Fsc Part 1 Chapter 7

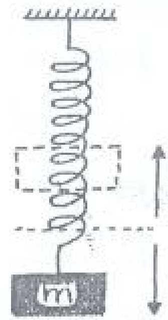
OSCILLATIONS

VIBRATORY OR OSCILLATORY MOTION

The to and fro motion of a body about a mean position is called vibratory or oscillatory motion.

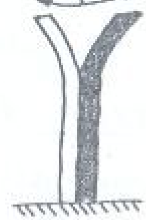
EXAMPLES

(1) A mass suspended from a spring, when pulled down and then released, the mass starts oscillating about its mean position as shown.



(2) When the bob of a simple pendulum displaced from its mean position and released, then simple pendulum execute oscillatory motion.

(3) A steel ruler clamped at one end to a bench oscillates when the free end is displaced sideways.

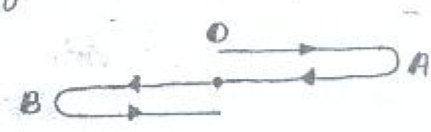


(4) A steel ball rolling in a curved dish, oscillates about its rest position.

VIBRATION OR OSCILLATION

One complete round trip of a body about its mean position is called one vibration or oscillation.

For example, a body move from O to A, then A to B and finally from B to O.



Time Period

Time to complete one vibration is called time period.

- ⇒ It is denoted by T.
- ⇒ Its SI-unit is second (S).
- ⇒ Its dimension is $[T]$ or $[M^0L^0T]$

Frequency

The number of vibrations completed by a body in one second is called frequency.

- ⇒ It is denoted by f or ν.
- ⇒ Its SI-units are (a) Vibration per second (vib/s) (b) Revolution per second (rev/s) (c) Cycle per second (C/s) (d) 1/second or S^{-1} (e) Hertz (Hz)
- ⇒ Its dimension is $[\frac{1}{T}]$ or $[T^{-1}]$ or $[M^0L^0T^{-1}]$

Hertz (Hz)

The frequency of a body is said to be one hertz if it complete one vibration in one second.

Relation Between Time Period And Frequency

Time period is equal to reciprocal of frequency and vice-versa.

$$\boxed{T = \frac{1}{f}} \quad \text{or} \quad \boxed{f = \frac{1}{T}}$$

Displacement

The distance covered by a body on either side of its mean position at any instant is called displacement.

- ⇒ It is denoted by x.
- ⇒ Its SI-unit is meter (m).

➡ It is a vector quantity.

➡ Its dimension is $[L]$ or $[M^0LT^0]$.

AMPLITUDE

The magnitude of maximum displacement of a body on either side of its mean position is called amplitude.

➡ It is denoted by x_0 and a scalar quantity.

➡ Its unit and dimension is meter (m) and $[L]$ respectively.

HOOKE'S LAW

It states that the applied force on an elastic body is directly proportional to the displacement from mean position within the elastic limit.

Consider a force F acts on a body and move it through a certain displacement x, then

$$F \propto x$$

$$\boxed{F = Kx}$$

Where K is spring constant. Its value depends upon the nature of the material of the spring.

SPRING CONSTANT

It is defined as "The ratio between force applied on an elastic body to the displacement produced from mean position is constant, called spring constant."

$$\boxed{K = \frac{F}{x}}$$

➡ Its SI-unit is N/m or $N \cdot m^{-1}$. or $Kg \cdot S^{-2}$.

➡ Its dimension is $[MT^{-2}]$ or $[ML^0T^{-2}]$

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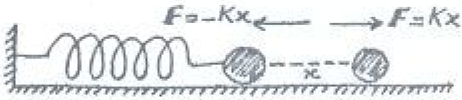
Elastic Restoring Force

A force which opposes the applied force due to elasticity is called elastic restoring force.

In case of a body attached with a spring of spring constant K, the elastic restoring force is



$$F = -Kx$$

Where negative sign shows that elastic restoring force is always directed towards the mean position.

Simple Harmonic Motion (SHM)

A type of vibratory motion in which acceleration produced is directly proportional to the displacement and always directed towards the mean position is called simple harmonic motion.

If "a" is the acceleration produced and "x" be the displacement from mean position, then

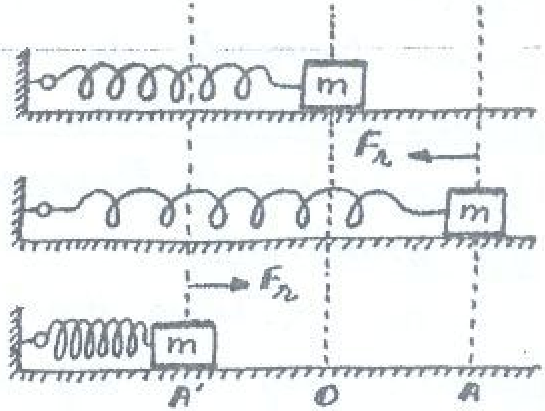
$$\boxed{a \propto -x}$$

Explanation

or Acceleration Of A Body Executing SHM

Consider a mass m is attached with one end of an elastic spring of spring constant K whose other end is attached with a rigid support as shown. The mass m can move freely on a frictionless horizontal surface.

Let a force F acts on mass m and displaced it towards right through a distance x from its mean position as shown in figure. According to Hooke's law, the applied force is



$$F = Kx$$

Due to elasticity, spring opposes the applied force, called elastic restoring force F_r given as

$$F_r = -Kx \longrightarrow ①$$

Where negative sign shows that F_r is directed towards the mean position.

When the mass m is released, then it vibrate about its mean position and acceleration produced in mass m due to restoring force. The force producing acceleration is given by using Newton's second law of motion

$$F = ma \longrightarrow ②$$

Comparing equation ① and ②

$$ma = -Kx$$

$$a = -\frac{K}{m}x$$

Where K and m are constants, so

$$a = -(\text{Constant})\,x$$

$$\boxed{a \propto -x}$$

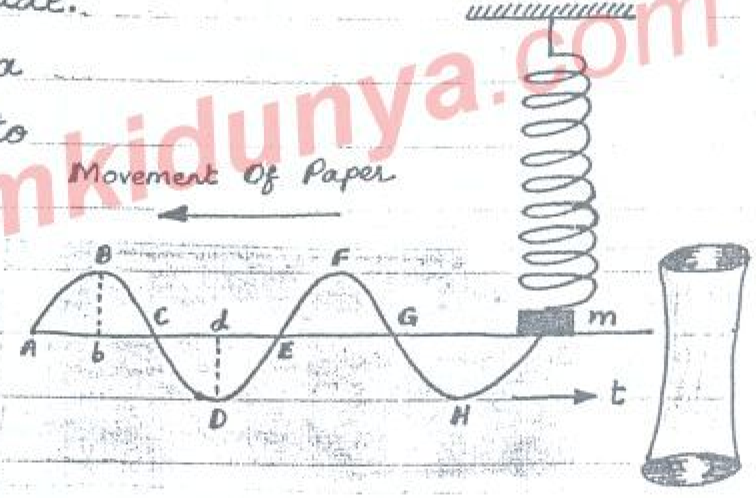
Hence, the acceleration produced in a body at any instant executing SHM is directly proportional to the displacement and always directed towards its mean position.

VARIOUS TERMS USED IN DESCRIBING SHM.

i) INSTANTANEOUS DISPLACEMENT AND AMPLITUDE OF VIBRATION

As we know that the distance covered by a vibrating body at any instant from its mean position is known as instantaneous displacement. At the mean position, the displacement is zero and is maximum at the extreme position. The maximum displacement is known as amplitude.

Now arrange a mass-spring system to study the variations in displacement with time. For this, consider a small mass m is attached with a vertical spring. A



sheet of paper is placed behind the mass and is moving with constant speed from right to left as shown in figure. A pen is also attached with the mass which touches the paper.

The mass m is raised at a certain height and then released, it perform SHM and a waveform is produced on the paper which is a sine wave as shown. This waveform is usually known as waveform of SHM. The points B and D represents extreme positions of vibrating mass and points A, C and E represents its mean position. Hence, the line ACE represents the level of mean position of the mass. From figure, the amplitude of vibration is measure of the line Bb or Dd.

(ii) **VIBRATION**

One complete round trip of a body about its mean position is called a vibration.

In figure, the curve ABCDE represents the different positions of the pen during one complete vibration.

Vibration can also be defined as "motion of the body from its one extreme position back to the same extreme position". This will corresponds to the portion of the curve from points B to F or from points D to H.

(iii) **TIME PERIOD**

Time required to complete one vibration is called time period. It is denoted by "T".

Mathematically

$$\boxed{T = \frac{1}{f}}$$

(iv) **FREQUENCY**

The number of vibrations completed by a body in one second is called frequency. It is denoted by f or ν.

Mathematically

$$\boxed{f = \frac{1}{T}}$$

) **ANGULAR FREQUENCY**

If T is the time period of a body, then its angular frequency ω is

$$\omega = \frac{2\pi}{T}$$

or $$\omega = 2\pi\left(\frac{1}{T}\right)$$

or $$\boxed{\omega = 2\pi f} \qquad \left(\because \frac{1}{T} = f\right)$$

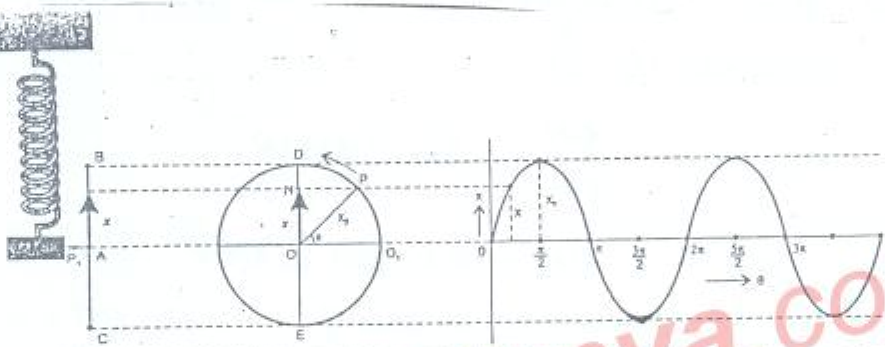
It is a characteristic of circular motion and it provides an easy method to calculate value of instantaneous displacement and instantaneous velocity of a body executing SHM.

SHM AND UNIFORM CIRCULAR MOTION

Consider a mass m having a pointer P_1 is attached with the one end of a vertically suspended spring as shown. When the mass m is raised and then released, it starts vibrating and execute SHM with time period T, frequency f and amplitude x_o. The motion of the mass is represented by the pointer P_1 on the line BC with A as its mean position and points B, C as its extreme positions as shown in figure.

Let A is the position of the pointer at $t = 0$. It will move so that it is at B, A, C and back to A at instants $T/4$, $T/2$, $3T/4$ and T respectively. Hence it will complete one vibration with amplitude $x_o = AB = AC$.

Now consider a point P which is moving along a circular path of radius x_o equal to the amplitude of the pointer's motion. The point P moves with uniform angular frequency $\omega = 2\pi/T$, where T is the time period of the vibration of the pointer. Now consider the motion of point N and draw projection from point P on the diameter DE parallel to the line of vibration of the pointer as shown in figure.



The levels of points D and E are same as the points B and C. As point P move along circular path with constant angular speed ω, so point N oscillates to and fro on the diameter DE with time period T. Let O_1 be the position of point P at $t = 0$ and position of point N at the instants 0, $T/4$, $T/2$, $3T/4$ and T will be at the points O, D, O, E and O respectively. of pointer P_1. Hence, the motion of point N is related with the motion of pointer P_1 i-e; circular motion is related with vibratory motion. Thus the expressions of displacement, velocity and acceleration for the motion of point N also hold good for the motion of pointer P_1 executing SHM.

Displacement

Consider point P is passing through O_1 at $t=0$, en the angle subtended by radius OP in time t om figure is given as

$$\angle O_1OP = \theta = \omega t$$

Also $\angle O_1OP = \angle OPN = \theta = \omega t$

Now from triangle OPN, the displacement x f N at instant t is

$$\frac{ON}{OP} = \sin \angle O_1OP$$

or $ON = OP \sin \angle O_1OP$

or $x = x_0 \sin\theta$ $\quad (\because ON = x,\ OP = x_0)$

or $\boxed{x = x_0 \sin \omega t} \longrightarrow (1)$

It will be also the displacement of the pointer P_1 at instant t.

The value of x as a function of θ is shown n figure which is the waveform of SHM. This wav-form is same as traced experimentally. The angle θ ives the positions of the system in its vibrational ycle. For example, at the start of the cycle $\theta = 0°$. When quarter cycle is completed, $\theta = 90°$ and $\theta = 180°$ when half cycle is completed. Thus we call θ s the phase of vibration. Phase of vibration is 0° or $\pi/2$ radian when quarter of the cycle is completed. Hence phase is also related with the circular motion.

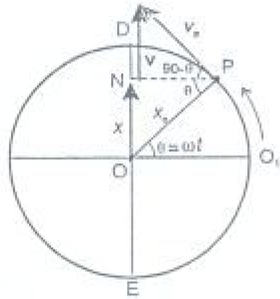
(ii) Instantaneous Velocity

The velocity of point P at any instant t will be directed along the tangent to the circle at point P and its magnitude given as

$$V_P = r\omega$$

For circular path, $r = x_o$

$$\therefore \quad V_P = x_o\omega \longrightarrow ②$$



As the motion of point N on the diameter DE is due to motion of P on the circle and velocity of N is actually the component of the velocity V_P parallel to the diameter DE as shown in figure.

The component of V_P parallel to DE is

$$V_P \sin(90° - \theta) = V_P \cos\theta \qquad (\because \sin(90°-\theta) = \cos\theta)$$

So the magnitude of velocity N is

$$V = V_P \cos\theta \qquad (\because V_P = x_o\omega)$$

or $$V = x_o\omega\cos\theta \qquad (\because \theta = \omega t)$$

or $$V = x_o\omega\cos\omega t \longrightarrow ③$$

Where the direction of V depends upon θ.

When θ varies from 0° to 90°, then its direction is from O to D. If θ varies from 90° to 270°, then its direction is from D to E and when θ varies from 270° to 360°, then its direction is from E to O.

Now from figure, in ΔOPN, we have

$$\cos\theta = \cos\angle OPN = \frac{NP}{OP}$$

or $\quad \cos \omega t = \frac{NP}{OP} \longrightarrow (4)$

From figure, it is clear that

$OP = r = x_0$ and $ON = x$

Using Pythagorean's theorem on ΔOPN

$$(Hyp)^2 = (Perp)^2 + (Base)^2$$

or $\quad (OP)^2 = (ON)^2 + (NP)^2$

or $\quad x_0^2 = x^2 + (NP)^2$

or $\quad (NP)^2 = x_0^2 - x^2$

Taking square root on both sides

$$NP = \sqrt{x_0^2 - x^2}$$

Put values in equation (4), we get

$$\cos \omega t = \frac{\sqrt{x_0^2 - x^2}}{x_0}$$

Put this value in equation (3), so

$$V = x_0 \omega \frac{\sqrt{x_0^2 - x^2}}{x_0}$$

$$\boxed{V = \omega \sqrt{x_0^2 - x^2}} \longrightarrow (5)$$

MAXIMUM VELOCITY

The velocity will be maximum at mean position, so

$V = V_0 =$ Maximum velocity at $x = 0$

Put these values in equation (5)

$$\therefore V_0 = \omega \sqrt{x_0^2 - 0} = \omega \sqrt{x_0^2}$$

$$\boxed{V_0 = \omega x_0} \longrightarrow (6)$$

MINIMUM VELOCITY

The velocity will be minimum at extreme position, so

$V = V_{min}$ at $x = x_0$

Put these values in equation ⑤

$\therefore \quad V_{min} = \omega\sqrt{x_o^2 - x_o^2} = \omega(0)$

$$\boxed{V_{min} = 0}$$

(iii) **ACCELERATION IN TERMS OF ω**

As point P moves along circular path, so it has centripetal acceleration a_p and it always directed towards the centre of the circle O as shown.

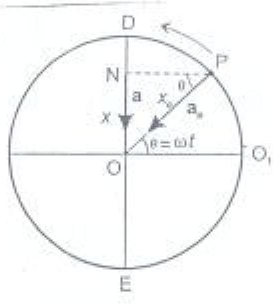
The magnitude of acceleration a_p is given as

$$a_p = r\omega^2 \qquad (\because r = x_o)$$

$$a_p = x_o\omega^2$$

At any instant t, the direction of a_p is along PO. The acceleration "a" of point N is the component of acceleration "a_p" along the diameter DE on which point N moves due to the motion of point P.

The component of a_p along DE is given as



$$a_p \sin\theta = x_o\omega^2 \sin\theta$$

Hence, the acceleration of N is

$$a = a_p \sin\theta$$

or $\quad a = x_o\omega^2 \sin\theta \longrightarrow ⑦$

It is directed towards mean position O.

From figure, in ΔOPN, we have

$$\sin\theta = \frac{ON}{OP} \qquad (\text{Where } ON = x,\ OP = x_o)$$

or $\quad \sin\theta = \frac{x}{x_o}$

Put in equation ⑦

$\therefore \quad a = x_o\omega^2 \sin\theta = x_o\omega^2 \times \frac{x}{x_o}$

or $\boxed{a = \omega^2 x} \longrightarrow ⑧$

In vector form

$\boxed{\vec{a} = -\omega^2 \vec{x}} \longrightarrow ⑨$

Where negative sign shows that its direction is towards the mean position O. This equation also shows that the acceleration is directly proportional to the displacement and directed towards the mean position i-e;

$\boxed{a \propto -x}$ $(\because \omega^2$ is constant)

Hence, point N execute S.H.M.

CONDITIONS FOR VIBRATORY MOTION

The conditions for a body to execute vibratory motion are given as

(i) The body should have inertia.

(ii) The body should have elastic restoring force.

CONDITIONS FOR S.H.M.

A system execute S.H.M. must obey following conditions

(i) The system should have vibratory motion.

(ii) The system should have instantaneous acceleration.

(iii) The system should be frictionless.

(iv) The magnitude of acceleration is directly proportional to the displacement from mean position.

(v) The direction of acceleration is always towards the mean position i-e;

$\boxed{a \propto -x}$

PHASE

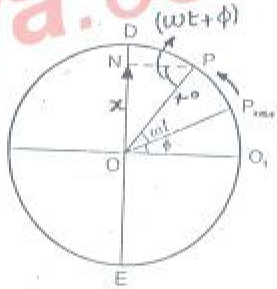
DEFINITION: The angle $\theta = \omega t$ which specifies the displacement as well as the direction of motion of the point executing S.H.M. is known as phase.

OR The angle subtended by a rotating body at any instant with the refrence direction is also known as phase.

EXPLANATION:

The phase gives us the information about the state of motion of a vibrating body. The waveform of S.H.M can be traced by applying the concept of phase.

Consider a body is moving along a circular path and at any instant t, it is at point P as shown in figure.



Let at $t=0$, the position of the rotating radius OP is along refrence axis OO_1, so that the point N is at mean position O and its displacement $x=0$ at $t=0$. Now we assume that at $t=0$, the rotating radius OP make certain angle ϕ with the refrence axis OO_1 as shown in figure. After time t, let the radius OP make an angle ωt. Now, the radius OP make an angle $(\omega t+\phi)$ at any instant t and displacement $ON = x$ can be obtained from figure (in ΔOPN) as

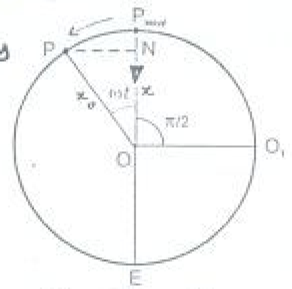
$$\frac{ON}{OP} = \sin(\omega t+\phi)$$

or $\quad ON = OP \sin(\omega t + \phi)$

Where $\quad ON = x$ and $OP = r = x_o$

$\therefore \quad x = x_o \sin(\omega t + \phi) \longrightarrow (1)$

Here phase angle is given as



$$\theta = \omega t + \phi$$

When $t = 0$, $\theta = \omega(0) + \phi = 0 + \phi$

$$\theta = \phi$$

So, ϕ is called initial phase as $t = 0$.

If we take initial phase $\phi = \pi/2$ or $90°$, the displacement x from equation (1) is given as

$$x = x_o \sin(\omega t + 90°)$$

or $\quad x = x_o \cos \omega t \longrightarrow (2) \quad (\because \sin \omega t + 90° = \cos \omega t$

Equation (2) also gives the displacement of S.H.M. but in this case, the point N starts its motion from the extreme position instead of mean position.

A Horizontal Mass Spring System

Consider a vibrating mass m is attached with a spring of spring constant K as shown in figure.

Let a force F acts on mass m and displaced it through a distance x as shown. According to Newton's second law of motion, the applied force is

$$F = ma \longrightarrow (1)$$

Due to elasticity, spring opposes the applied force called elastic restoring force given as

$$F = -Kx \longrightarrow (2)$$

Comparing equation ① and ②

$$\therefore \quad ma = -Kx$$

or $$a = -\frac{K}{m}.x \longrightarrow ③$$

Where K and m are constants, so

$$a = -(\text{Constant})\,x$$

or $$\boxed{a \propto -x}$$

Hence, a mass attached with a spring execute S.H.M. Now for a body executing S.H.M., the acceleration is given as

$$a = -\omega^2 x \longrightarrow ④$$

Comparing equation ③ and ④

$$\therefore \quad -\omega^2 x = -\frac{K}{m}x$$

or $$\omega^2 = \frac{K}{m}$$

Taking square root on both sides

$$\boxed{\omega = \sqrt{\frac{K}{m}}} \longrightarrow ⑤$$

This is the angular frequency of a mass attached with a spring.

Now we discuss different features of a mass spring system.

TIME PERIOD OF MASS

Time period of a mass executing SHM is

$$T = \frac{2\pi}{\omega}$$

Put value of ω from eq. ⑤

$$T = \frac{2\pi}{\sqrt{K/m}}$$

or $$\boxed{T = 2\pi\sqrt{\frac{m}{K}}} \longrightarrow ⑥$$

Frequency Of Mass

As, frequency is equal to reciprocal of time period, so

$$f = \frac{1}{T}$$

Put value of T from eq. ⑥

$$\therefore \quad f = \frac{1}{2\pi\sqrt{m/K}}$$

or $$\boxed{f = \frac{1}{2\pi}\sqrt{\frac{K}{m}}} \longrightarrow ⑦$$

Instantaneous Displacement

The instantaneous displacement x of mass is

$$x = x_o \sin \omega t$$

Put value of ω from eq. ⑤

$$\therefore \quad \boxed{x = x_o \sin\sqrt{\frac{K}{m}} \cdot t} \longrightarrow ⑧$$

Instantaneous Velocity

The instantaneous velocity V of the mass m is

$$V = \omega\sqrt{x_o^2 - x^2}$$

or $$V = \omega\sqrt{x_o^2\left(1 - \frac{x^2}{x_o^2}\right)}$$

or $$V = x_o\omega\sqrt{1 - \frac{x^2}{x_o^2}}$$

Put value of ω from eq. ⑤

$$\therefore \quad V = x_o\sqrt{\frac{K}{m}}\sqrt{1 - \frac{x^2}{x_o^2}}$$

or $$\boxed{V = x_o\sqrt{\frac{K}{m}\left(1 - \frac{x^2}{x_o^2}\right)}} \longrightarrow ⑨$$

MAXIMUM VELOCITY

Velocity of mass is maximum at mean position i-e; $V = V_o$ = Maximum Velocity at $x = 0$.

Put these values in equation ⑨

$$\therefore \quad V_o = x_o \sqrt{\frac{K}{m}\left(1 - \frac{0}{x_o^2}\right)}$$

$$\text{or} \quad V_o = x_o \sqrt{\frac{K}{m}(1-0)} = x_o \sqrt{\frac{K}{m}(1)}$$

$$\text{or} \quad \boxed{V_o = x_o \sqrt{\frac{K}{m}}} \longrightarrow (10)$$

Put this value in equation ⑨, we get another form of instantaneous velocity V as

$$\boxed{V = V_o \sqrt{1 - \frac{x^2}{x_o^2}}} \longrightarrow (11)$$

ENERGY CONSERVATION IN SHM

As total energy of a system is equal to the sum of P.E. and K.E. of the system at any instant, so

$$E_{TOTAL} = P.E. + K.E. \longrightarrow (12)$$

POTENTIAL ENERGY

Consider that the vibrating mass m attached with a spring is pulled through displacement x_o against the elastic restoring force F. It is assumed that mass m is pulled slowly, so that acceleration is zero.

According to Hooke's law, the stretching force is

$$F = Kx_o$$

When displacement $x = 0$, Force $= 0$

When displacement $x = x_o$, Force $= Kx_o$

Hence, average force F is

$$F = \frac{0 + Kx_o}{2} = \frac{1}{2}Kx_o$$

So, work done in displacing the mass m through displacement x_o is given as

$$\text{WORK} = \text{FORCE} \times \text{DISTANCE}$$

or $$W = Fd = \frac{1}{2}Kx_o \cdot x_o$$

or $$W = \frac{1}{2}Kx_o^2$$

This work done appear as elastic P.E. of the spring, so

$$P.E. = W$$

or $$\boxed{P.E. = \frac{1}{2}Kx_o^2} \longrightarrow (13)$$

If the mass m is displaced through x at any instant t, then P.E. at that instant is

$$\boxed{P.E. = \frac{1}{2}Kx^2} \longrightarrow (14)$$

MAXIMUM P.E.

P.E. of the mass is maximum at the extreme position, so

$P.E. = P.E._{max}$, when $x = x_o$

Put these values in equation (14)

$$\therefore \boxed{P.E._{max} = \frac{1}{2}Kx_o^2} \longrightarrow (15)$$

MINIMUM P.E.

P.E. of the mass is minimum at the mean position, so

$P.E. = P.E._{min}$, when $x = 0$

Put these values in equation (14)

$$\therefore P.E._{min} = \frac{1}{2}K(0)^2$$

or $$\boxed{P.E._{min} = 0}$$

KINETIC ENERGY

K.E. of mass m is

$$K.E. = \frac{1}{2}mV^2$$

Where the velocity of the mass is

$$V = x_o \sqrt{\frac{K}{m}\left(1-\frac{x^2}{x_o^2}\right)}$$

So

$$K.E. = \frac{1}{2} m \left[x_o \sqrt{\frac{K}{m}\left(1-\frac{x^2}{x_o^2}\right)}\right]^2$$

or $$K.E. = \frac{1}{2} m x_o^2 \left[\frac{K}{m}\left(1-\frac{x^2}{x_o^2}\right)\right]$$

or $$\boxed{K.E. = \frac{1}{2} K x_o^2 \left(1-\frac{x^2}{x_o^2}\right)} \longrightarrow (16)$$

MAXIMUM K.E.

K.E. of the mass is maximum at the mean position, so

K.E. = K.E.max , when $x = 0$

Put these values in equation (16)

$$\therefore \quad K.E._{max} = \frac{1}{2} K x_o^2 \left(1-\frac{0}{x_o^2}\right)$$

or $$K.E._{max} = \frac{1}{2} K x_o^2 (1-0)$$

or $$\boxed{K.E._{max} = \frac{1}{2} K x_o^2} \longrightarrow (17)$$

MINIMUM K.E.

K.E. of the mass is minimum at the extreme position, so

K.E. = K.E. min , when $x = x_o$

Put these values in equation (16)

$$\therefore \quad K.E._{min} = \frac{1}{2} K x_o^2 \left(1-\frac{x_o^2}{x_o^2}\right)$$

or $$K.E._{min} = \frac{1}{2} K x_o^2 (1-1) = \frac{1}{2} K x_o^2 (0)$$

or $$\boxed{K.E._{min} = 0}$$

TOTAL ENERGY

As the total energy is the sum of K.E. and P.E.

$$\therefore \quad E_{TOTAL} = P.E. + K.E.$$

Put values from equation (14) and (16)

$$\therefore \quad E_{TOTAL} = \frac{1}{2} K x^2 + \frac{1}{2} K x_o^2 \left(1-\frac{x^2}{x_o^2}\right)$$

or $$E_{TOTAL} = \frac{1}{2}Kx^2 + \frac{1}{2}Kx_o^2 - \frac{1}{2}Kx^2$$

or $$\boxed{E_{TOTAL} = \frac{1}{2}Kx_o^2} \longrightarrow (18)$$

From equation (15), (17) and (18), it is clear that he total energy of the vibrating mass and spring is onstant. It shows that when K.E. of mass is maximum hen the P.E. of the spring is zero and when the K.E. of the mass is zero, then the P.E. of the spring is naximum. Hence, K.E. can be converted into P.E. and P.E. can be converted into K.E. continuously as the spring is compressed and released alternately.

For example, when simple pendulum is displaced, hen the gravitational P.E. of the mass is converted into K.E. at the mean position and K.E. is converted into P.E. as the mass rises to extreme top position. Usually, a vibrating system do not oscillate continuously because energy is dissipated against frictional forces.

Simple Pendulum

A simple pendulum consists of a single isolated particle suspended from a frictionless support with a light, inextensible string.

Prove That Simple Pendulum Execute S.H.M.

Consider a simple pendulum having a small, heavy particle of mass 'm' suspended by a light, inextensible string of length 'l' fixed at its upper end with a rigid support as shown in figure.

Let pendulum is displaced from its mean position A to B through a small angle 'θ' and displacement 'x' as shown. When pendulum is released, then it starts vibrating to and fro over the same path.

The forces acting on the particle are given as

(i) Weight $W = mg$ of the particle acting vertically downward towards the centre of earth.

(ii) Tension T, acting upward along the string.

At position B, weight mg resolved into two rectangular components given as

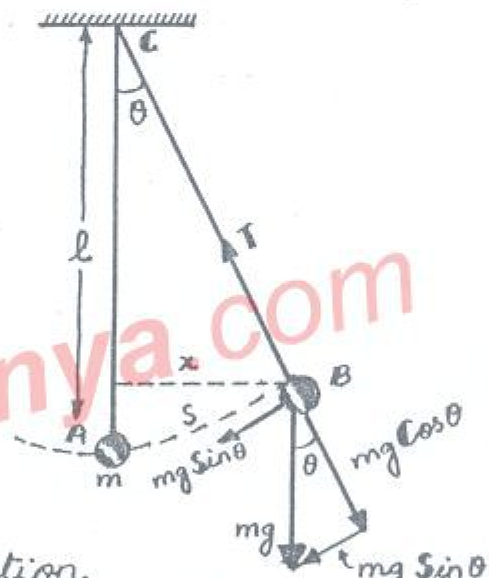
(a) $mg\cos\theta$, acting along the string.

(b) $mg\sin\theta$, acting perpendicular to the string.

As there is no motion along the string, so the component $mg\cos\theta$ balances the tension T.

$$\therefore \quad T = mg\cos\theta$$

The component $mg\sin\theta$ acting perpendicular to the string, so it is responsible for the vibratory motion of the pendulum. Also we take negative sign to it as it directed towards the mean position.



So net restoring force acting on the pendulum is

$$F = -mg\sin\theta$$

As θ is very small, so $\sin\theta \approx \theta$

$$\therefore \quad F = -mg\theta \longrightarrow (1)$$

Using Newton's second law, we have

$$F = ma \longrightarrow (2)$$

Comparing equation (1) and (2)

$$ma = -mg\theta$$

or $$a = -g\theta \longrightarrow (3)$$

As

$$S = r\theta$$

or $$\theta = \frac{S}{r} = \frac{Arc\ AB}{r}$$

Where $r = l$ and when θ is small, then

$$Arc\ AB = OB = x$$

So

$$\theta = \frac{Arc\ AB}{r}$$

or $$\theta = \frac{x}{l}$$

Put this value in equation (3)

$\therefore$ $$a = -g\left(\frac{x}{l}\right)$$

or $$a = -\frac{g}{l}\cdot x \longrightarrow (4)$$

As g and l are constants, so

$$a = -(\text{CONSTANT})\, x$$

or $$\boxed{a \propto -x}$$

This equation shows that acceleration of particle is directly proportional to the displacement and directed towards the mean position. Hence, it is prove that the motion of simple pendulum is SHM.

As the acceleration of body executing SHM is

$$a = -\omega^2 x \longrightarrow (5)$$

Comparing equation (4) and (5)

$\therefore$ $$-\omega^2 x = -\frac{g}{l}\cdot x$$

or $$\omega^2 = g/l$$

Taking square root on both sides

$\therefore$ $$\boxed{\omega = \sqrt{g/l}} \longrightarrow (6)$$

TIME PERIOD OF SIMPLE PENDULUM

As we know that

$$T = \frac{2\pi}{\omega}$$

Put value of ω from equation (6)

$\therefore$ $$T = \frac{2\pi}{\sqrt{g/l}}$$

or $$\boxed{T = 2\pi\sqrt{\frac{l}{g}}} \longrightarrow ⑦$$

This shows that the time period of a simple pendulum depends only on the length of the pendulum and the acceleration due to gravity. It is independent of its mass.

Frequency Of Simple Pendulum

As frequency is equal to the reciprocal of time period, so

$$f = \frac{1}{T} \qquad \because T = 2\pi\sqrt{l/g}$$

or $$f = \frac{1}{2\pi\sqrt{l/g}}$$

or $$\boxed{f = \frac{1}{2\pi}\sqrt{\frac{g}{l}}} \longrightarrow ⑧$$

Second's Pendulum

A pendulum whose time period is two seconds is called second's pendulum.

OR

A pendulum which complete one vibration in two seconds is called second's pendulum.

So

$$\boxed{T = 2 \text{ second}}$$

The frequency of second's pendulum is

$$f = \frac{1}{T} = \frac{1}{2} Hz$$

or $$\boxed{f = 0.5 \ Hz}$$

Use Of Simple Pendulum

With the help of simple pendulum, we can find the value of 'g' by measuring T and l as

$$T = 2\pi\sqrt{\frac{l}{g}} \Rightarrow T^2 = 4\pi^2\left(\frac{l}{g}\right)$$

or $$\boxed{g = \frac{4\pi^2 l}{T^2}}$$

Free And Forced Oscillations

Free Oscillations

A body is said to be executing free oscillations if it oscillates with its natural frequency without the interference of an external force.

For example, when a simple pendulum is slightly displaced from its mean position, then it vibrates freely with its natural frequency which depends upon the length of the pendulum. As

$$f = \frac{1}{2\pi}\sqrt{\frac{g}{l}} \Rightarrow \boxed{f \propto 1/\sqrt{l}}$$

Forced Oscillations

If a freely oscillating system is subjected to an external force, then forced oscillations are produced.

For example, when the mass of a pendulum is struck repeatedly, then forced oscillations are produced. Also vibrations produced in the factory floor due to running of heavy machinery is an example of forced vibrations.

Another example of forced vibrations is loud music produced by sounding wooden boards of strings instruments.

Driven Harmonic Oscillator

A physical system undergoing forced vibration is known as Driven Harmonic Oscillator.

Natural Frequency And Natural Time Period

When a body is disturbed from its mean position with a particular frequency known as natural frequency and its time period is known as natural time period.

As we know that frequency of a simple pendulum depends upon the length of the pendulum as $f \propto 1/\sqrt{l}$, so when a simple pendulum of certain length is disturbed, it always vibrate with particular frequency, called natural frequency. Also those pendulums which have equal lengths, have same natural frequencies.

As time period T is equal to reciprocal of frequency f i-e; $T = 1/f$, such time period is called natural time period.

RESONANCE

When the frequency of the applied force is equal to the one of the natural frequencies of vibration of the forced or driven harmonic oscillator, then we said that resonance occurs in the system.

OR

When the time period of the applied periodic force becomes equal to the natural time period of the vibrating body, then body vibrates with a greater amplitude is called resonance.

EXPLANATION:

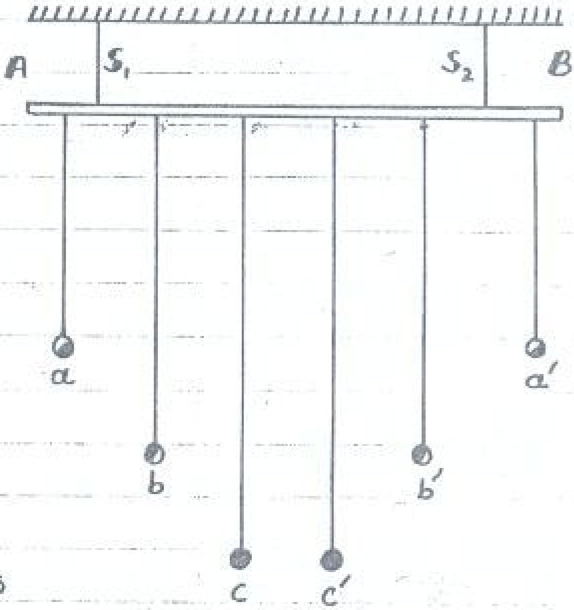
In many mechanical systems, vibrations can be produced when an object is repeatedly displaced from its mean position by some external force. If applied force becomes equal to the natural frequency of the object, then resonance occur and vibration may build up to dangerous levels.

In 1939, The Tacoma bridge disaster caused due to wind conditions in the valley.

A tragic accident took place in Angers (France) in 1850 when soldiers marching over a bridge which breakes due to vibrations and about 200 soldiers were killed

EXPERIMENT

To explain resonance effect, perform an experiment whose apparatus is shown in figure. Apparatus consists on a horizontal rod AB supported by two strings S_1 and S_2. Three pairs of pendulums aa', bb' and cc' are suspended to this rod. The length of each pair of pendulums is same but is different for different pairs.



If one of these pendulums say c, is displaced in a direction perpendicular to the plane of the rod, then its resultant oscillatory motion produces a very slight disturbing motion in rod AB. Due to this slight motion of the rod, the pendulums aa', bb' and cc' start to move with a slight periodic motion. This causes the pendulum c' oscillate back and forth with steadily increasing amplitude as its length and hence period is exactly equal as that of c. Whereas, the amplitudes of other pendulums aa' and bb' remain small because their natural periods are not the same as that of disturbing force due to the rod AB.

EXAMPLES

1. A swing is a good example of resonance. It is a pendulum having fixed length and a single natural frequency. If force applied on the swing in regular intervals, its amplitude increases continuously. If force applied irregularly, the swing will hardly vibrate.

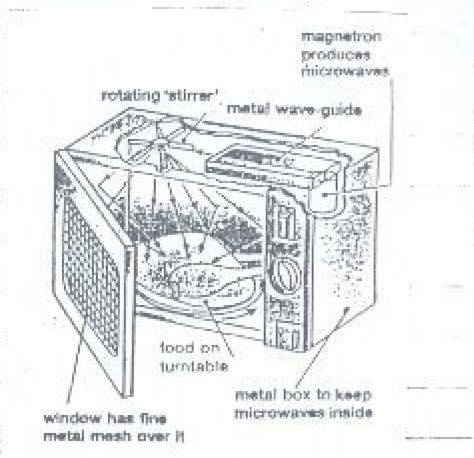
2. If there is a long bridge capable of vibrating with a certain natural frequency. If the frequency of the regular steps of soldiers marching on the bridge becomes equal to the natural frequency of the bridge, then vibrations of dangerously large amplitude may build up. In order to avoid this danger, the soldiers are ordered to break their steps while crossing the bridge.

3. Tuning a radio is another good example of electrical resonance. When we turn the knob of a radio, to tune a station, then the natural frequency of the electric circuit of the receiver is made equal to the transmission frequency of the radio station. When the two frequencies concides, energy absorption is maximum and this is the only station we hear.

4. Many cities having tall buildings have refused to allow supersonics to fly over them to avoid resonance in buildings.

5. Another good example of resonance is the heating and cooking of food very efficiently and evenly by microwave oven as shown in figure.

The waves produced in this type of oven have a wavelength of 12 cm at a frequency of 2450 MHz. At this frequency the waves are absorbed due to resonance by water and fat molecules in the food which heating the food and so cooking it.



RESONANT FREQUENCY

The particular frequency with which the vibrations of maximum amplitude are produced, called the resonant frequency.

Applications Of Resonance

1. It is used to determine the frequency of a body.
2. Speed of sound can be determined with the help of resonance apparatus.
3. Microwave oven also make use of resonance.

Damped Oscillations

Definition: A process in which energy is dissipated from the oscillating system is called damped oscillations.

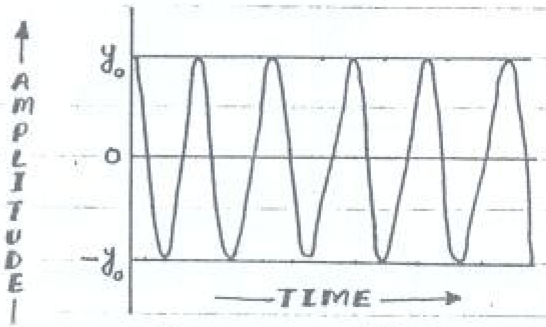
OR

Such oscillations in which the amplitude of vibrations decreases steadily with time are called damped oscillations.

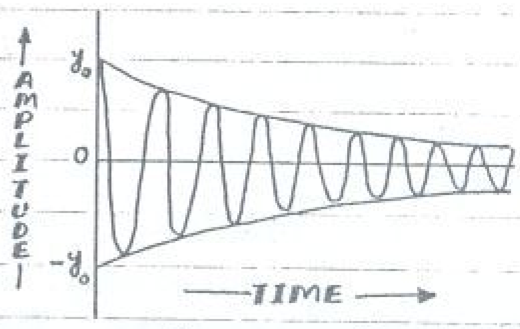
Explanation

It is a common observation that amplitude of a vibrating simple pendulum decreases gradually with time till it becomes zero. Such type of oscillations in which amplitude decreases steadily with time are called damped oscillations.

It is also our every day experience that any macroscopic system is effected by friction. In case of a simple pendulum, this effect is ignored. In actual practice, the amplitude of motion of simple pendulum gradually becomes smaller and smaller due to the force of friction and air resistance. Figure shows the damped and undamped harmonic wave changes with time as given below.



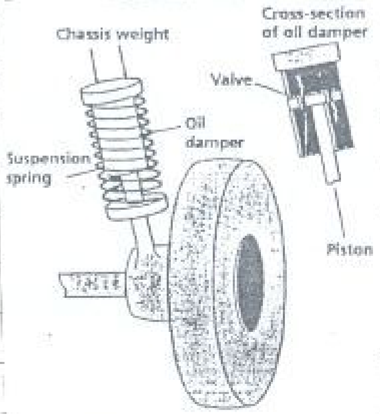
UNDAMPED WAVE



DAMPED WAVE

Applications Of Damped Oscillations

1. Shock absorber of a car is an application of damped oscillations which provides a damping force to prevent excessive oscillations as shown in figure. The shock absorber provide a velocity-dependent damping force. If the shock absorbers are not good, then the car becomes too much bouncy and this is really uncomfortable.



2. Human body system is another application of damped oscillations. If a skier moves over bumpy snow, then skier's body moves smoothly while his or her thighs and calves act like a damping spring That is why skiing instructors always saying "Bend Zee Knees".

Sharpness Of Resonance

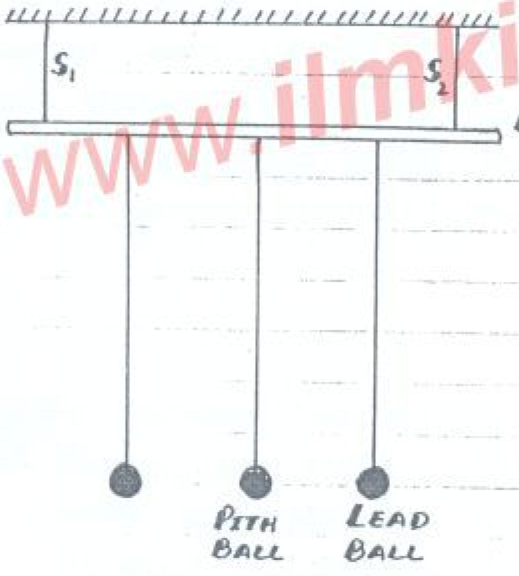
As the amplitude of vibration becomes very large at resonance if damping force is small. Hence damping prevents the amplitude of vibration becoming excessively large. The amplitude decreases rapidly at a frequency slightly different from the resonant frequency. Whereas

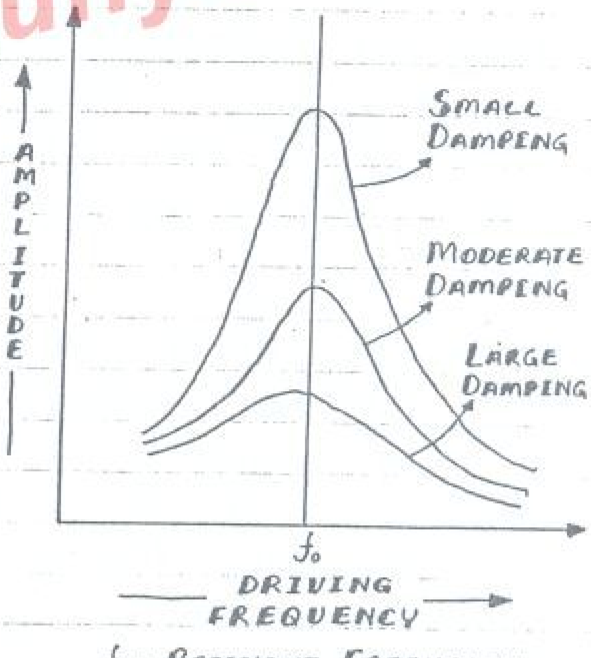
"A heavily damped system has a fairly flat resonance curve as shown in amplitude frequency graph."

The effect of damping can be observed as

Consider a small mass like pith-ball is attached with a string and a heavy mass like lead ball is also attached with a string of same length suspended from a rod AB as shown. They are set into vibrations by a third pendulum of equal length attached to the same rod. It is observed that the amplitude of lead-ball is much more greater than the pith-ball. The damping effect for the pith-ball due to air resistance is much greater than for the lead-ball.

"Hence, the sharpness of the resonance curve of a resonating system depends on the frictional loss of energy."





f_0 = RESONANT FREQUENCY

Fsc Part 1 Chapter 7

SHORT QUESTIONS & ANSWERS

7.1 Name two characteristics of simple harmonic motion (SHM).

Ans. The characteristics of SHM are:

(i) Acceleration is directly proportional to the displacement from the mean position.

(ii) Acceleration is always directed towards mean position.

i.e., $a \propto -x$

7.2 Does frequency depend on amplitude for harmonic oscillators?

Ans. The frequency of harmonic oscillator is;

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

From this expression it is clear that the frequency of a harmonic oscillator does not depend on amplitude of vibration but it depends on its mass. By increasing mass of harmonic oscillator, its frequency decreases.

7.3 Can we realize an ideal simple pendulum?

Ans. No, we cannot realize an ideal simple pendulum. An ideal simple pendulum consists of a heavy but small metallic bob suspended from a rigid, frictionless support by means of a long, weightless and inextensible string.

7.4 What is the total distance traveled by an object moving with SHM in a time equal to its period, if its amplitude is 'A'?

Ans. The time period of a SHM is the time to complete one complete to and fro motion about the mean position. If 'A' is the amplitude of vibration, then during this time, the object moves a distance equal to '4A'.

7.5 What happens to the time period of a simple pendulum if its length is doubled? What happens if the suspended mass is doubled?

Ans. The time period T of a simple pendulum is given by

$$T = 2\pi\sqrt{\frac{l}{g}}$$

Where 'l' is the length of the pendulum and 'g' is acceleration due to gravity at the place. Then, if its length is doubled, the time period increases to 1.4 times i.e. the square root of 2. As above-mentioned expression is independent of suspended mass, hence, time period is not affected when suspended mass is doubled.

7.6 Does the acceleration of a simple harmonic oscillator remain constant during its motion? Is the acceleration ever zero? Explain.

Ans. The acceleration 'a' of a simple harmonic oscillator is given by

$a = -\text{constant} \times x$

where 'x' is the displacement from the mean position. As the displacement 'x' changes during a SHM, its acceleration does not remain constant. The acceleration becomes zero at the mean position ($x = 0$) and acceleration becomes maximum at the extreme position.

7.7 What is meant by phase angle? Does it define angle between maximum displacement and the driving force?

Ans. The phase angle is the angle, which specifies the displacement as well as the directions of motion of the point executing SHM. Thus, it determines the state of motion of the vibrating point.

It does not define angle between maximum displacement and the driving force. It is the angle θ, which the rotating radius OP makes with the reference direction OO_1 at any instant as shown in the figure.

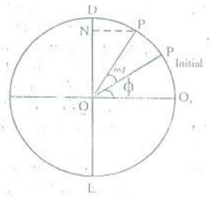


Fig. 7.14

7.8 Under what conditions does the addition of two simple harmonic motions produce a resultant, which is also simple harmonic?

Ans. If the SHM are parallel and of the same frequency and have a constant phase difference, then their resultant motion will also be SHM.

7.9 Show that in SHM, acceleration is zero when the velocity is greatest and the velocity is zero when acceleration is greatest?

Ans. The acceleration 'a' and velocity 'v' of SHM are given by

$a = -\omega^2 x$ and $v = \omega\sqrt{x_0^2 - x^2}$

where 'x' is the instantaneous displacement, 'x_o' is the maximum displacement and 'ω' is the constant of SHM. It is seen that when

$x = 0$ then $a = 0$, but 'v' has maximum value, i.e. $v = \omega x_0$

At the extreme position $x = x_o$, then accelerations will be greatest i.e. $a = -\omega^2 x_0$ but velocity becomes zero.

7.10 In relation to SHM, explain the equation;

(i) $y = A\sin(\omega t + \phi)$

(ii) $\vec{a} = -\omega^2\vec{x}$

Ans. (i) $y = A\sin(\omega t + \phi)$

In this equation 'y' is the instantaneous displacement of a particle executing SHM with amplitude 'A'. φ represents the initial phase and ωt is the angle subtended in time 't' with angular frequency 'ω' starting from initial phase 'φ'. Thus (ωt + φ) is the phase angle made with the reference direction or mean position.

(ii) $\vec{a} = -\omega^2\vec{x}$

In this equation $\vec{a}$ represents the acceleration of particle executing SHM with constant angular frequency 'ω' at instantaneous displacement '$\vec{x}$' from the mean position.

7.11 Explain the relation between the total energy, potential energy and kinetic energy for a body oscillating with SHM.

Ans. The total energy of a body who oscillates with SHM remains constant if frictional forces are not present.

During SHM, P.E. and K.E. are interchange but total energy remains constant if frictional forces are absent. At mean position, the energy is totally K.E. and at extreme positions the K.E. is completely changed to P.E. In between, it is partly P.E. and partly K.E but total remains equal to max K.E. or max P.E.

7.12 Describe some common phenomena in which resonance plays an important role.

Ans. Resonance plays an important role in the following phenomena.

1. In the musical instruments where air column resonates in the wooden boxes under the strings with string frequencies and loud music is produced.
2. In producing electrical resonance in our radio set with transmission frequency of a particular radio station, which amplifies the transmitted signal, which is received and this enables us to hear that transmission.
3. In a microwave oven, where the water and fat molecules in the food absorb energy by resonance resulting in efficient and evenly heating and cooking of the food.

7.13 If the mass spring system is hung vertically and set into oscillations, why does the motion eventually stop?

Ans. A SHM eventually stops due to friction and air resistance. Because of these frictional forces energy is dissipated into heat and consequently the system does not oscillate indefinitely and eventually stops.