Rs. 30

FLUID DYNAMICS

Fluid Dynamics:-

The branch of mechanics which deals with the fluids in motion.

Fluid:-

Matter which can flow from one place to another and has no shape of its own is known as fluid.

The word fluid comes from a Latin word meaning "to flow". Liquids and gases together are called fluids. In a fluid the atoms can move relative to one another.

Ideal Fluid:-

A fluid which can flow without giving any internal resistance.

OR

A fluid which is incompressible and non viscous.

Viscosity:-

The property of fluids due to which they oppose their flow is called viscosity. It is sometime called fluid friction.

Coefficient Of Viscosity:-

The ratio between stress and strain in the fluid is called coefficient of viscosity.

Symbol:-

It is denoted by "η" (Greek letter eta).

Formula:-

$$\eta = \frac{FD}{VA}$$

where F = The opposing force acting on the fluid.
D = The distance between the layers.
V = The relative velocity between the layers.
A = Surface area of layer.

UNIT:-

The SI unit of coefficient of viscosity is $N.S/m^2$ or $Kg/m.S$.

DIMENSION:-

Dimension of η is $[ML^{-1}T^{-1}]$.

DRAG FORCE:-

The force experienced by a body while moving through a fluid is called drag force.

It is retarding force and increases as the speed of the body increases. In case of a sphere of radius r moving slowly with speed V through a fluid of viscosity η, the drag force F is given by Stoke's law as:

$$F = 6\pi\eta r v$$

At high speeds the drag force is proportional to speed, i-e; $F \propto V$ if $6\pi\eta r$ = Constant.

For example, a fast moving car experiences large retarding force i-e; drag force by air even of the small viscosity of air.

TERMINAL VELOCITY:-

"The maximum constant velocity gained by a spherical body falling through a fluid when the drag force becomes equal to weight of the body". i-e; if $F_D = W$ then body will fall with constant velocity V_t called terminal velocity.

MATHEMATICALLY:-

$$V_t = \frac{2gr^2\rho}{9\eta}$$

where V_t = Terminal Velocity, g = Acceleration due to gravity, r = Radius of spherical body, ρ = Density of the body, η = Viscosity of the medium.

PROOF:-

Consider a water droplet falling vertically. The drag force of air on water droplet increases with speed of falling due to gravity. The upward drag force which is the reaction force of gravity increases as the speed of droplet increases. Thus, the net force on droplet is:

Net Force = Weight − Drag force.

When the magnitude of drag force becomes equal to weight of the droplet, the net force acting on the droplet is zero and the droplet will fall with constant speed called terminal velocity V_t.

$\Rightarrow$ if $F_D = W$

then Net force $= 0$

$$\Rightarrow \quad 0 = F_D - W$$

According to Stoke's law for the drag force;

$$F_D = 6\pi\eta r V_t$$

$$\Rightarrow \quad 6\pi\eta r V_t - mg = 0 \qquad \because W = mg$$

or

$$6\pi\eta r V_t = mg$$

or

$$V_t = \frac{mg}{6\pi\eta r}$$

As density of droplet is $\rho = \frac{m}{V}$

$\Rightarrow$ Mass of droplet is $m = \rho V$

where V = Volume of spherical droplet $= \frac{4}{3}(\pi r^3)$

$$\Rightarrow \quad m = \frac{4}{3}\pi r^3 \rho$$

and

$$V_t = \frac{4\pi r^3 \rho g}{(3)\,6\pi\eta r}$$

or $$V_t = \frac{2gr^2\rho}{9\eta}$$

Let $\frac{2g\rho}{9\eta}$ = Constant, then

$$V_t = \text{Constant}\,(r^2)$$

or $$V_t \propto r^2$$

Hence the terminal velocity of the droplet is directly proportional to the square of its radi

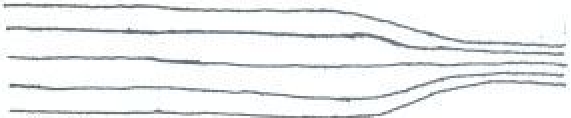
FLUID FLOW:-

To study the behaviour of the fluids in motion, consider their flow through pipes. When a fluid is in motion, its flow can be streamline or turbulent.

STREAMLINE FLOW:-

The flow is said to be streamline if each particle of the fluid passi through a certain point flows the same path as that followed by the particles whic passed the same point earlier.

Streamline flow is also called laminar flow or steady flow.



(Streamline flow)

STREAM LINE:-

The path followed by all the parti of the fluid passing through the same point is called stream line. It may be straight or curve

The direction of the stream lines is the same as the direction of velocity of the fluid at that point.

(Stream Line)

CONDITION FOR STEADY FLOW:-

The condition for steady flow is that different streamlines cannot cross each other.

EXAMPLES OF STREAMLINE FLOW:-

(i) Formula one racing cars have a streamlined design.

(ii) Dolphins have streamlined bodies to assist their movement in water.

TURBULENT FLOW:-

The flow is said to be turbulent if particles of the fluid which pass through a certain point do not follow the same path as that followed by the particles earlier.

OR

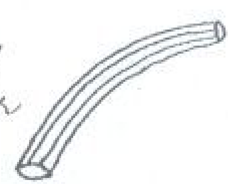
The irregular or unsteady flow of the liquid is called turbulent flow.

TUBE OF FLOW:-

If we select a finite number of streamlines to form a series of tubes through which the fluid can pass, then such tubes are called tubes of flow.

OR

The streamlines passing normally through finite areas assume a tubular form called tube of flow.



(Tube of flow)

In case of turbulent flow, the velocity of the fluid changes suddenly and the exact path of the particles of the fluid cannot be forecasted.

CRITICAL VELOCITY:-

That value of the velocity of fluid

flow below which the flow is steady and above which the flow is unsteady or irregular (turbulen

The critical velocity of a fluid varies directly as its viscosity and inversely as its density and radius of the tube through which it flows.

CONDITIONS FOR IDEAL FLUID:-

(i) The fluid is non-viscou i-e; there is no internal frictional force between adjacent layers of fluid.

(ii) The fluid is incompressible i-e; its density is constant.

(iii) The fluid motion is steady.

EQUATION OF CONTINUITY:-

It states that the product of cross sectional area of the pipe and the fluid speed at any point along the pipe is a constant. This constant equals the volume flow per second of the fluid or simply flow rate.

MATHEMATICALLY:-

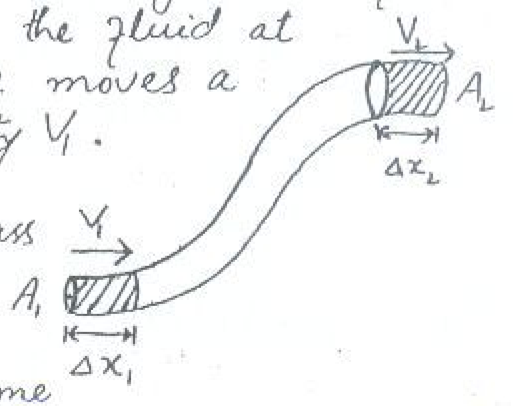
$$V_1 A_1 = V_2 A_2$$

$$\text{or} \quad VA = \text{Constant}$$

PROOF:-

Consider a fluid flowing through a pipe of non-uniform size. The particles in the fluid move along the streamlines in a steady state flow.

In a small time Δt the fluid at the lower end of the tube moves a distance Δx_1 with a velocity V_1. If A_1 is the area of cross section of this end then the mass of the fluid in this shaded region is :



$$\text{mass} = \text{density} \times \text{volume}$$

$$\text{or} \quad m = \rho \times V$$

or $$\Delta m_1 = \rho_1 \Delta V_1$$

where ρ_1 is the density of the fluid and

$$\Delta V_1 = A_1 \Delta x_1$$

$$\Rightarrow \quad \Delta m_1 = \rho_1 A_1 \Delta x_1$$

or $$\Delta m_1 = \rho_1 A_1 \frac{\Delta x_1}{\Delta t} \times \Delta t$$

or $$\Delta m_1 = \rho_1 A_1 V_1 \times \Delta t \qquad \because \frac{\Delta x_1}{\Delta t} = V_1 \text{ (velocity)}$$

Similarly, the fluid that moves with velocity V_2 through the upper end of the pipe of cross sectional area A_2 in the same time Δt has a mass :

$$\Delta m_2 = \rho_2 A_2 V_2 \times \Delta t$$

If the fluid is incompressible and the flow is steady then the mass of the fluid is conserved. That is the mass that flows into the bottom of of the pipe through A_1 in a time Δt must be equal to the mass of the fluid that flows out through A_2 in the same time.

$$\Rightarrow \quad \Delta m_1 = \Delta m_2$$

or $$\rho_1 A_1 V_1 \times \Delta t = \rho_2 A_2 V_2 \times \Delta t$$

$$\Rightarrow \quad \rho_1 A_1 V_1 = \rho_2 A_2 V_2$$

This equation is called the equation of continuity. Since density is constant for steady flow of incompressible fluid, the equation of continuity becomes :

$$A_1 V_1 = A_2 V_2$$

or $$VA = \text{Constant}$$

Inward rate of flow = Outward rate of flow

RATE OF FLOW:-

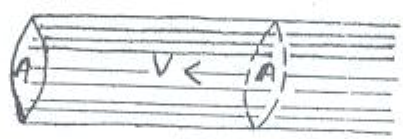
It is defined as the volume of fluid flowing per unit time across any section of the tube of flow.

The fluid is taken to be incompressible and non-viscous. Rate of flow is also called rate of discharge of fluid.

Consider an incompressible and non-viscous fluid in steady flow in a tube of length L and face area A. If a fluid particle covers the tube length in time t, the volume of fluid flowing out of the opposite face will be equal to the volume of the tube i-e; $V = L \times A$. Thus

$$\text{Rate of flow} = \frac{\text{Volume of fluid}}{\text{Time}}$$

$$= \frac{L \times A}{t}$$



$$= \frac{L}{t} \times A$$

$$\therefore \text{Rate of flow} = VA \qquad \because \frac{L}{t} = \text{Average velocity of fluid}$$

or Rate of flow = Average velocity of fluid × Area of cross section

Rate of flow can also be represented by the dot product of velocity and vector area, i-e;

$$\text{Rate of flow} = \vec{V} \cdot \vec{A}$$

Rate of flow is a scalar quantity and is measured in m^3/s (Cubic meter per second) or ft^3/s (Cubic foot per second). One cubic foot per second is called 'cusec'.

BERNOULLI'S EQUATION:-

It states that for an incompressible and non-viscous fluid in steady flow the sum of pressure and kinetic energy per unit volume and potential energy per unit volume is constant.

MATHEMATICALLY:-

$$\text{Pressure} + \frac{\text{Kinetic energy}}{\text{Volume}} + \frac{\text{Potential energy}}{\text{Volume}} = \text{Constant}$$

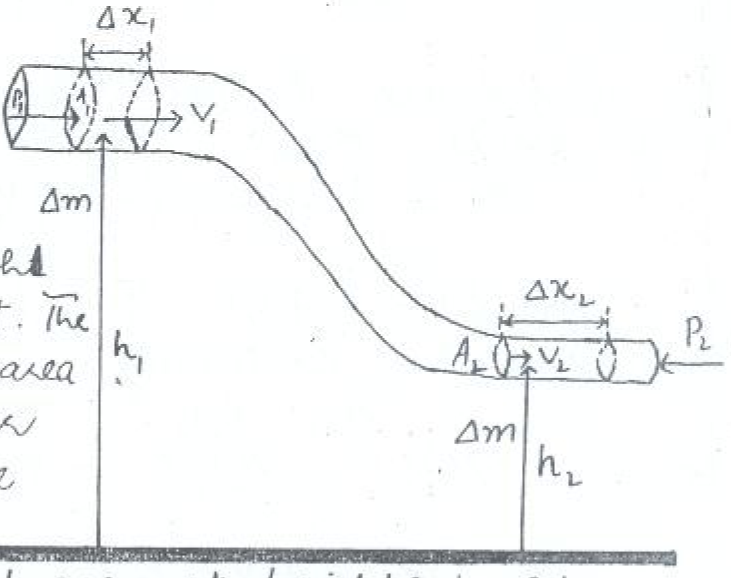
$$\text{or} \quad P + \frac{\frac{1}{2}mv^2}{\text{Volume}} + \frac{mgh}{\text{Volume}} = \text{Constant}$$

$$\text{or} \quad P + \frac{1}{2}\rho v^2 + \rho g h = \text{Constant} \quad \left[\because \frac{\text{Mass}}{\text{volume}} = \text{Density} \; (\rho)\right]$$

It was stated by swiss physicist, Daniel Bernoulli (1700–1782). It is a particular form of the law of conservation of energy.

PROOF:-

To derive Bernoulli's equation, consider the fluid is incompressible, non-viscous and flows in a steady flow. Let the flow of the fluid through the pipe in time t. The pipe has uniform area of cross section over some length at the two ends which



are horizontal and are at heights h_1 & h_2 from some horizontal reference level.

The force on the upper end of the fluid is,

$$F_1 = P_1 A_1 \qquad \because P_1 = \frac{F_1}{A_1}$$

where P_1 is the pressure and A_1 is the area of cross-section at the upper end. The work done on the fluid by the fluid behind it in moving it through a distance Δx_1 will be:

$$W_1 = F_1 \Delta x_1 = P_1 A_1 \Delta x_1$$

Similarly, the work done on the fluid at the lower end is:

$$W_2 = -F_2 \Delta x_2 = -P_2 A_2 \Delta x_2 \quad \because F_2 = P_2 A_2$$

where P_2 is the pressure and A_2 is the area of cross-section at the lower end and Δx_2 is the distance moved by the fluid in the same time interval. The work W_2 is negative as this work is done against fluid force.

The net work done $W = W_1 + W_2$

or $\quad W = P_1 A_1 \Delta x_1 + P_2 A_2 \Delta x_2$

If V_1 & V_2 are the velocities at the upper and lower ends, then

$$W = P_1 A_1 V_1 t + (-P_2 A_2 V_2 t) \quad \left[\because \Delta x = Vt \Rightarrow \Delta x_1 = V_1 t, \; \Delta x_2 = V_2 t\right.$$

$$\Rightarrow W = P_1 A_1 V_1 t - P_2 A_2 V_2 t$$

From equation of continuity:

$$A_1 V_1 = A_2 V_2$$

Hence $\quad A_1 V_1 t = A_2 V_2 t = V$ (volume) $\left\{ \because A_1 V_1 t = A_1 \frac{\Delta x_1}{t} \times t = V, \; A_2 V_2 t = A_2 \frac{\Delta x_2}{t} \times t = V \right.$

$$\Rightarrow W = P_1 V - P_2 V$$

$$W = (P_1 - P_2) V$$

If m is the mass and ρ is the density then

$$\rho = \frac{m}{V} \Rightarrow V = m/\rho$$

$$\text{Thus} \quad W = (P_1 - P_2)\, m/\rho$$

Part of this work is used by the fluid in changing its K.E and a part is used in changing its gravitational P.E. Where

$$\text{Change in K.E} = \Delta(K.E) = \frac{1}{2}mV_2^2 - \frac{1}{2}mV_1^2$$

$$\text{and change in P.E} = \Delta(P.E) = mgh_2 - mgh_1$$

Applying law of conservation of energy to this volume of the fluid, we get

$$\text{Work} = \Delta(K.E) + \Delta(P.E)$$

$$\text{or} \quad (P_1 - P_2)\frac{m}{\rho} = \frac{1}{2}mV_2^2 - \frac{1}{2}mV_1^2 + mgh_2 - mgh_1$$

$$\text{or} \quad (P_1 - P_2)\frac{\cancel{m}}{\rho} = \cancel{m}\left(\frac{1}{2}V_2^2 - \frac{1}{2}V_1^2 + gh_2 - gh_1\right)$$

$$\text{or} \quad P_1 - P_2 = \rho\left(\frac{1}{2}V_2^2 - \frac{1}{2}V_1^2 + gh_2 - gh_1\right)$$

$$\text{or} \quad P_1 - P_2 = \frac{1}{2}\rho V_2^2 - \frac{1}{2}\rho V_1^2 + \rho gh_2 - \rho gh_1$$

On rearranging above equation, we have

$$P_1 + \frac{1}{2}\rho V_1^2 + \rho gh_1 = P_2 + \frac{1}{2}\rho V_2^2 + \rho gh_2$$

This is Bernoulli's equation or called Bernoulli's Theorem. In general

$$P + \frac{1}{2}\rho V^2 + \rho gh = \text{Constant}$$

Here the term $(P + \rho gh)$ is called the 'static pressure of the fluid and the term $\frac{1}{2}\rho v^2$ is called the 'dynamic pressure' or 'velocity pressure'.

APPLICATIONS OF BERNOULLI'S EQUATION:-

(i) TORRICELLI'S THEOREM:-

It states that the speed of efflux is equal to the velocity gained by the fluid in falling through the distance $(h_1 - h_2)$ under the action of gravity.

Suppose a large tank of fluid has two small holes A and B on it. Let us find the speed with which the fluid flows from hole A. As the holes are so small, the efflux speeds V_2 & V_3 will be much larger than the speed V_1 of the top surface of the fluid. Thus V_1 is approximately zero.

Applying Bernoulli's equation:

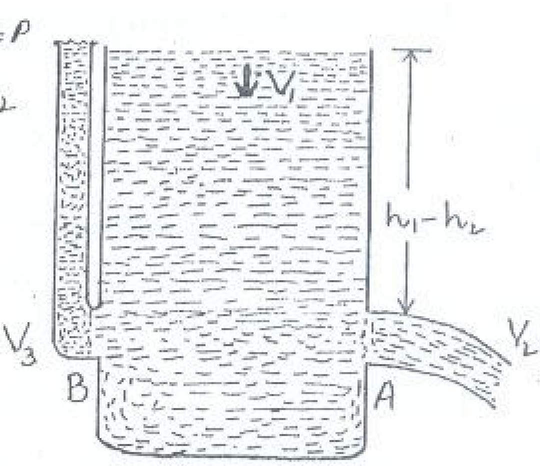
$$P_1 + \frac{1}{2}\rho V_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho V_2^2 + \rho g h_2$$

but $V_1 = 0 \Rightarrow \quad P_1 + \rho g h_1 = P_2 + \rho g h_2$

Also

$P_1 = P_2 =$ Atmospheric pressure $= P$

$\Rightarrow P + \rho g h_1 = P + \frac{1}{2}\rho V_2^2 + \rho g h_2$



or $\quad \frac{1}{2}\rho V_2^2 = \rho g h_1 - \rho g h_2$

$\Rightarrow \quad \frac{1}{2}\rho V_2^2 = \rho g (h_1 - h_2)$

$\Rightarrow \quad V_2^2 = 2g(h_1 - h_2)$

$\Rightarrow \quad V_2 = \sqrt{2g(h_1 - h_2)} \qquad \because V_2 = V_{efflux}$

This result is known as Torricelli's Theorem or 'the law of efflux'.

EFFLUX VELOCITY:-

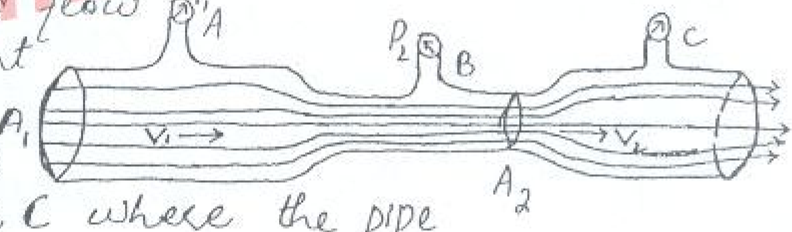
The speed with which the fluid flows out of the hole is known as efflux velocity.

$$V_{eff} = \sqrt{2gh} \qquad \text{where } h_1 - h_2 = h$$

(ii) RELATION BETWEEN SPEED AND PRESSURE OF THE FLUID:-

By applying Bernoulli's equation we can find the relation between the speed and pressure of the fluid, that is 'when the speed is high, the pressure will be low'.

Consider a fluid is flowing through a horizontal pipe having a narrow area of cross-section at its middle. Since the rate of flow of fluid must remain constant at all points, the fluid flow is faster at point B where the pipe is narrow than that A or C where the pipe is wide.



Applying Bernoulli's equation to fluid flowing from A to B and noting that average P.E. is the same at both places, we have.

$$P_1 + \frac{1}{2}\rho V_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho V_2^2 + \rho g h_2$$

As $\rho g h_1 = \rho g h_2 = \rho g h$

$$\therefore P_1 + \frac{1}{2}\rho V_1^2 + \cancel{\rho g h} = P_2 + \frac{1}{2}\rho V_2^2 + \cancel{\rho g h}$$

$$\Rightarrow P_1 + \frac{1}{2}\rho V_1^2 = P_2 + \frac{1}{2}\rho V_2^2$$

$$P_1 - P_2 = \frac{1}{2}\rho V_2^2 - \frac{1}{2}\rho V_1^2$$

$$P_1 - P_2 = \frac{1}{2}\rho\left(V_2^2 - V_1^2\right)$$

$\Rightarrow \quad P_1 > P_2$ and $V_1 < V_2$

This shows that the pressure in the narrow pipe where streamlines are closer is much smaller than in the wider pipe. Thus,

"Where the speed is high,
the pressure is low."

It can explain the action of scent sprayer and Bunsen burner.

EXAMPLES:-

(i) The lift on an aeroplane is due to this effect. The aeroplane wings are so designed to deflect the air so that streamlines are closer above the wings than below. Where the streamlines are closer, the speed is faster. Thus air is travelling faster on the upper side of the wings than on the lower. The pressure will be lower at the top of the wings and the wings will be forced upward.

(ii) When a tennis ball is hit by a racket in such a way that it spins as well as moves forward, the velocity of air on one side of the ball increases due to spin and pressure decreases. This gives an extra curvature to the ball known as swing which deceives an opponent player.

(iii) Venturi Relation:-

$$P_1 - P_2 = \frac{1}{2}\rho V_2^2$$

is known as Venturi relation used in Venturi-meter (a device used to measure speed of fluid flow).

Consider two pipes in which one pipe has a much smaller diameter than the other as shown in figure. Let the pipes are horizontal so that ρgh terms become equal. Thus applying Bernoulli's Equation:

$$\Rightarrow P_1 + \frac{1}{2}\rho V_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho V_2^2 + \rho g h_2$$

where $\rho g h_1 = \rho g h_2 = \rho g h$

$$\Rightarrow P_1 + \frac{1}{2}\rho V_1^2 = P_2 + \frac{1}{2}\rho V_2^2$$

or $$P_1 - P_2 = \frac{1}{2}\rho V_2^2 - \frac{1}{2}\rho V_1^2$$

or $$P_1 - P_2 = \frac{1}{2}\rho (V_2^2 - V_1^2)$$

As the cross-sectional area A_2 is small as compared to the area A_1, then from equation of continuity:

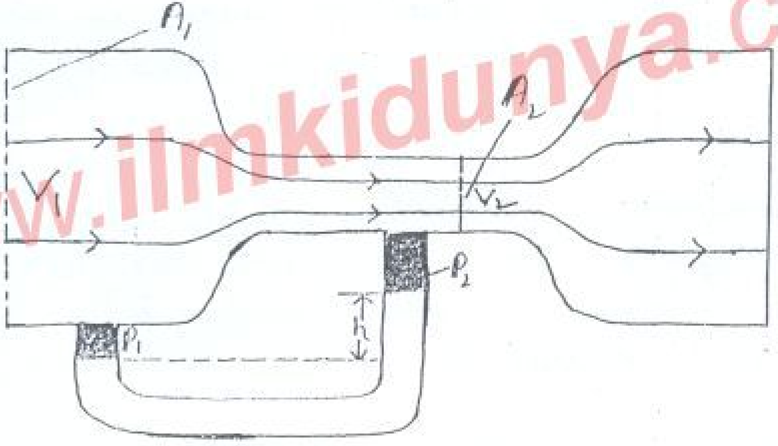
$$A_1 V_1 = A_2 V_2$$

or $V_1 = \frac{A_2 V_2}{A_1}$ will be small as compared to V_2, i-e; $V_1 \ll V_2$

Thus for flow from a large pipe to a

small pipe we can neglect V_1.

$$\Rightarrow P_1 - P_2 = \frac{1}{2}\rho V_2^2 \text{ (Venturi Relation)}$$



Fsc Part 1 Chapter 6

SHORT QUESTIONS & ANSWERS

6.1 Explain what do you understand by the term viscosity?

Ans. The frictional effect between different layers of a flowing fluid is described in terms of viscosity of the fluid. Viscosity measures, how much force is required too slide one layer of the liquid over another layer.

6.2 What is meant by drag force? What are the factors upon which drag force acting upon a small sphere of radius r, moving down through a liquid, depend?

Ans. An object moving through fluid experiences a retarding force called a drag force. This force increases as the speed of the object through the fluid increases. In case of a spherical object moving through a fluid, the drag force depends upon the radius ' r ' of the spherical object, its speed ' v ' and the coefficient of viscosity 'η' of the fluid. The drag force ' F ' is given by 'Stokes Law

$$F = 6\pi\eta rv$$

6.3 Why fog droplets appear to be suspended in air?

Ans. A fog droplet is a tiny drop with very small weight. As it falls, very soon, the drag force acting on it becomes equal to its weight and hence, it attains terminal velocity quickly which has a very small value. Thereafter it drops very slowly with that terminal velocity and appears to be suspended in air.

6.4 Explain the difference between laminar flow and turbulent flow.

Ans. The fluids flow is said to be laminar if every particle of the fluid that passes a point moves along the same path, as followed by particles, which passed that point earlier.

The irregular or unsteady flow of the fluid is called turbulent flow. In this case the exact path of the particles of the fluid cannot be predicted as the velocity of the fluid may change abruptly.

6.5 State Bernoulli's relation for a liquid in motion and describe some of its applications.

Ans. Bernoulli's equation is the fundamental equation in fluid dynamics which states that the sum of pressure and kinetic and potential energies per unit volume is a steady flow of an incompressible and non viscous liquid has the same value. Thus;

$$P = \frac{1}{2}\rho v^2 + \rho gh = \text{constant}$$

where ' ρ ' is the densitv of the fluid and ' g ' is acceleration due to gravity.

Some of its applications include

(i) Operation of paint sprayer or perfume sprayer

(ii) Operation of a chimney for smoke exhaust

(iii) Lift in an aeroplane

(iv) Swing of a cricket or tennis ball

(v) Measurement of speed of liquid or gas flow through a pipe

(vi) Working of a carburettor of a car

(vii) Working of a filter pump

6.6 A person is standing near a fast moving train. Is there any danger that he will fall towards it?

Ans. Yes, there is a danger that the man may fall towards a fast moving train. Near the fast moving train, the pressure decreases. Hence, the greater pressure behind the man may push him towards low-pressure side and he may fall towards the train.

6.7 Identify the correct answer. What do you infer from Bernoulli's theorem?

(i) Where the speed of the fluid is high the pressure will be low.

(ii) Where the speed of the fluid is high the pressure is also high.

(iii) This theorem is valid only for turbulent flow of the liquid.

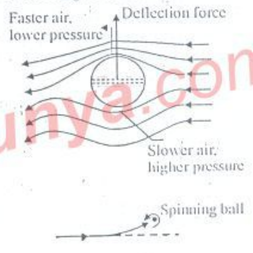
Ans. The correct statement is number (i) where the speed of the fluid is high the pressure will be low.

6.8 Two rowboats moving parallel in the same direction are pulled towards each other. Explain.

Ans. When two bodies are moving parallel in the same direction, the water between them is also dragged along with them. Hence, water between the boats has greater speed as compared to water on the other sides of the boats. As a result, the pressure of the water between the boats is reduced and greater pressure on the other sides pushes the boats towards each other.

6.9 Explain how the swing is produced in a fast moving cricket ball?

Ans. When a fast moving cricket ball moves in such a way that it spins as well as moves forward, the velocity of air on one side of the ball increases due to spin and air speed in the same direction and hence, the pressure decreases. The greater pressure on the other side deflects the path of ball and gives it an extra curvature to the ball known as swing, which deceives a batsman.



6.10 Explain the working of carburetors of a motorcar using Bernoulli's equation.

Ans. The carburetor of a car engine uses a Venturi duct to feed the correct mix of air and petrol to the cylinders. Air is drawn through the duct and along a pipe to the cylinders. A tiny inlet at the side of the duct is fed with petrol. The air through the duct moves very fast, creating low pressure in the duct, which draws petrol vapours into the air stream.

6.11 For which position will the maximum blood pressure in the body have the smallest value? (a) Standing up right (b) Sitting (c) Lying horizontally (d) Standing on one's head?

Ans. Maximum blood pressure in the body has the smallest value while lying horizontally. In this position, all parts of the body are in level with the heart; hence heart doesn't have to work quite as hard, pumping against gravity.

6.12 In an orbiting space station, would the blood pressure in major arteries in the leg ever be greater than the blood pressure in major arteries in the neck?

Ans. Blood pressure will be equal due to state of weightlessness in an orbiting space station.