec{L} = \vec{r} \times \vec{P}$

$\Rightarrow L = rp\sin\theta$

$= r(P\sin\theta)$

$\boxed{L = rP_{\perp}}$

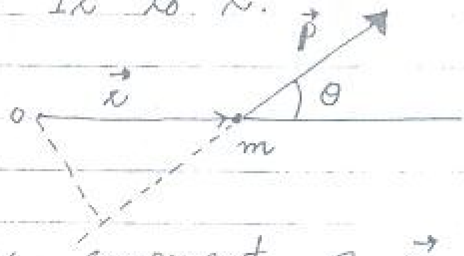


Where, $P_{\perp} = P\sin\theta$ is perpendicular ~~concept~~ component of $\vec{P}$ $\perp r$ to $\vec{r}$.

Also,

$L = P(r\sin\theta)$

$\boxed{L = Pr_{\perp}}$



Where, $r_{\perp} = r\sin\theta$ is component of $\vec{r}$ $\perp r$ to $\vec{P}$.

Case (i) If $\theta = 0°$, $L = rp\sin 0° = 0$

Case (ii) If $\theta = 180°$, $L = rp\sin 180° = 0$

Case (iii) If $\theta = 90°$, $L = rp\sin 90° = rp$

$= rmV$

UNITS:

As,

$$L = mVr$$

$$= Kg.\frac{m}{sec}.m$$

$$L = Kg\ m^2/sec$$

or $L = Kg\frac{m}{sec^2}.m \times sec = N.m.sec^2$

$L = Joul.\ sec^2$

DIMENSION:

As,

$$L = mVr$$

Dimension of 'L' is,

$$[L] = \left[M.\frac{L}{T}.L\right]$$

$$[L] = [M L^2 T^{-1}]$$

ANGULAR MOMENTUM AND MOMENT OF INERTIA:

As, the magnitude of angular momentum:

$$L = mVr$$

$$L = m(r\omega)r \qquad \text{as } (V = r\omega)$$
$$L = mr^2\omega$$

or Angular momentum $= L = (mr^2)\,\omega$

where,

$mr^2 =$ Moment of Inertia $= I$

$\therefore$ Angular Momentum $= L = I\omega$

This is the relation between angular momentum and moment of inertia.

If a body is made of a large number of particles, then total angular momentum of the rigid body is,

$$L = \left(\sum_{i=1}^{n} m_i r_i^2\right)\omega$$

The moment of inertia of rigid body is,

$$I = \sum_{i=1}^{n} m_i r_i^2$$

Where "r_i" is the distance of different particles from axis of rotation.

Types Of Angular Momentum:

(i) Spin angular momentum.

(ii) Orbital angular momentum.

Spin Angular Momentum:

It is the angular momentum of spining body.

Orbital Angular Momentum:

It is the angular momentum associated with motion of a body along a circular path.

NOTE:

We have seen that "$\vec{T}$" plays same role in rotatory motion as "$\vec{F}$" in linear motion. In linear motion if 'F' is the rate of change of linear momentum,

$$\vec{F} = \frac{\Delta \vec{P}}{\Delta t}$$

Then "$\vec{T}$" is the rate of change of angular momentum in rotatory motion.

$$\vec{T} = \frac{\Delta \vec{L}}{\Delta t}$$

LAW OF CONSERVATION OF ANGULAR MOMENTUM:

STATEMENT:-

According to this law,

"If no external torque acts on a system, the total angular momentum of the system remains constant."

As we know that,

$$\vec{T} = \frac{\Delta \vec{L}}{\Delta t}$$

If no external torque acts on a system. Then,

$$\vec{T} = 0$$

$$\therefore \quad \frac{\Delta \vec{L}}{\Delta t} = 0$$

$$\Rightarrow \quad L = \text{constant}$$

As the angular momentum is constant so

$$I_1 \omega_1 = I_2 \omega_2$$

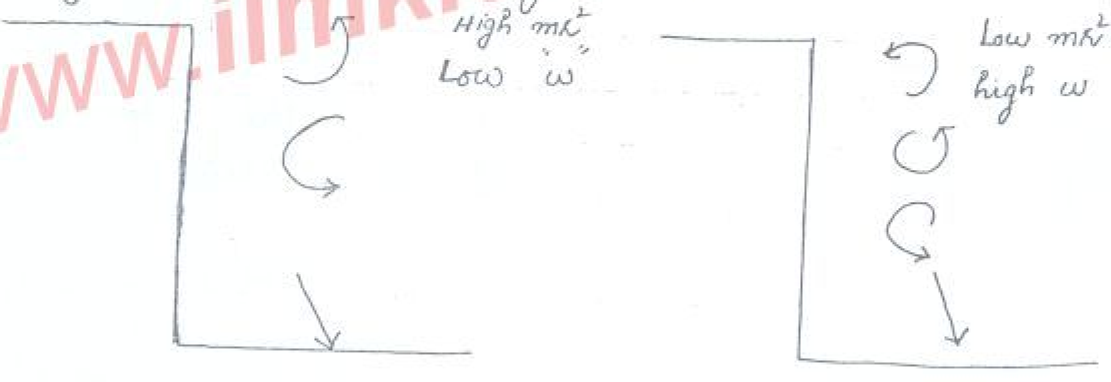
EXPLANATION:

For explanation, let us consider the following examples.

Examples:

(1) Let us take the example of a diver who jumps from a swimming board. If he curls his body, as the radius of his curled body decreases, the moment of inertia about his centre of gravity decreases and angular velocity increases to keep the angular momentum conserved.

In this way, he is able to perform certain number of ______ before touching the water surface.



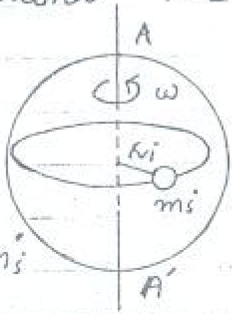
(2) The evolution of stars is considered to be the result of the law of conservation angular momentum.

ROTATIONAL KINETIC ENERGY:

The K.E possessed by a body due to its spin about its axis is known as "rotational kinetic energy"

EXPLANATION:

Consider a body is spinning about an axis with constant angular velocity 'ω', then each point of the body is moving in a circular path and has rotational K.E. Consider the spinning body is composed of tiny pieces of masses,



$m_1, m_2 \text{ ------- } m_n$

If a piece of mass "m_i" is at a distance "r_i" from axis of rotation. Then its K.E is,

$$K.E = \frac{1}{2} m_i V_i^2$$

where,

$$V_i = r_i \omega$$

$$\therefore \quad K.E = \frac{1}{2} m_i (r_i \omega)^2$$

$$K.E = \frac{1}{2} m_i r_i^2 \omega^2$$

The rotational K.E of the spinning body is the sum of all the pieces.

$$\therefore (K.E)_{rot} = \frac{1}{2} m_1 r_1^2 \omega^2 + \frac{1}{2} m_2 r_2^2 \omega^2 + \text{------} + \frac{1}{2} m_n r_n^2 \omega^2$$

$$= \frac{1}{2} \left(m_1 r_1^2 \omega^2 + m_2 r_2^2 \omega^2 + \text{------} + m_n r_n^2 \omega^2 \right)$$

$$= \frac{1}{2} \left(\sum_{i=1}^{n} m_i r_i^2 \right) \omega^2$$

$$\boxed{(K.E)_{rot} = \frac{1}{2} I \omega^2}$$

where $I = \sum_{i=1}^{n} m_i r_i^2$ = Moment of inertia of Body

Advantage Of Rotational K.E.

Rotational K.E has practical uses. For example, the rotational K.E of flywheels is used in many engines for keeping smooth rotation. A flywheel stores energy between the power stokes of the pistons, so that the energy is distributed over the full revolution of the crankshaft and hence rotation remains smooth.

Rotational K.E Of Disc And Hoop:

As, we know that rotational K.E of a spinning or rolling body is,

$$(K.E)_{rot} = \frac{1}{2} I \omega^2$$

The moment of inertia of a disc is given by,

$$I = \frac{1}{2} mr^2$$

$$\therefore (K.E)_{rot} = \frac{1}{2} (\tfrac{1}{2} mr^2) \omega^2$$

$$= \frac{1}{4} mr^2 \omega^2$$

where,

$$V^2 = r^2 \omega^2$$

Hence,

$$\boxed{(K.E)_{rot} = \frac{1}{4} mV^2}$$

This is expression for rotational K.E of a disc.

Also, the moment of inertia of a hoop is given by,

$$I = mr^2$$

$$\therefore (K.E)_{rot} = \frac{1}{2}(mr^2)\omega^2$$

where,

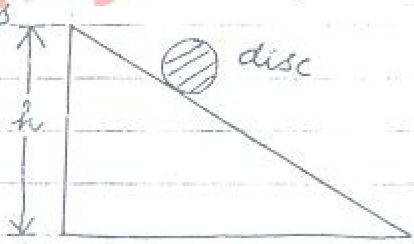
$$V^2 = r^2\omega^2$$

$$\boxed{(K.E)_{rot} = \frac{1}{2}mV^2}$$

This is expression for rotational K.E of a hoop.

Velocity Of A Disc And Hoop Rolling On An Incline Plane:

Consider a disk starts moving down on incline plane of height 'h'. The motion of disc consists of both rotational and translational motions. If no energy is lost due to friction, then total energy of the disc on reaching the bottom of incline plane must be equal its P.E at the top.



$\therefore$ Loss in P.E = Gain in Trans. K.E + Gain in rot. K.E

$$mgh = \frac{1}{2}mV^2 + \frac{1}{4}mV^2$$

Dividing B.S by 'm'

$$gh = \frac{1}{2}V^2 + \frac{1}{4}V^2$$

$$gh = \frac{3}{4}V^2$$

or $$V = \sqrt{4/3\, gh}$$

This is expression for velocity of disc on reaching bottom of incline plane.

For Hoop:

$$\text{Loss in P.E} = \text{Gain in Trans. K.E} + \text{Gain in rot. K.E}$$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}mv^2$$

$$mgh = mv^2$$

$$v = \sqrt{gh}$$

This is expression for velocity of a hoop on reaching bottom of incline plane.

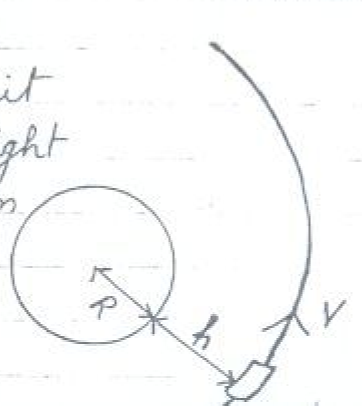
Artificial Satellites:

Artificial satellites are the objects that revolve around the earth in certain orbits and the gravitational pull of earth provides the necessary centripetal force.

They are put into orbits by rockets.

Expression For Orbital Velocity:

Consider a satellite is revolving around the earth in an orbit of radius 'r' at very low height above the surface of earth. When a satellite is moving in a circular orbit, it has centripetal acceleration.



$$a_c = \frac{v^2}{r}$$

where, $r = R + h$ (R = radius of earth)

$r \approx R$

The centripetal acceleration is produced due to force of gravity

$$\therefore \quad a_c = g$$

$$\boxed{V = \sqrt{gR}}$$

This is expression for the orbital velocity of the satellite.

where,

$g =$ acc. due to gravity $= 9.8 m/s^2$

$R =$ radius of earth $= 6.4 \times 10^6 m$

$$V = \sqrt{9.8 \times 6.4 \times 10^6}$$

$$= 7.9 \times 10^3 \; m/sec$$

or $V = 7.9 \; Km/sec$

This is the minimum orbital velocity necessary to put a satellite into a stable orbit and is called centripetal velocity.

Time Period For Circular Motion Of A Satellite:

Time taken by satellite to complete one revolution around the earth is called "Time Period."

It is denoted by 'T' and given as,

$$T = \frac{Distance}{Velocity}$$

When a satellite revolve around earth at very low height, distance covered by it is,

$$Distance = 2\pi R$$

$$= \text{circumference of earth}$$

$$T = \frac{2\pi R}{V}$$

$$= \frac{2(3.14)(6.4\times10^6)}{7.9\times10^3}$$

$$T = 5060 \text{ sec} = 84 \text{ min approximately.}$$

When a satellite revolve around in an orbit at large distance 'h' above the surface of earth, then slower obital velocity is required and it will take long time to complete one revolution.

ADVANTAGE OF CLOSE ORBITING SATELLITES:

Close orbiting satellites orbit the earth at a height of above 400km. Twenty four such satellites form the Global Positioning System.

An airline pilot, sailor or any person can used a pocket size instrument or mobile phone to find his position on the earth surface to within 10m accuracy.

REAL AND APPARENT WEIGHT:

(i) REAL WEIGHT:

The real weight of an object is equal to the gravitational pull of earth on the object.

Mathematically,

$$W = mg$$

where,

m = Gravitational mass of an object
g = acc. due to gravity $= 9.8\ m/sec^2$.

(ii) APPARENT WEIGHT:

The weight of an object in a certain frame of reference is equal and opposite to the force needed to prevent the object from falling from rest in that frame of reference.

Its value depends upon the acceleration of the frame of reference.

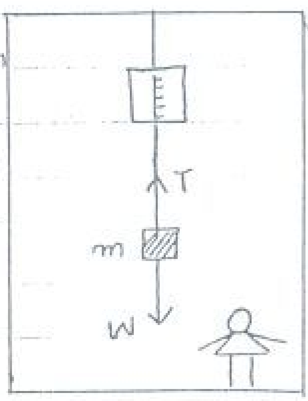
EXPLANATION:

In order to explain the idea of apparent weight, let us consider the following example.

EXAMPLE:

Consider a body of mass 'm' tied to a spring balance which is attached to the ceiling of a lift as shown in figure.

The reading of the spring balance shows the tension in the string and is called apparent weight of the body. Its value depends upon the acceleration of the lift.



LIFT AT REST OR MOVING WITH UNIFORM VELOCITY:

When lift is at rest or moving with uniform velocity. Then, According to Newton's 2nd law,

$$\text{acc. of object} = a = 0$$

Also, Net force on object $= F = 0$

If 'W' is gravitational force acting on object and tension in string is 'T'. Then,

$$T - W = ma$$
$$T - W = m(0)$$
$$T - W = 0$$
$$T = W = mg$$

Hence apparent weight is equal to real weight.

Lift Moving Upward With Acceleration:

When lift is moving upward with an acceleration 'a' Then, according to Newton's 2nd law,

$$T - W = F$$

As,

$$F = ma \quad \text{and} \quad W = mg$$
$$\therefore T - mg = ma$$
$$T = ma + mg$$
$$T = m(g + a)$$

The apparent weight of an object is more than its real weight by an amount ma.

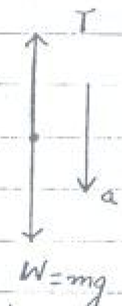
Lift Moving Downward With Acceleration:

Suppose the lift is moving downward with an acc. 'a' Then, according to Newton's 2nd law

$$W - T = F$$

As

$$F = ma \quad \text{and} \quad W = mg$$



$$\therefore \quad mg - T = F$$

$$mg - T = ma$$

$$\Rightarrow \quad T = mg - ma$$

$$T = m(g - a)$$

So, apparent weight of an object is less than its real weight by an amount ma.

Lift Is Falling Freely Under Gravity:

Suppose the lift is falling freely under gravity with acceleration $a = g$ then the apparent weight of an object is given by

$$T = W - F$$

As,

$$W = mg \quad \text{and} \quad F = ma$$

$$\therefore \quad T = mg - ma$$

$$T = m(g - a)$$

But here,

$$a = g$$

$$\therefore \quad T = m(g - g)$$

$$T = m(0)$$

$$T = 0$$

So, apparent weight becomes zero. Thus according to this observation, the body appears to weight less.

Weightlessness In Satellite And Grvity Free System:

When a satellite or

space-craft is falling freely in space, everything within this freely falling system will appear to weightless. Thus no force is required to prevent an object from falling in the frame of refrence of the space-craft.

Such a system is called gravity free system.

Reason For Weightlessness:

When we stand on the ground we feel the upward contact force or reaction force of ground. This falling tells us that our feet are pressing against the ground. If the ground is taken away from under our feet so we are falling through air then we have no experience of any contact force on us and so we feel weightless. The astronaut who is in a space-craft orbiting the earth is in a permanent state of free fall towards the earth. This can be explain as follow.

Explanation:

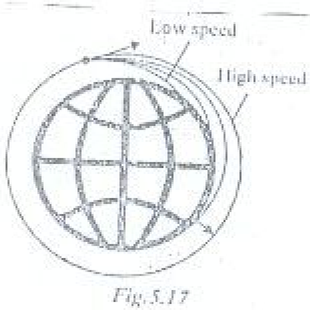


Fig.5.17

Consider a projectile is thrown parallel to earth surface at increasing speeds then curvature of path of projectile decreases during its free fall on earth. If the projectile (Space-craft) is thrown with certain large velocity parallel to earth, the curvature of its path will match the curvature of earth. In this case, the space-ship will simple circle the earth.

The astronauts and their space-ships is falling freely towards the earth.

with acceleration due to gravity ($g = 9.8\ m/sec^2$) all the time but the curvature of earth prevents the space-craft from hitting. So there is no contact force between the astronaut and space-ship. As, he feel no support or contact force from the space-ship around him, he experiences apparent weightlessness.

Orbital Velocity:

The artificial satellites revolve around the earth in nearly circular orbits. The instantaneous velocity of the satellite is along the tangent to circular orbit and is called orbital velocity.

Expression For Orbital Velocity:

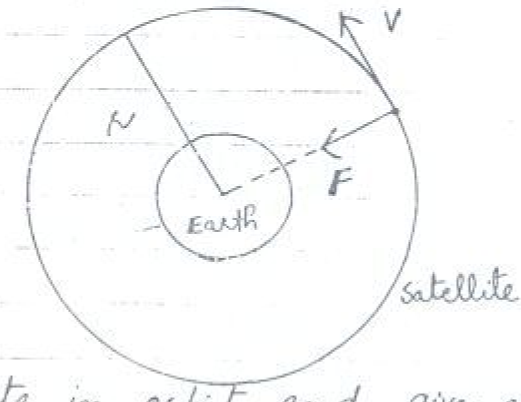
Consider a satellite revolving around the earth in a circular orbit of radius 'r'.

Let,

M = mass of earth

m_s = mass of satellite

V = Orbital velocity of satellite



The centripetal force is required to hold the satellite in orbit and given as,

$$F_c = \frac{m_s V^2}{r} \longrightarrow (i)$$

This force is provided by the gravitational force of attraction between the earth and the satellite.

$$\text{Gravitational Force} = \frac{G m_s M}{r^2} \longrightarrow (ii)$$

Comparing expression (i) and (ii)

$$\frac{m_s V^2}{r} = G\frac{m_s M}{r^2}$$

$$\Rightarrow \quad V = \sqrt{\frac{GM}{r}}$$

This is expression for orbital velocity of satellite. This shows that orbital velocity is independent of the mass of satellite.

Significance Of Orbital Velocity:

A satellite orbiting the earth in a circular orbit must have the orbital velocity given by,

$$V = \sqrt{\frac{GM}{r}}$$

where, r = Distance of satellite from centre of earth

The orbital velocity is the minimum velocity necessary to hold a satellite into a stable orbit. It is also called "critical velocity". Any velocity less than critical value will bring the satellite tumbling back to the earth.

Artificial Gravity:

When a satellite moves on a circular orbit around the earth in deep space, the astronauts in it is in the state of weightlessness. If the satellite is a space laboratory then it is very difficult to the astronaut to stay long in that state and perform experiments. In order to overcome this difficulty, artificial gravity is created. It is done by rotating the satellite about its own axis with certain frequency.

DETERMINATION OF FREQUENCY:

Consider a space ship rotating about its axis with constant speed 'V'. The outer radius of space-ship is R.

Any person who is standing at the outer rim will experience centripetal acceleration

$$a_c = \frac{V^2}{R}$$

$$a_c = \frac{(R\omega)^2}{R} \qquad \because (V = R\omega)$$

$$\Rightarrow \quad a_c = R\omega^2$$

Also, time period of revolution of space-ship is,

$$T = \frac{2\pi}{\omega}$$

$$\text{or} \quad \omega = \frac{2\pi}{T}$$

Hence,

$$a_c = R\left(\frac{2\pi}{T}\right)$$

$$a_c = R\,\frac{4\pi^2}{T^2}$$

As,

$$f = \frac{1}{T}$$

$$\therefore a_c = 4\pi^2 R f^2$$

$$f^2 = \frac{1}{4\pi^2}\,\frac{a_c}{R}$$

$$f = \frac{1}{2\pi}\sqrt{\frac{a_c}{R}}$$

As, we know that, when satellite moves in the orbit around the earth, the force of the necessary centripetal acceleration so that,

$$a_c = g$$

$$\therefore \quad f = \frac{1}{2\pi}\sqrt{\frac{g}{R}}$$

When the space-ship rotates with this frequency, the artificial gravity like earth is provided to astronauts in space-ship.

MECHANISM OF ARTIFICIAL GRAVITY:

When we stood on the ground we feel the upward constant force or reaction force of the ground. This feeling teels us that our feet are pressing against the ground and there is a weight force acting on us.

When the space-ship is set into rotation about its own axis, the astronaut is pressed towards the outer rim and exerts a force on the floor of space-ship in same way as on the earth. This way artificial gravity like earth is created in space-ship.

GEOSTATIONARY ORBITS:

Satellites are used for international communicational links. Communicational satellites are often placed in a high orbit over the equator. The orbit is called a geosynchronous or geostationary orbit.

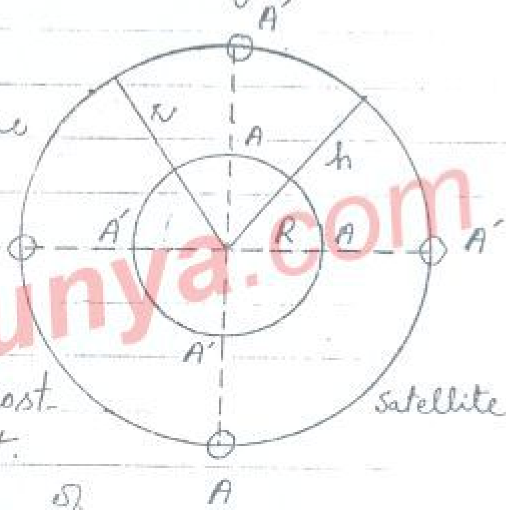
In this orbit, the orbital motion of

satellite is synchronized with the rotation of the earth. So the satellite remains always over the same point on the equator as the earth spins on its axis. The high orbit allows a large area of earth to be seen by the satellite.

Expression For Radius Of Geostationary Orbit:

Let us find the expression for radius of geostationary orbit.

As we know that the orbital speed for circular orbit is given by,



$$V = \sqrt{\frac{GM}{r}} \longrightarrow (i)$$

where r = Radius of geostationary orbit.

The average speed of satellite in one day is given by,

$$V = \frac{S}{t}$$

$$V = \frac{2\pi r}{T} \longrightarrow (ii)$$

where T = Time period of motion of satellite = one day.

As, the earth rotates in one day and the satellite revolve around the earth in one day. The satellite at A' will always stay over the same point A on the earth as shown in fig.

Comp (i) and (ii), we get.

$$\frac{2\pi r}{T} = \sqrt{\frac{GM}{r}}$$

Squaring B.S, we get.

$$\frac{4\pi^2 r^2}{T^2} = \frac{GM}{r}$$

$$r^3 = \frac{GMT^2}{4\pi^2}$$

$$\boxed{r = \left(\frac{GMT^2}{4\pi^2}\right)^{1/3}}$$

This is expression for orbital radius of geostationary orbit.

where,

G = Universal gravitational constant $= 6.67 \times 10^{-11}\ N \cdot m^2/kg^2$

M = Mass of earth $= 6 \times 10^{24}$ kg

T = Time period of orbital motion of satellite

= one day

= 24 hours

= $24 \times 60 \times 60$ Seconds

Substitute these value in above eq. we get.

$r = 4.23 \times 10^7 m$

$r = 4.23 \times 10^4 Km$

The height of geostationary orbit above the eq. is,

$$h = r - R = 4.23 \times 10^4 - 6.4 \times 10^3 \approx 36000 \ Km$$

COMMUNICATION SATELLITES:

Satellites are used for international communication links. A satellite communication system can be set up by placing satellite in a high orbit over the equator at different points. This orbit is called geostationary orbit.

One such geostationary satellite covers 120 of longitude so that whole of the populated earth's surface can be covered by three correctly positioned satellites as shown in figure. Since the geostationary satellites above the same point on the earth, continuous communication with any place on the surface of earth is possible. Microwaves are set up to a satellite at one frequency say 4 GHZ. and are then amplified and converted to another frequency say 11 GHZ before being sent back down to a receiving station on the ground.

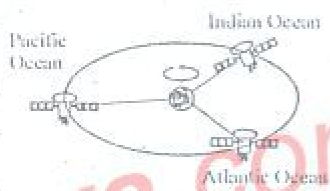


Fig. 3.21

The range of frequencies available is between 3 and 30 G.HZ.

Microwaves are used in satellite communication due to following reasons.

(i) They travel in a narrow beam
(ii) They travel in a straight line.
(iii) They can easily pass through the atmosphere of earth. The energy needed to amplify and retransmit signals is provided by large solar cell panels fitted on the satellites.

There are over 200 earth stations which transmit signals to satellites and receive signals Via satellites from other countries.

LARGEST SATELLITE SYSTEM:

The largest satellite system is managed by 126 countries, International Telecommunicational satellite organization. (INTELSAT)

An intelsat VI satellite appears at microwave frequencies of 4, 6, 11 and 14 GHz and has a capacity of 30,000 two way telephone circuit plus three TV channels.

Newton's And Einstein's Views Of Gravitation:

According to Newton, every two particles of matter attract each other with a force called force of gravitation.

The gravitation is the intrinsic property of the matter and is given by Newton's "law of gravitation".

According to this law, "Every particle of matter attracts every other particle with a force which is directly proportional to the product of their masses and inversely proportional to the square of the distance between their centres."

Einsten's View Of Gravitation:

"According to Einstein's theory "Space time is curved near massive bodies".

Explanation:

In order to explain Einstein's idea, let us consider space as a thin rubber sheet. If a heavy weight is hung from thin rubber sheet, it curves as shown in figure.

The weight corresponds to a huge mass that causes space itself to curve. In Einstein's theory we do not speak of force of gravity acting on bodies, rather we say that, "Material particles and light rays move along geodesics in curved space time. So a body at rest or moving slowly near the great mass would follow a geodesic towards that body.

Comparison Of Eistein's And Newton's Theory Of Gravitation:

Einstein's theory gives us a physical picture of how gravity works.

Newton discovered the inverse square law of gravity but did not offer an explanation of why gravity follows an inverse square law.

Einstein's theory says that gravity follows an inverse square law (except in strong gravitational fields). It also tells us why gravity follows an inverse square law. Therefore, we can conclude that,

Einstein's theory is better than Newton's even through it includes Newton's theory within itself and gives the same answers, as Newton's theory every-where except where the gravitational field is very strong.

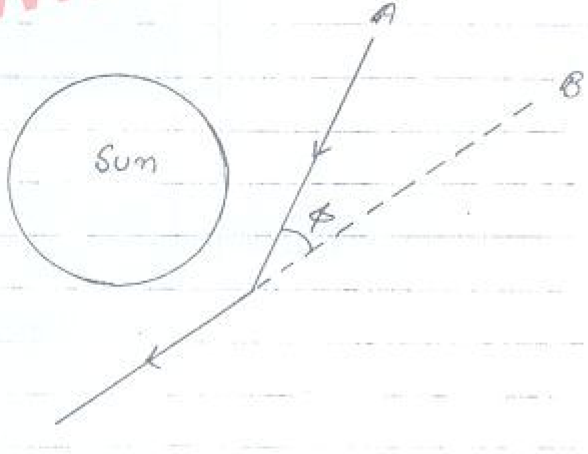
Success Of Einstein's Theory Of Gravitation:

According to Einstein "If gravity and acceleration are precisely equivalent, gravity must bend light by a precise amount which can be calculated".

This was not entirely new idea, the same idea was given by Newton in his corpuscular theory of light. According to Newton's corpuscular theory of light light consists of a stream of tiny particles called corpuscles. These tiny particles would be deflected by gravity.

But in Einstein's theory, the deflection of light by gravity is predicted to be twice than that predicted by Newton's theory.

The bending of star light by gravity of sun was measured during solar eclipse in 1919. and found to match Einsteins prediction rather than Newton's. Light from the star A is deflected as it passes close to the sun on its way to direction B, shifted by the angle ϕ. Einstein predicted that $\phi = 1.745$ seconds of angle which was found to the same during the solar eclipse of 1919.



SHORT QUESTIONS & ANSWERS

5.1 Explain the difference between tangential velocity and the angular velocity. If one of these is given for a wheel of known radius, how will you find the other?

Ans: The tangential velocity is the linear velocity of a particle moving along a curve or a circle directed along the tangent at any point on the curve.

The angular velocity is the rate of change of angular displacement of a particle moving along a curved path. The tangential velocity v, angular velocity ω and the radius r of the wheel are related the equation.

$$v = r\,\omega$$

5.2 Explain what is meant by centripetal force and why it must be furnished to an object if the object is to follow a circular path?

Ans: To move an object in a circular path, its direction needs to be changed at every point. To change the direction of motion continuously, a continuous perpendicular force is required. This force is known as centripetal force. It is directed along the radius towards the centre of the circle.

5.3 Explain what is meant by moment of inertia? Explain its significance.

Ans: Moment of inertia is a characteristic of the configuration of the body with respect to a chosen axis. It depends on the mass of the body and its distribution with respect to axis of rotation of the body.

For a rigid body consisting of particles of masses $m_1, m_2, m_3, \ldots, m_n$ situated at distance $r_1, r_2, r_3, \ldots r_n$ from the axis of rotation, the moment of inertia is given be

$$I = \sum_{i=1}^{n} m_i r_i^2$$

The moment of inertia plays the same role in angular motion as that of mass in linear motion. As mass is a measure of linear inertia in linear motion, similarly, moment of inertia is a measure of rotational inertia in angular motion.

5.4 What is meant by angular momentum? Explain the law of conservation of angular momentum.

Ans: Angular momentum $\vec{L}$ is the moment of linear momentum $\vec{p}$.

$$\vec{L} = \vec{r} \times \vec{p} \qquad \text{where } \vec{r} \text{ is the position vector}$$

It is a quantity similar to linear momentum for angular motion. As linear momentum is a product of mass and linear velocity, similarly angular momentum can also be defined as the product of their counterparts i.e., moment of inertia I and angular velocity $\vec{\omega}$. The total angular momentum of all the bodies in a system remains constant if no external torque acts on the system. This is known as law of conservation of angular momentum. It can be expressed as

$$L = I_1\omega_1 = I_2\omega_2 = \text{constant}$$

If momentum of inertia I of a body decreases, its angular velocity ω increases, so that their product remains constant.

5.5 Show that orbital angular momentum $L_o = mvr$

Ans: From the definition of angular momentum, its magnitude $\vec{L}$ is given by

$$L = mrv\sin\theta$$

where θ is the angle between position vector r and velocity v. In case of circula orbital motion, the angle between radius and tangential velocity is 90°, hence

$$L_o = mrv\sin 90^\circ \qquad \text{as } \sin 90^\circ = 1$$

Hence $L_o = mrv$

5.6 Describe what should be the minimum velocity, for a satellite, to orbit close to th Earth around it.

Ans: When a satellite is moving in a circular orbit, it has an accelerator $\frac{v^2}{r}$ which i supplied by gravity. The low flying Earth satellite have acceleration $g = 9.8\ ms^{-2}$.

Hence $\frac{v^2}{r} = g$ or $v = \sqrt{gr}$

or $v = \sqrt{gR}$

where R is the radius of the earth = 6400 km

Putting the values $v = \sqrt{9.8 \times 6400} = 7.9\ kms^{-1}$

This is the minimum velocity necessary to put a satellite into orbit around tl Earth.

5.7 State the direction of the following vectors in simple situations; angular momentum and angular velocity.

Ans: **Direction of Angular momentum**

As Angular momentum L is defined by

$$\vec{L} = \vec{r} \times \vec{p} = m(\vec{r} \times \vec{v})$$

Being cross product of vector $\vec{r}$ and $\vec{v}$, its direction is perpendicular to the plane formed by r and v and its sense is given by right hand rule of cross product. In general, for a body having counter clockwise rotation, the angular momentum is directed outward along the axis of rotation.

Direction of Angular Velocity

The direction of angular velocity is taken along the axis of rotation given by right hand rule. If rotation is along the curl of the fingers of right hand, then the outward stretched thumb gives the direction of angular velocity. According to this rule, for counterclockwise rotation, the direction of angular velocity is outward along the axis of rotation.

5.8 Explain why an object, orbiting the Earth, is said to be freely falling. Use your explanation to point out why objects appear weightless under certain circumstances.

Ans: An object such as an artificial satellite orbits the Earth due to force of gravity. Its centripetal acceleration is equal to the acceleration due to gravity directed towards the centre of the Earth. Hence, a satellite is always falling towards the centre of Earth with acceleration equal to "g" and is a freely falling object. It does not hit ground because of its tangential velocity and curvature of the Earth.

In the fame of reference of a freely falling object the bodies appears weightless because both, the body inside and the frame of reference, are falling with the same acceleration. The relative acceleration of an inside body with respect to its frame of reference is zero. Hence, it seems that no force is acting on the body and when it is suspended to a spring balance, the spring balance shows its weight as zero.

5.9 When mud flies off the tyre of a moving bicycle, in what direction does it fly? Explain.

Ans: The mud clinging to the tyre of a moving bicycle flies in a direction tangent to the tyre.

When the bicycle is moving, each point on the tyre, including the mud clinging to the tyre has a tangential velocity v, as well as angular velocity ω. A centripetal force mv^2/r must be provided to the mud to keep it on to the tyre. The adhesion force of mud to the surface of the tyre provides this inward force. When the cyclist moves faster and faster, v increases and the required force necessary to keep the mud on the tyre increases. When required centripetal force exceeds the adhesion force, the mud breaks and flies off the tyre in a direction tangent to the surface of the tyre.

5.10 A disc and a hoop start moving down from the top of an inclined pla same time. Which one will have greater speed on reaching the bottom?

Ans: The velocity of the disc on reaching the bottom of the inclined plane is give the following equation.

$$v = \sqrt{\frac{4gh}{3}}$$

or

$$v = \sqrt{\frac{4}{3}} \times \sqrt{gh}$$

and for a hoop, its velocity is

$$v = \sqrt{gh}$$

As

$$\sqrt{\frac{4}{3}} = 1.15 > 1$$

Hence, the speed of the disc will be greater.

5.11 Why does a diver change his body positions before diving in the pool?

Ans: Before lifting off the diving board, the diver's legs and arms are fully extended which means that the diver has a large moment of inertia I_1 about an axis. The moment of inertia is considerably reduced to a new value I_2, when the legs and arms are drawn into closed tuck position. As angular momentum is conserved, so

$$I_1\omega_1 = I_2\omega_2$$

where ω_1 and ω_2 are the angular speeds before and after diving. Hence the diver spins faster when its moment of inertia becomes smaller.

5.12 A student holds two dumb-bells with out-stretched arms while sitting on a turn table. He is given a push until he is rotating at certain angular velocity. The student then pulls the dumbbell towards his chest (Fig. 5.24). What will be the effect on rate of rotation?

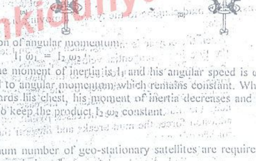


Fig. 5.24

Ans: Using the law of conservation of angular momentum,

$$I_1\omega_1 = I_2\omega_2$$

With out-stretched arms, the moment of inertia is I_1 and his angular speed is ω_1. Their product $I_1\omega_1$ is equal to angular momentum which remains constant. When he pulls the dumbbells towards his chest, his moment of inertia decreases and he spins faster as ω_2 increases to keep the product $I_2\omega_2$ constant.

5.13 Explain how many minimum number of geo-stationary satellites are required for global coverage of T.V transmission.

Ans: A geo-stationary satellite covers 120° of longitude, so the whole of the Earth surface for global transmission can be covered by the three correctly positioned geo-stationary satellites.